2. Quantum Langevin approach.

Donnerstag, 11. Oktober 2018 08:35

Literature: Non-equilibrium statistical mechanics

- Enck, Liu: The Logic of Thermostatistical Physics
- Oono: Perspectives on Statistical Thermodynamics
- Sekimoto: Stochastic Energetics
- Oxenius: Kinetic Theory of particles and photons
- Kubo: Statistical Physics I & II

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for questions or wishes concerning the lecture

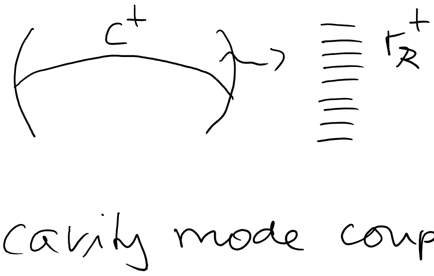
$$m\dot{v} = -\gamma m v + R(t) \qquad \begin{array}{l} (i)\ \langle R(t)\rangle = 0 \\ (ii)\ \langle R(t) v(t)\rangle = 0 = \langle R(t)\, x(t)\rangle \\ (iii)\ \langle R(t) R(t')\rangle = R_0\, 2\pi\, \delta(t-t') \end{array}$$

$$\Rightarrow \langle v^2(t)\rangle = e^{-2\gamma t}\left[\langle v^2(0)\rangle + \frac{\pi R_0}{m^2\gamma}\left(e^{2\gamma t}-1\right)\right]$$

$$\langle x^2(t)\rangle = \frac{2\pi R_0}{m^2\gamma^2}\left[t - \frac{1}{\gamma} + \frac{e^{-\gamma t}}{\gamma}\right] \qquad (\langle x(0)\rangle = 0 = \langle x^2(0)\rangle)$$

$\hat{=}$ classical Brownian motion

now, find a physical model for $R(t)$!



$$H = \hbar\omega_0 c^+ c + \sum_k \hbar\omega_k r_k^+ r_k + \hbar \sum_k \left(g_k c^+ r_k + r_k^+ c\, g_k^*\right)$$

cavity mode coupled to a dissipative medium

$$[c, c^+] = \mathbb{1}, \qquad [r_k, r_{k'}^+] = \delta(k-k')$$

let's assume a very simple coupling: $g_k = \sqrt{\frac{\Delta k\, \gamma c}{\pi}} = g_k^*$ $= g_0$

$\Delta k = \frac{2\pi}{L}$ (one-dimensional problem)

Heisenberg's equation: $-i\hbar \frac{d}{dt} A = [H, A] + \underbrace{\partial_t A}_{=0}$

$$-i\hbar \dot{c} = [c^\dagger c, c] + \sum_k \hbar \underbrace{g_k}_{=g_0} [c^\dagger r_k + r_k^\dagger c, c]$$

$$= \underline{\underline{i}}\,\omega_0 c - i \sum_k g_0 \underline{\underline{r_k}}$$

$$\dot{r}_k = -i\omega_k r_k - i g_0 c$$

Solve this equation $\quad r_k(t) = e^{-i\omega_k t}\left[r_k(0) - i g_0 \int_0^t dt'\, e^{i\omega_k t'} c(t')\right]$

$$\dot{c} = -i\omega_0 c - i g_0 \sum_k r_k(0) e^{-i\omega_k t} \underbrace{- i \sum_k g_0(-i g_0) \int_0^t dt'\, e^{i\omega_k (t'-t)} c(t')}_{= -\int_0^t dt'\, c(t') \underbrace{\sum_k g_0^2 e^{i\omega_k (t'-t)}}}$$

$$= \sum_k \Delta k \frac{\gamma c}{\pi} e^{i\omega_k (t'-t)}$$

$$= \frac{\gamma c}{\pi} \int dk\, e^{i\omega_k (t'-t)}$$

$$= \frac{\gamma c}{\pi} 2\pi \delta(ct - ct')$$

$$= 2\gamma\, \delta(t-t')$$

$$\dot{c} = -i\omega_0 c - i g_0 \sum_k r_k(0) e^{-i\omega_k t} - 2\gamma \underbrace{\int_0^t dt'\, \delta(t-t')\, c(t')}_{= \frac{1}{2}\gamma c(t)}$$

$$= (-i\omega_0 - \gamma)\, c(t) + \underbrace{\Delta R(t)}_{= -i g_0 \sum_k r_k(0) e^{-i\omega_k t}}$$

(quantum Langevin equation, very similar to the classical one)

$$c(t) = e^{-[i\omega_0 + \gamma]t}\left[c(0) + \int_0^t dt'\, e^{[i\omega_0+\gamma]t'} \Delta R(t')\right]$$

we need to average quantum mechanically

$$\varrho(0) = |1\rangle\langle 1| \otimes \frac{1}{Z} \exp\left[-\beta \sum_k \hbar\omega_k r_k^\dagger r_k\right]$$

cavity mode has a single excitation

reservoir in the thermal state

$$\langle c^\dagger(t)\rangle = tr\left(\varrho(0)\, c(t)\right) = e^{-(i\omega_0+\gamma)t} \sum_n \underbrace{\langle n|1\rangle\langle 1| c(0) |n\rangle}_{= \langle 1| c(0) |1\rangle = \langle 1|0\rangle = 0}$$

vanishes because the initial state is a Fock state

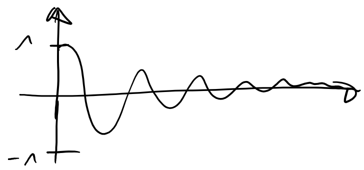
$$-i\int_0^t dt'\, e^{[i\omega_0+\gamma](t'-t)} \sum_\lambda g_0\, e^{-i\omega_\lambda t'}\, \underbrace{\mathrm{tr}\left(\rho(0)\, r_\lambda(0)\right)}_{=0}$$

we assumed thermal equilibrium

$$=0$$

but when we choose a different initial condition, such as a cavity prepared in a coherent state $c\,|\alpha\rangle = \alpha\,|\alpha\rangle$, $|\alpha\rangle = \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}\, e^{-|\alpha|^2}\,|n\rangle$

$$\langle c(t)\rangle\Big|_{coh} = \alpha\, e^{-(i\omega_0+\gamma)t}$$



So do we need really the noise contribution?

$[c(t), c^\dagger(t)] = 1\!\!1$ **must** hold for all times.

$$\left[c(0)\,\underline{e^{-i\omega_0 t-\gamma t}},\; c^\dagger(0)\,\underline{e^{i\omega_0 t-\gamma t}}\right] = e^{-2\gamma t}\,\underbrace{[c(0), c^\dagger(0)]}_{=1\!\!1} \xrightarrow{t\to\infty} 0$$

don't neglect the noise!! otherwise, this is wrong

__but__: with noise it works.

$$[c(t), c^\dagger(t)] = \Big[e^{-i\omega_0-\gamma t}\Big\{c(0) - i g_0 \sum_\lambda \int_0^t dt'\, e^{i\omega_0 t'+\gamma t'}\, e^{-i\omega_\lambda t'}\, r_\lambda(0)\Big\},$$
$$e^{i\omega_0-\gamma t}\Big\{c^\dagger(0) + i g_0 \sum_\lambda \int_0^t dt'\, e^{-i\omega_0 t'+\gamma t'}\, e^{i\omega_\lambda t'}\, r_\lambda^\dagger(0)\Big\}\Big]$$

$$=\{\text{note } [c(0), r_\lambda^\dagger(0)] = 0 = [r_\lambda(0), c^\dagger(0)]\}$$

$$= e^{-2\gamma t}\,\underbrace{[c(0), c^\dagger(0)]}_{=1\!\!1} + \sum_\lambda \sum_{\lambda'} \underbrace{g_0^2}_{=\frac{\Delta\lambda c\gamma}{\pi}} \int_0^t dt' \int_0^t dt''\, e^{(i\omega_0+\gamma)t' - i\omega_0 t'' + \gamma t'' - i\omega_\lambda t' + i\omega_{\lambda'} t''}\, \underbrace{[r_{\lambda'}(0), r_{\lambda''}^\dagger(0)]}_{=\delta(\lambda'-\lambda'')}$$

$$= e^{-2\gamma t}\Big\{1\!\!1 + \frac{\gamma c}{\pi} \int d\lambda'' \underbrace{\sum_{\lambda'} \delta(\lambda'-\lambda'')}_{=1\!\!1} \int_0^t dt' \int_0^t dt''\, e^{\gamma(t'+t'') + i\omega_0(t'-t'')}\, \underbrace{e^{-i\omega_{\lambda''}(t'-t'')}}_{\to \frac{2\pi}{c}\delta(t'-t'')}$$

$$= e^{-2\gamma t}\Big\{1\!\!1 + 1\!\!1\, 2\gamma \int_0^t dt'\, e^{2\gamma t'}\Big\} = e^{-2\gamma t}\Big\{1 + 2\gamma\,\frac{1}{2\gamma}\,[e^{2\gamma t} - 1]\Big\}\, 1\!\!1$$

$$= \left[e^{-2\gamma t} + 1 - e^{-2\gamma t}\right] 1\!\!1 = \underline{1\!\!1}$$

✓ commutation relation is a conserved quantity

relation to
a conserved quantity

we have a very good model for $R(t)$.

but noise does not only guarantee the commutation relation, also the thermal equilibrium.

$$\langle c^+(t)\,c(t)\rangle = e^{-2\gamma t}\langle c^+(0)\,c(0)\rangle$$

$$+ \sum_{k'}\sum_{k''} \underbrace{g_0^2}_{=\frac{\Delta k \gamma c}{\pi}} e^{-2\gamma t}\int_0^t dt' \int_0^t dt'' \, e^{-i\omega_{k'}t' + i\omega_{k''}t'' - i\omega_0(t'-t'')} e^{\gamma(t'+t'')}$$

$$\mathrm{tr}\left(\rho(0)\, r^+_{k'}(0)\, r_{k''}(0)\right)$$

$$= \underbrace{\bar{n}_{k'}}_{=[\exp[\beta\hbar\omega_k]-1]^{-1}} \delta(k'-k'')$$

$$\approx \bar{n}_{k_0}\,\delta(k'-k'')$$

$$= e^{-2\gamma t}\langle c^+(0)c(0)\rangle + \frac{\gamma c}{\pi}\int_0^t dt' \int_0^t dt'' \, e^{-2\gamma t + \gamma t' + \gamma t''} e^{-i\omega_0(t'-t'')}\,\bar{n}_{k_0}$$

$$\int dk\, e^{i\omega_k(t'-t'')} = \frac{2\pi}{c}\delta(t''-t')$$

$$= e^{-2\gamma t}\langle c^+(0)c(0)\rangle + 2\gamma e^{-2\gamma t}\,\frac{e^{2\gamma t}-1}{2\gamma}\,\bar{n}_{k_0}$$

$$= e^{-2\gamma t}\langle c^+(0)c(0)\rangle + [1-e^{-2\gamma t}]\,\bar{n}_{k_0} \xrightarrow{t\to\infty} \bar{n}_{k_0}$$

therefore, Langevin both classically and quantum mechanically rely on similiar approaches.

<u>but</u>: in the classical version, we don't know $R(t)$, now, in the quantum regime, we at least know the underlying substructure.

$$R(t) = \int_0^t dt' \, e^{(-i\omega_0-\gamma)(t-t')} \sum_k g_k\, r_k(0)\, e^{-i\omega_k t'} \;!!$$