9. Quantum feedback. MPS.

Mittwoch, 24. Oktober 2018 16:59

1905 : Einstein's description of Brownian motion [phenomenological derived]

1910 : Langevin generalizes Einstein's idea to non-equilibrium physics via stochastic calculus

Development of a theoretical framework which allows to explain dissipation [Schrödinger's equation, von-Neumann]

1954/1957 : Green-Kubo relations for non-equilibrium statistical mechanics fluctuation-dissipation theorem

1960 : Nakajima-Zwanzig theory of quantum master equation

1965 : Feynman path integrals propagator method

1969 : Mollow's exact solution of a open quantum system model in a Markovian environment

1976 : Lindblad „On the generators of quantum dynamical semigroup"

1981 : Caldeira / Leggett's exact solution of a non-Markovian open quantum system via the Feynman-Vernon influence functional

1980's : Quantum optics, cavity QED success of Markovian, white noise description

1990's : Haroche / Kimble quantum optical control of qubits, generation of Fock state

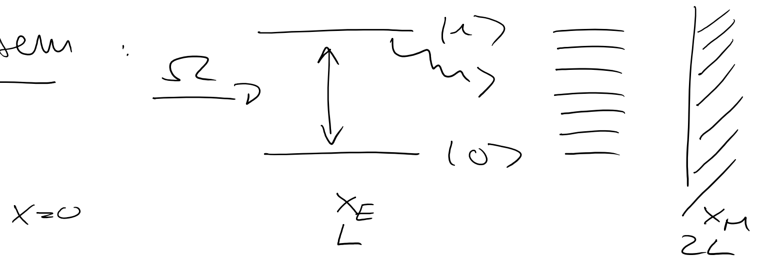
2009 : Wiseman / Milburn „Quantum measurement and control" => Markovian quantum

Control of cQED

2010 — : exploration of non-Markovian quantum control
$\Rightarrow$ feedback, dynamical decoupling, quantum non-demolition measurement, Zeno physics in the presence of structured reservoirs!

goal: calculate non-Markovian numerically exact via the matrix product state approach

system:



$$H = \hbar\omega_0 \sigma_{11} + 2\hbar\Omega \cos(\omega_L t)\,[\sigma_{10}+\sigma_{01}]$$
$$+ \hbar\int d\omega\, \omega\, r_\omega^\dagger r_\omega + \int d\omega\, \hbar\,[g^*(\omega)\, r_\omega^\dagger \sigma_{01} + g(\omega)\,\sigma_{10}\, r_\omega]$$

we choose a rotating $U_{RF} = \exp[-i(\omega_L \sigma_{11} + \int d\omega\, \omega\, r_\omega^\dagger r_\omega)t]$

$$H' = \hbar(\omega_0-\omega_L)\sigma_{11} + \Omega[\sigma_{01}+\sigma_{10}] + \int d\omega\, \hbar\,[g^*(\omega)\, r_\omega^\dagger(t)\,\sigma_{01}(t) + h.a.]$$

$g(k) = e^{ikx}\sin[k(x_m - x)]$, using $\omega = ck$

$g(\omega) = g_0\, e^{i\omega\frac{x}{c}} \sin[\omega \frac{L}{c}]$, $x_E = L$, $x_m = 2L$

roundtrip $\tau = \frac{2L}{c}$

$$= \frac{g_0}{2i}\left[e^{i\omega\frac{\tau}{2}} e^{i\omega\frac{\tau}{2}} - e^{i\omega\frac{\tau}{2} - i\omega\frac{\tau}{2}}\right] = \frac{1}{2i}\left[e^{i\omega\tau} - 1\right]$$

$$H'/\hbar = H_{sys} + \sigma_{10}\left(\frac{g_0}{2i}\right)\int d\omega \left[e^{i\omega\tau} - 1\right] r_\omega\, e^{-i\omega t + i\omega_0 t}$$
$$+ \sigma_{01}\left(\frac{g_0}{2i}\right)\int d\omega \left[1 - e^{-i\omega\tau}\right] r_\omega^\dagger\, e^{i\omega t - i\omega_0 t}$$

$$= H_{sys} + \Big\{ \sigma_{10}\left(\frac{g_0}{2i}\right)\int d\omega\, r_\omega \Big[e^{-i\omega(t-\tau)}\, e^{i\omega_0(t-\tau+\tau)} - e^{-i(\omega-\omega_0)t}\Big]$$

$+ h.a. \}$

$$R(t) = \int d\omega\, e^{-i(\omega-\omega_0)t}\, r_\omega$$

$$= H_{sys} + \left\{ \sigma_{10} \left(\frac{g_0}{2i}\right) \left[e^{i\omega_0 \tau} R(t-\tau) - R(t) \right] + h.a. \right\}$$

$$[R(t), R^\dagger(t')] = \int d\omega \int d\omega'\, e^{-i(\omega-\omega_0)t + i(\omega-\omega_0)t'} \underbrace{[r_\omega, r_{\omega'}^\dagger]}_{=\delta(\omega-\omega')}$$

$$= \int d\omega\, e^{-i\omega(t-t')}\, e^{i\omega_0(t-t')}$$

$$= \frac{2\pi}{c}\, \delta(t-t')\, e^{i\omega_0(t-t')}$$

not a c-number

$$\int_0^{t_1} dt \int_0^{t_2} dt' \, [R(t), R^\dagger(t')] = \int_0^{t_1} dt \int_0^{t_2} dt' \, \frac{2\pi}{c}\, \delta(t-t')\, e^{i\omega_0(t-t')}$$

$$= \int_0^{t_1} dt \int_0^{\infty} dt' \, \Theta(t_2 - t')\, \frac{2\pi}{c}\, \delta(t-t')\, e^{i\omega_0(t-t')}$$

$$= \frac{2\pi}{c} \int_0^{t_1} dt\, \Theta(t_2 - t) = \frac{2\pi}{c}\, t \quad \text{with } t = \min[t_1, t_2]$$

$$|\psi(t)\rangle_I = T\left\{ \exp\left[-i \int_0^t dt' \frac{H'(t')}{\hbar} \right] |\psi(0)\rangle \right\}$$

therefore, our collective reservoir operators are very convenient in the interaction picture.

but it is still a very hard problem!

the number of excitations is not limited

$$|\psi(t)\rangle = c_0(t)\, |0, vac\rangle + c_1(t)\, |1, vac\rangle$$

$$+ \sum_{i=0,1} \int d\omega\, c_i^{\omega}\, |i, \ldots 1_\omega \ldots\rangle$$

$$+ \sum_{i=0,1} \int d\omega \int d\omega'\, c_i^{\omega\omega'}\, |i, \ldots 1_\omega \ldots 1_{\omega'} \ldots\rangle$$

$+ \ldots$ numerically intractable!!

$t = N\Delta t$, so let's choose small enough that only one excitation per Δt is created

$$|\psi(N\Delta t)\rangle = \sum_{i=0,1} \sum_{n_0=0,1} \sum_{n_1=0,1} \cdots \sum_{n_N=0,1} c^i_{n_0 \ldots n_N}\, |i\rangle_S\, |n_0\rangle_0 \cdots |n_N\rangle_N$$

so we switch to stroboscopic time evolution
(we do numerics anyway)

$$|\psi(n)\rangle = T\left\{ \exp\left[-i\int_{(n-1)\Delta t}^{n\Delta t} dt' \, \frac{H(t')}{\hbar}\right] |\psi(n-1)\rangle \otimes |0\rangle_n \right\}$$

assuming vacuum for the future time bins

$$\int_{(n-1)\Delta t}^{n\Delta t} dt' \left\{ H_{sys} \{ - \sigma_{10} \left(\frac{g_0}{2i}\right) \left[e^{i\omega_0\tau} R(t'-\tau) - R(t')\right] + h.c. \right\}$$

$$= H_{sys}\,\Delta t - \left\{ \sigma_{10} \frac{g_0}{2i} \left[e^{i\omega_0\tau} \underbrace{\int_{(n-1)\Delta t}^{n\Delta t} dt' R(t'-\tau)}_{=:\Delta R(n)} - \int_{(n-1)\Delta t}^{n\Delta t} dt' R(t')\right] + h.c. \right\}$$

$$[\Delta R(n), \Delta R^\dagger(m)] = \int_{(n-1)\Delta t}^{n\Delta t} dt_1 \int_{(m-1)\Delta t}^{m\Delta t} dt_2 \; \underbrace{[R(t_1), R^\dagger(t_2)]}$$

$$= \int_{(n-1)\Delta t}^{n\Delta t} dt_1 \int_{-\infty}^{\infty} dt_2 \; \Theta(t_2 - (m-1)\Delta t)\, \Theta(m\Delta t - t_2)\, \frac{2\pi}{c}\, \delta(t_1 - t_2)\, e^{i\omega_0 (t_1 - t_2)}$$

$$= \int_{(n-1)\Delta t}^{n\Delta t} dt_1 \; \Theta(t_1 - (m-1)\Delta t)\, \Theta(m\Delta t - t_1)\, \frac{2\pi}{c}$$

$\Rightarrow$ only possible $n = m$

because $m = n+1$ $\quad \Theta(n\Delta t - t_1)\,\Theta(t_1 - n\Delta t)$

$$= \frac{2\pi}{c}\Delta t\, \delta_{nm}$$ (because of time evolution with fixed Δt!) $\quad \tau = m\Delta t$

$$|\psi(n)\rangle_I = T\left\{ \exp\left[-i H_{sys}\Delta t - \left\{ \left(\frac{g_0}{2i}\right) \sigma_{10} \left[\Delta R(n-m)\, e^{i\omega_0 m\Delta t} - \Delta R(n)\right] + h.c. \right\}\right]\right\} |\psi(n-1)\rangle_I$$

$$\bar{R}(n) := \sqrt{\frac{c}{2\pi\Delta t}}\, R(n) \implies [\bar{R}(n), \bar{R}^\dagger(m)] = \delta_{nm}$$

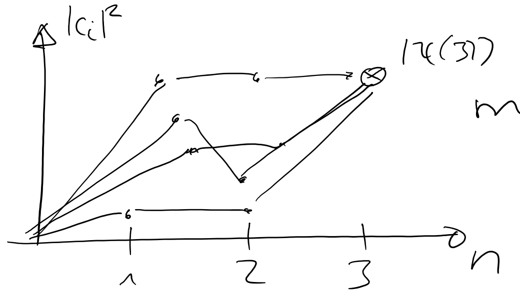
$$|\psi(n)\rangle_I = \exp\left[-i H_{sys}\Delta t - \left\{ \left(\frac{g_0}{2i}\right) \sqrt{\frac{2\pi\Delta t}{c}}\, \sigma_{10} \left[\Delta\bar{R}(n-m)\, e^{-i\omega_0 m\tau} - \Delta\bar{R}(n)\right] + h.c. \right\}\right] |\psi(n-1)\rangle_I$$

<u>but</u> still a very hard problem! $2 \cdot 2^N$ basis states if N time steps

$$|\psi(n)\rangle_I = U(n, n-1)\, |\psi(n-1)\rangle_I$$

$$|\Psi(3)\rangle_I = U(3,2)\,U(2,1)\,U(1,0)\,|\Psi(0)\rangle$$

Strongly entangled system-reservoir dynamics



many quantum paths, but all of them contribute equally

expand the time evolution operator up $O(\Delta t^{3/2})$

$$U(1,0)=\sum_{\substack{\alpha,\beta=0,1\\ n,m=0,1}} U^{\alpha\beta}_{nm}\, |\alpha\rangle_S\, {}_S\langle\beta| \otimes |n\rangle_1\, {}_1\langle m|$$

$$\begin{aligned}U(1,0) = \underline{\mathbb{1}} &- i\Omega\Delta t\,\{|1\rangle_S\,{}_S\langle 0| + |0\rangle_S\,{}_S\langle 1|\}\otimes \mathbb{1}_1\\
&- i\sqrt{\Gamma\Delta t}\,\{|1\rangle_S\,{}_S\langle 0|\otimes|0\rangle_1\,{}_1\langle 1| + |0\rangle_S\,{}_S\langle 1|\otimes |1\rangle_1\,{}_1\langle 0|\}\\
&- \frac{1}{2}\Gamma\Delta t\,\{|1\rangle_S\,{}_S\langle 1|\otimes|0\rangle_1\,{}_1\langle 0| + |0\rangle_S\,{}_S\langle 0|\otimes|1\rangle_1\,{}_1\langle 1|\}\\
&+ O(\Delta t^{3/2})\end{aligned}$$

$$|\Psi(1)\rangle = \Big\{\sum_{\substack{\alpha,\beta=0,1\\ n,m=0,1}} U^{\alpha\beta}_{nm}\,|\alpha\rangle_S\,{}_S\langle\beta|\otimes|n\rangle_1\,{}_1\langle m|\Big\}\Big\{|1\rangle_S\otimes|0\rangle_1\cdots|0\rangle_N\Big\}$$

$$\begin{aligned}= \Big[&\Big(|1\rangle_S - i\Omega\Delta t\,|0\rangle_S - \frac{\Gamma}{2}\Delta t\,|1\rangle_S\Big)\otimes|0\rangle_1\\
&- i\sqrt{\Gamma\Delta t}\;|0\rangle_S\otimes|1\rangle_1\Big]\otimes|0\rangle_2\cdots|0\rangle_N\end{aligned}$$

not separable state anymore

$$|1\rangle_S|0\rangle_1 \longrightarrow |1\rangle_S|0\rangle_1 + |0\rangle_S|1\rangle_1$$

$\Rightarrow$ use Schmidt decomposition to measure the entanglement

$$|\Psi(\Delta t)\rangle = |\Psi(1)\rangle = \sum_{\alpha,\beta=0,1} c_{\alpha\beta}\,|\alpha\rangle_S\,|\beta\rangle_1 \otimes |0\rangle_2\cdots|0\rangle_N$$

$$|\Psi(\Delta t)\rangle^T = \underbrace{\sum_{\alpha\beta=0,1} c_{\alpha\beta}\,|\beta\rangle_1\,{}_S\langle\alpha|}_{=M} \otimes |0\rangle_2\cdots|0\rangle_N$$

$$= M = \begin{pmatrix} -i\Omega\Delta t & -i\sqrt{\Gamma\Delta t}\\ 1-\frac{\Gamma}{2}\Delta t & 0\end{pmatrix}$$

$$M = U\Sigma V^{+} = \sum_{n,m=0,1} U^n_m |n\rangle_{1\,1}\langle m| \sum_{\lambda=0,1} S^\lambda_\lambda |\lambda\rangle_{1\,S}\langle \lambda| \sum_{i,j=0,1} V^{i*}_j |i\rangle_{S\,S}\langle j|$$

$$= \sum_n \sum_j \sum_\lambda \underbrace{U^n_\lambda}_{A^n_\lambda} \left(S^\lambda_\lambda V^{\lambda *}_j \right) |n\rangle_{1\,S}\langle j| .$$

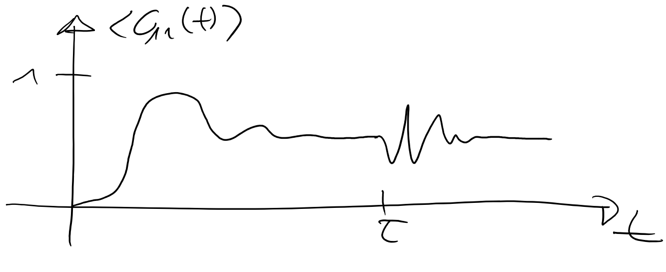
$$M \longrightarrow |M\rangle = \sum_n \sum_j \sum_\lambda A^n_\lambda \underbrace{\left(S^\lambda_\lambda V^{\lambda j *}\right)}_{= {}_\lambda A^j} |n\rangle_1 |j\rangle_S$$

Matrix product state presentation

$$|\psi(1)\rangle = \sum_{n=0,1} \sum_{\lambda=0,1} \sum_{j=0,1} A^n_\lambda \, {}_\lambda A^j_0 \, {}_0 A^{n_2 \dots n_N} |j\rangle_S |n\rangle_1 |0\rangle_2 \dots |0\rangle_N$$

$$|\psi(N)\rangle = \sum_{\substack{n_1 \dots n_N = 0,1 \\ \lambda_1 \dots \lambda_{N-1}}} A^{n_1}_{\lambda_1} \, {}_{\lambda_1} A^{n_2}_{\lambda_2} \dots {}_{\lambda_{N-1}} A^{n_N} |n_1\rangle_S |n_2\rangle_1 \dots |n_N\rangle_N$$

however $\lambda_1 \dots \lambda_{N-1}$ have different dimensions and values, some can be neglected. in this case typically in order of $\Omega\, 10^{-5}$
the matrix product state truncation is best done with libraries like LapacE or iTensor.org.



opens up new possibilities of quantum control

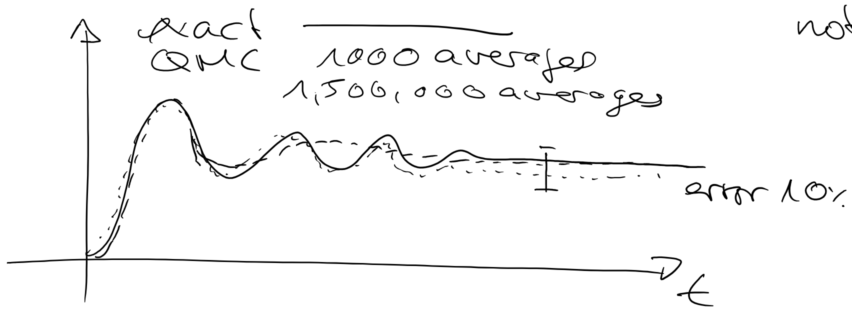
<u>recipe</u>: non-Markovian or not

MPS (disorder)
Feynman path integral (phonons)
or
perturbatively
Heisenberg operators
(adiabatic elimination)

- Lindblad master equation (but up to $N = 14$ degrees of freedom)
- Quantum Monte Carlo

(qualitative analysis but not for quantitative)

QMC :



Review Plenio & Knight QMC