

# Photon-statistics and entanglement in quantum light emission from semiconductor quantum dots

A. Carmele, M.-R. Dachner, M. Richter, and A. Knorr

## Strong coupling limit:

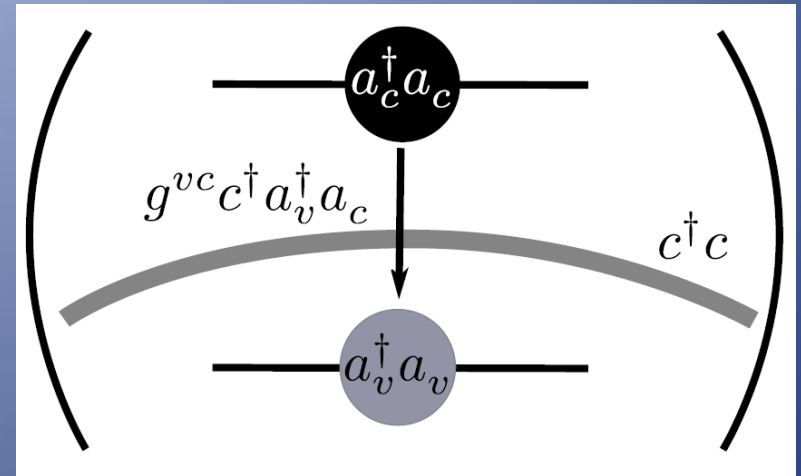
- The Jaynes-Cummings model (JCM)
- Photon-probability cluster expansion (PPCE)
- Electron-LO-phonon interaction: temperature dependent T1-times
- Electron-electron interaction: Electrically driven single-photon source

## Weak coupling limit:

- Polarization entangled photon pairs: temperature dependent concurrence

The Jaynes-Cummings model:<sup>1</sup>

- One-electron assumption
- One interaction: electron-light
- Closed system



„The simplest fully quantized model of interest“ (J.H. Eberly)

<sup>1</sup>E. Jaynes and F. Cummings, Proc.IEEE 51, 89 (1963)

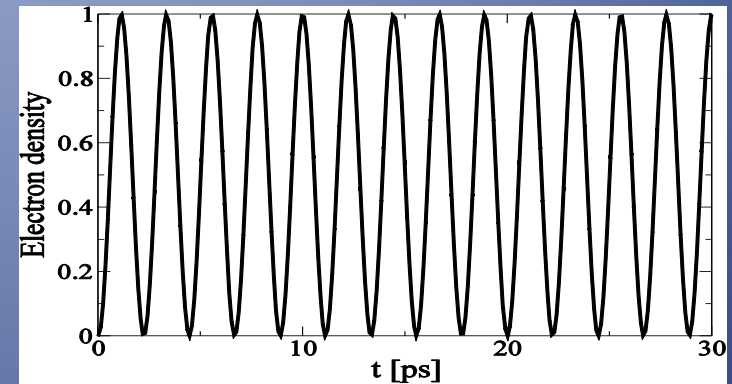
Brief notes about Atom-Cavity QED

Folie: 4

AG Seminar (Jahnke)

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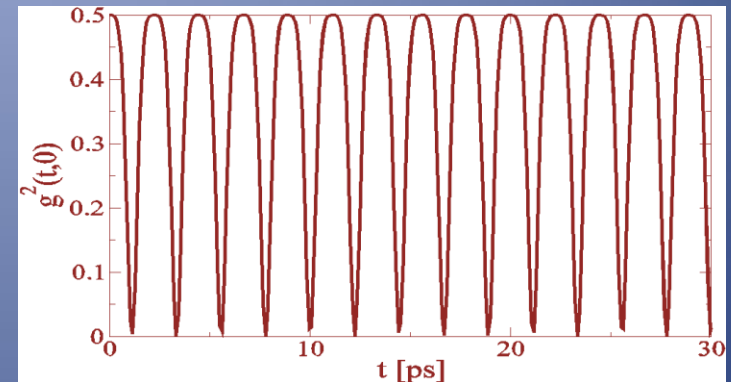
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Heisenberg equation of motion approach to treat more interactions

$$-i\hbar\partial_t\langle O \rangle = \langle [H_0 + H_{el-pt}, O] \rangle$$

To reproduce the Jaynes-Cummings model,  
 we introduce a factorization approach for strongly correlated systems:  
 the photon probability cluster expansion (PPCE).<sup>1</sup>

$$p_n = \langle |n\rangle \langle n| \rangle \quad f_n^h = p_n - \langle |n\rangle \langle n| a_v^\dagger a_v \rangle$$

$$\langle c^\dagger c \rangle = \sum_{n=1}^{\infty} n p_n \quad f_n^e = \langle |n\rangle \langle n| a_c^\dagger a_c \rangle$$

Heisenberg equation of motion approach to treat more interactions

$$-i\hbar\partial_t\langle O \rangle = \langle [H_0 + H_{el-pt}, O] \rangle$$

For given initial conditions, the analytical solution of the JCM is reproduced, e.g.:

$$f_0^e = f_0^h = 1, p_0 = 1, p_n = 0 \quad (n > 0)$$

Solution via differentiating with respect to time:

$$\partial_t^2(f_0^e - p_1) = -4(g^{vc})^2(f_0^e - p_1)$$



$$f_0^e(t) - p_1(t) = \cos(2g^{vc}t) = \cos^2(g^{vc}t) - \sin^2(g^{vc}t)$$

Heisenberg equation of motion approach to treat more interactions

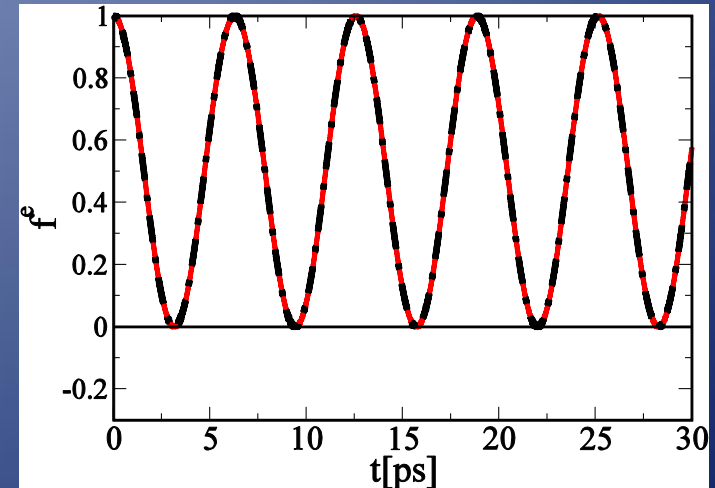
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Solution of the Jaynes-Cummings model for the case of vacuum Rabi oscillation:

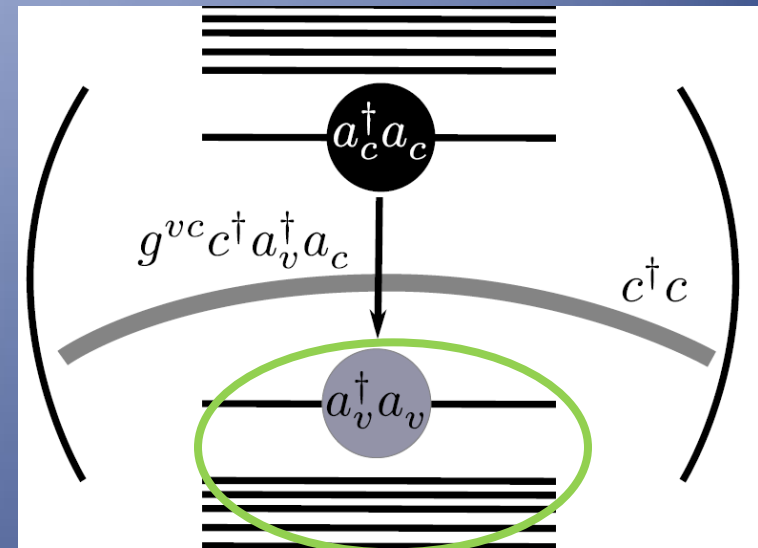
$$f_0^e(t) = \cos^2(g^{vc}t)$$





Due to the quantum dot-wetting layer interaction, the one-electron assumption not valid anymore.

$$\langle |n\rangle \langle n | a_1^\dagger a_2^\dagger a_3 a_4 \rangle \neq 0$$



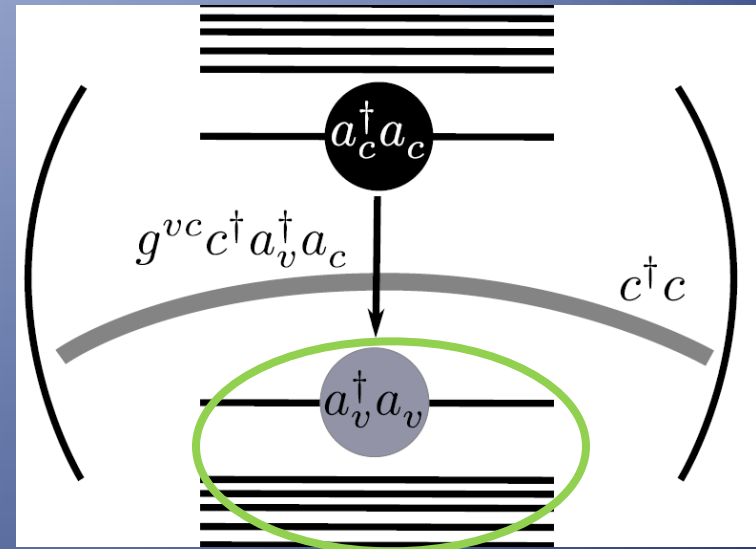
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$$\langle |n\rangle \langle n| a_1^\dagger a_2^\dagger a_3 a_4 \rangle \neq 0$$

This many-particle contribution is taken into account via the Hartree – Fock approximation

$$\langle a_1^\dagger a_2^\dagger a_3 a_4 \rangle \approx \langle a_1^\dagger a_4 \rangle \langle a_2^\dagger a_3 \rangle - \langle a_1^\dagger a_3 \rangle \langle a_2^\dagger a_4 \rangle$$



Here, we introduce a modified the Hartree – Fock approximation <sup>1</sup>

$$\langle |n\rangle \langle n| a_1^\dagger a_2^\dagger a_3 a_4 \rangle = p_n \langle \frac{|n\rangle \langle n|}{p_n} a_1^\dagger a_2^\dagger a_3 a_4 \rangle$$

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$$\langle |n\rangle \langle n| a_1^\dagger a_2^\dagger a_3 a_4 \rangle = p_n \langle \frac{|n\rangle \langle n|}{p_n} a_1^\dagger a_2^\dagger a_3 a_4 \rangle$$



$$\begin{aligned} \langle |n\rangle \langle n| a_1^\dagger a_2^\dagger a_3 a_4 \rangle &\approx p_n \left\{ \langle \frac{|n\rangle \langle n|}{p_n} a_1^\dagger a_4 \rangle \langle \frac{|n\rangle \langle n|}{p_n} a_2^\dagger a_3 \rangle - \langle \frac{|n\rangle \langle n|}{p_n} a_1^\dagger a_3 \rangle \langle \frac{|n\rangle \langle n|}{p_n} a_2^\dagger a_4 \rangle \right\} \\ &\approx \frac{1}{p_n} \left\{ \langle |n\rangle \langle n| a_1^\dagger a_4 \rangle \langle |n\rangle \langle n| a_2^\dagger a_3 \rangle - \langle |n\rangle \langle n| a_1^\dagger a_3 \rangle \langle |n\rangle \langle n| a_2^\dagger a_4 \rangle \right\} \end{aligned}$$

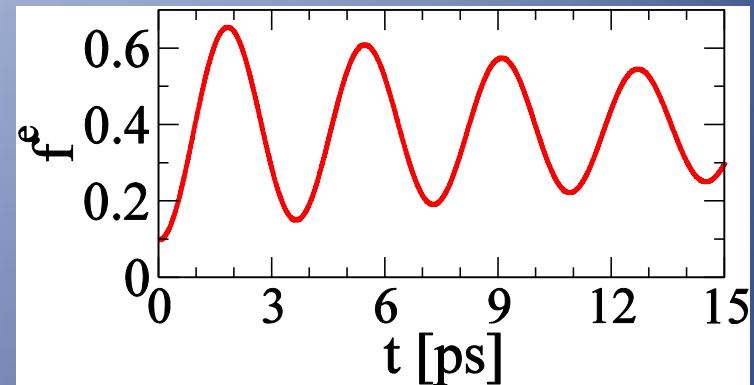
Quantum optics in the semiconductor

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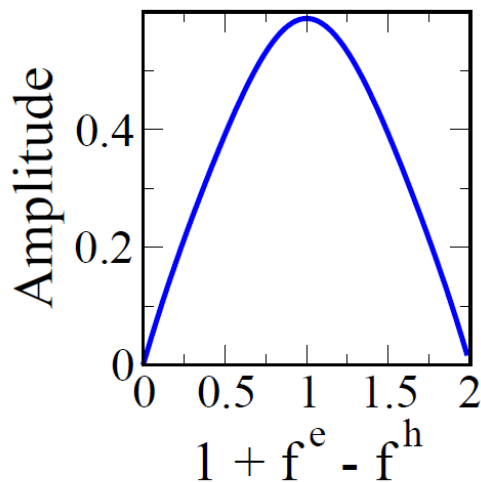
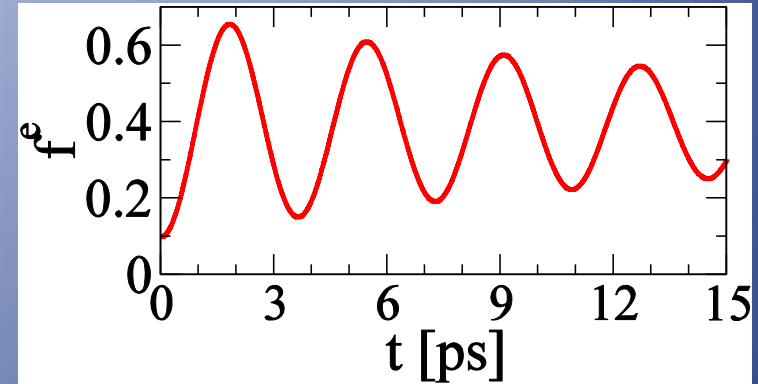
Few photon limit,  
the anti-bunching regime is simulated:

Amplitude of Rabi flops depends on the  
deviation from the one-electron assumption  
of electron and hole densities.



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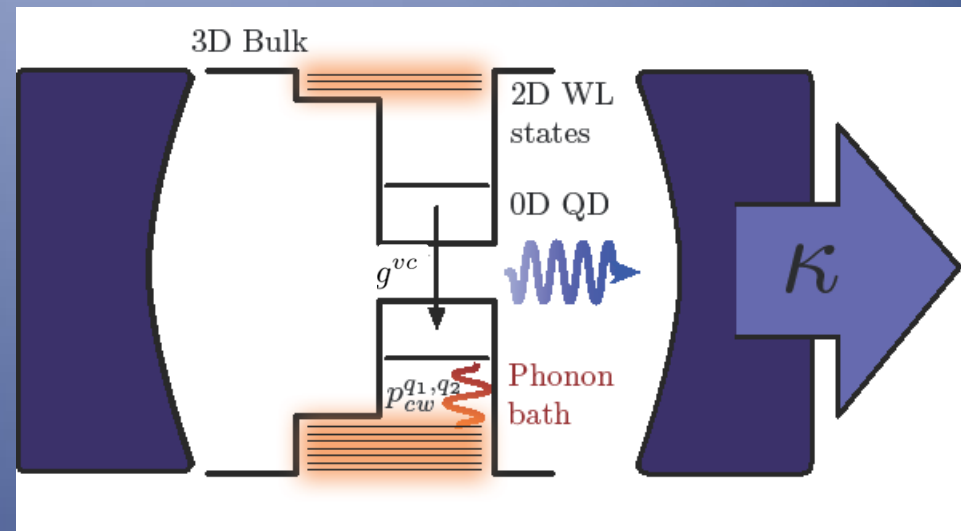
Amplitude of Rabi flops depends on the  
deviation from the one-electron assumption  
of electron and hole densities.



The amplitude of the Rabi flops  
might be used as a measure for  
the number of electrons in the  
actual quantum dot

Low carrier-density limit:

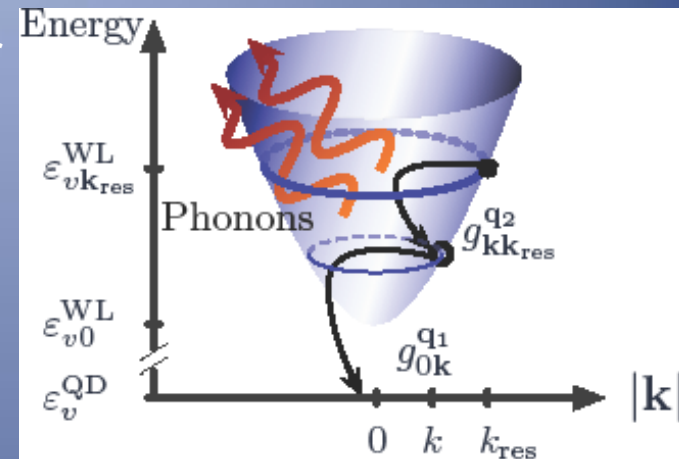
- 3D bulk: Electron-phonon interaction
- 2D WL: Electron-electron interaction
- 0D QD: Electron-photon



Phonon-assisted carrier scattering from the WL into the  
QD

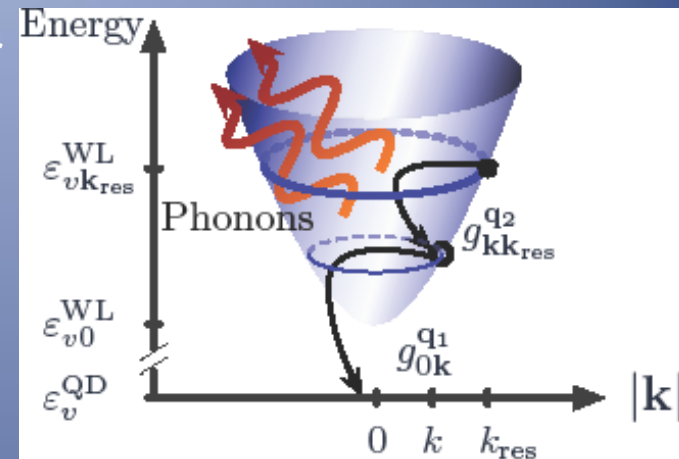
Folie: 16

- In an effective Hamiltonian approach a higher order Markovian process is assumed
- Probability of a subprocess depends on how strong the subprocess violates the energy conservation

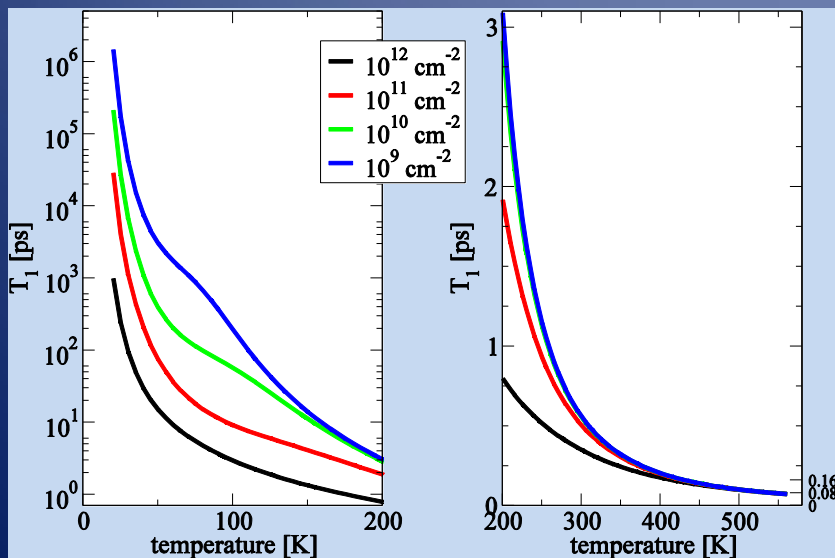




- In an effective Hamiltonian approach a higher order Markovian process is assumed
- Probability of a subprocess depends on how strong the subprocess violates the energy conservation



Relaxation rates<sup>1</sup>:



- The relaxation rates depend on the temperature of the bulk material
- Dependence corresponds to a square of the Bose-Einstein-distribution function
- The higher the temperature, the shorter the T1 - Time

Electrically driven QD is a source for single-photons on demand

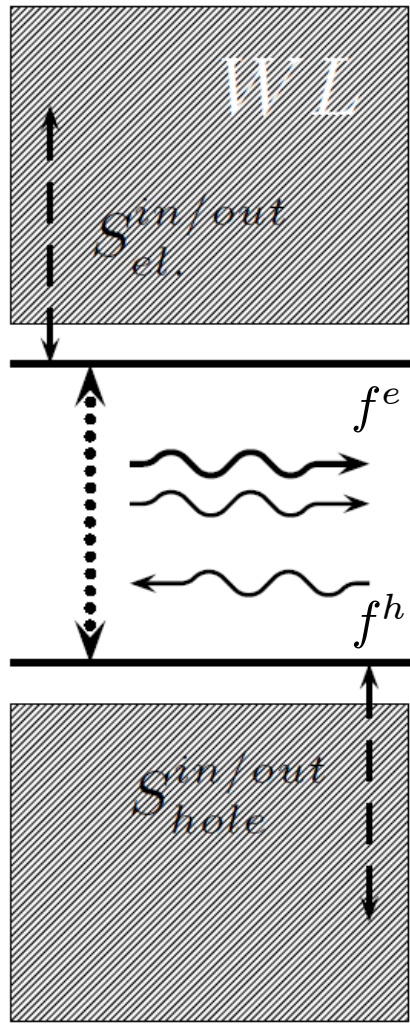
Electrons scatter into the QD

$$\partial_t f_n^e|_{pump} = S_e^{in}(p_n - f_n^e) - S_e^{out} f_n^e$$

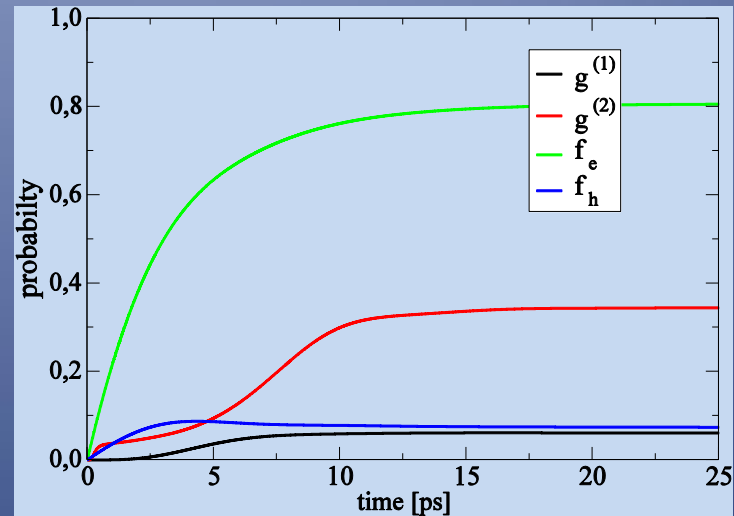
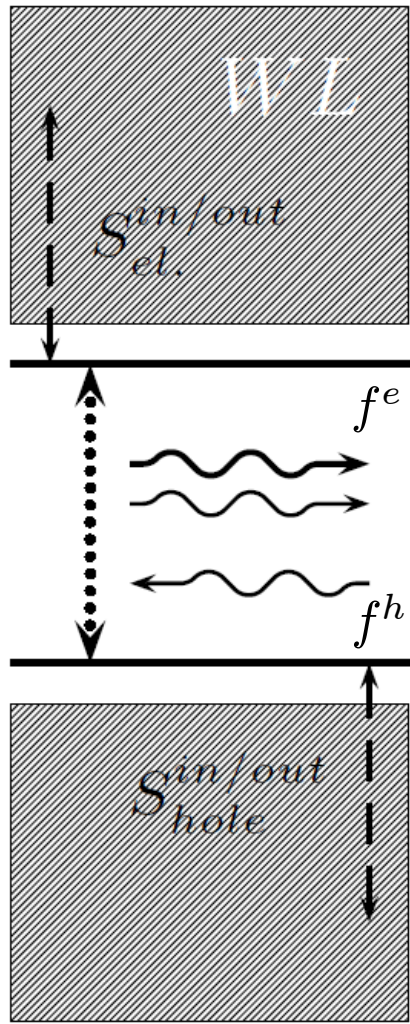
$$\partial_t f_n^h|_{pump} = S_h^{in}(p_n - f_n^h) - S_h^{out} f_n^h$$

$$\begin{aligned} & \partial_t \langle |n+1\rangle \langle n| a_v^\dagger a_c \rangle|_{pump} \\ &= - \left( \frac{S_e^{in}}{2} + \frac{S_e^{in}}{2} + \frac{S_e^{out}}{2} + \frac{S_h^{out}}{2} \right) \langle |n+1\rangle \langle n| a_v^\dagger a_c \rangle \end{aligned}$$

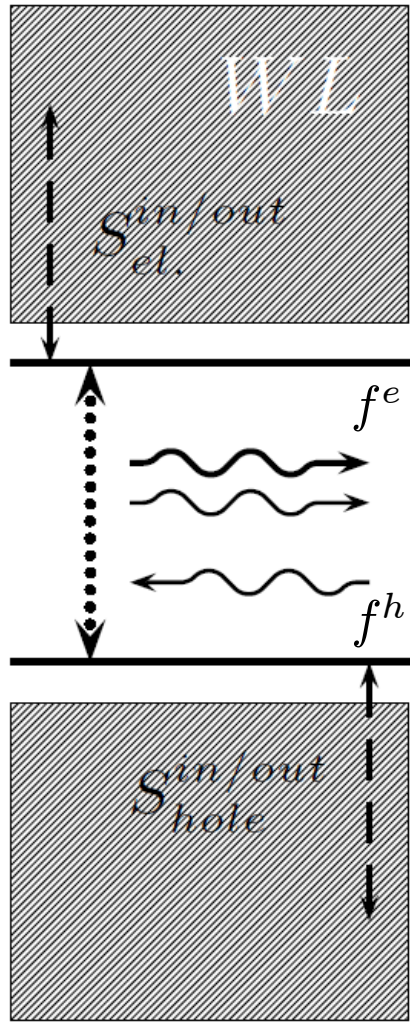
Derived via a bath assumption for electrons and holes in the wetting layer



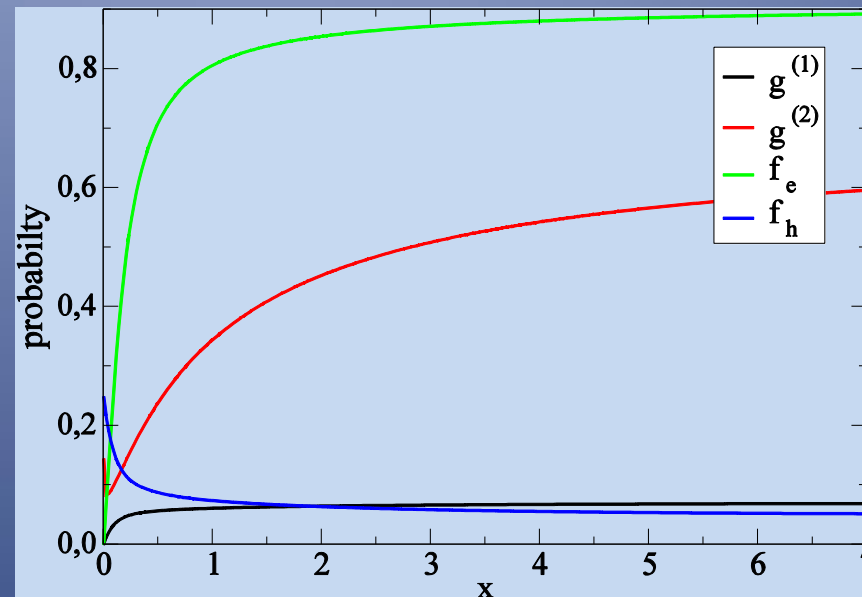
The single photon emitter in a steady state condition.



Material dependent photon-statistics



For weak pumping, a single-photon emitter is realized



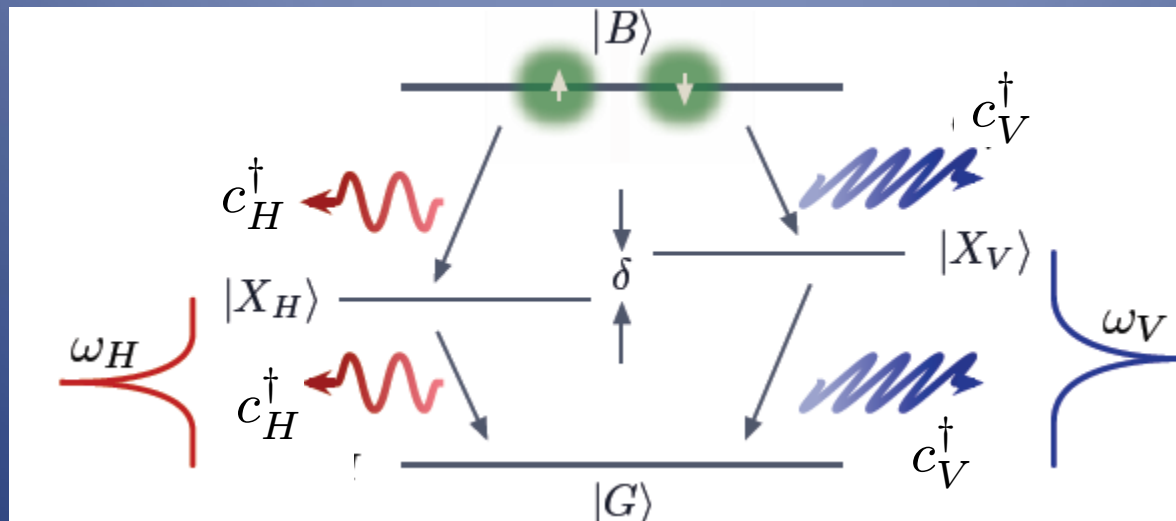
$$S_{in}^e = \frac{x}{4ps}, S_{out}^e = \frac{x}{40ps}$$

Application: Source of polarization entangled photon pairs

Folie: 21

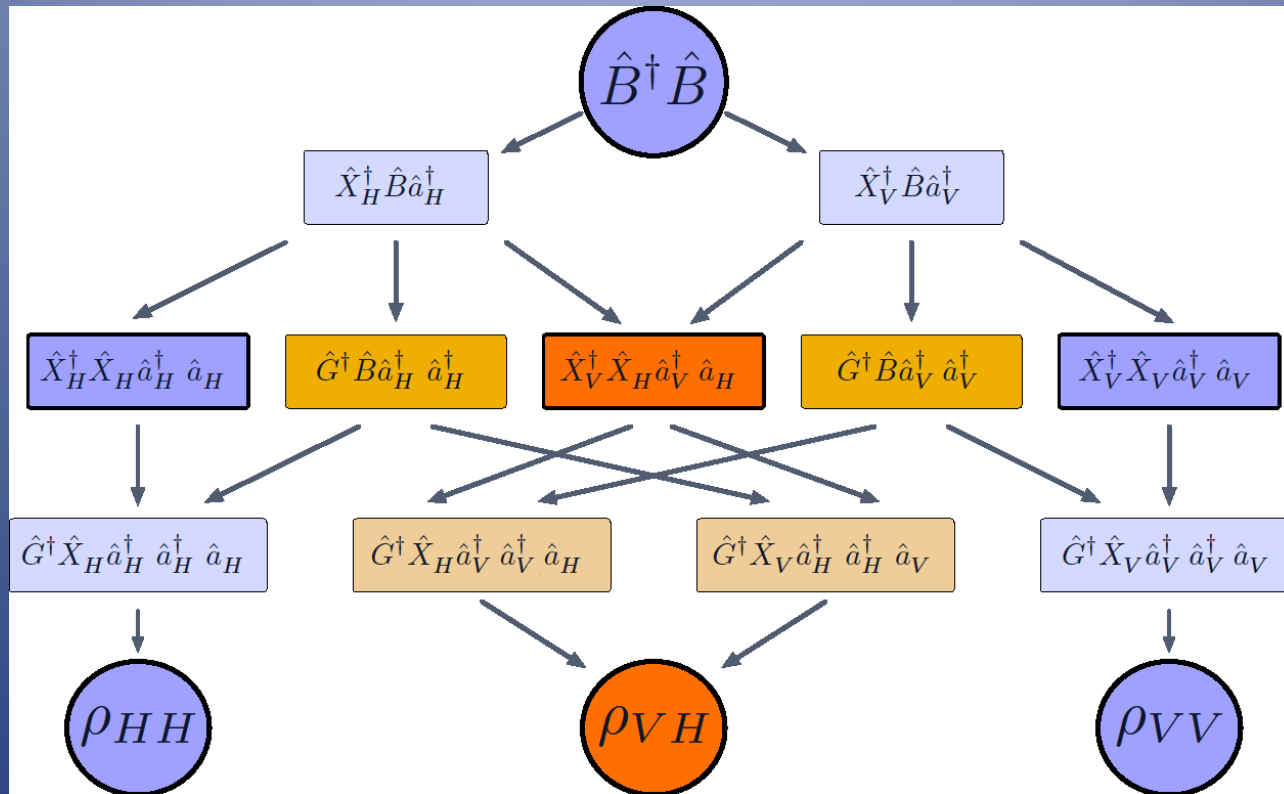
Source of polarization entangled photon pairs:

We assume a two-mode cavity with a quantum dot inside, coupled to the wetting layer via multi-phonon processes.



Biexciton  $|B\rangle$  decays via two intermediate exciton states  $|X_H\rangle, |X_V\rangle$  to the ground state  $|G\rangle$

# Weak coupling limit:

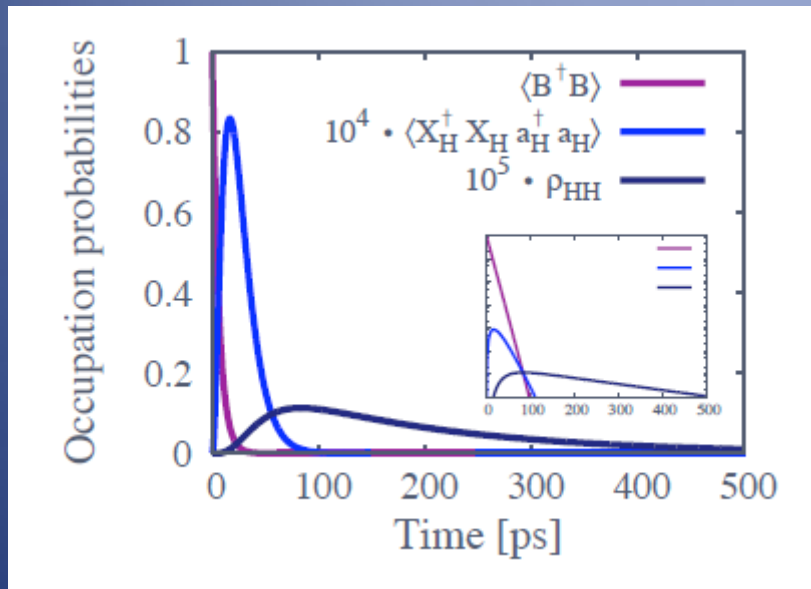


Cavity loss , radiative and phonon dephasing stronger than the electron-light coupling

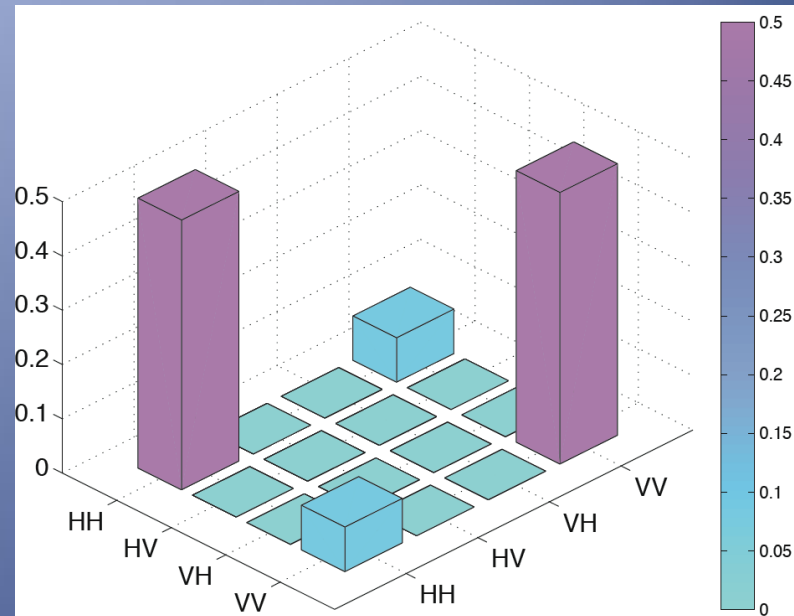
Application: Source of polarization entangled photon pairs

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The biexciton and exciton density decays with the radiative damping. The diagonal element of the polarization sub-space density matrix decays with the cavity loss.



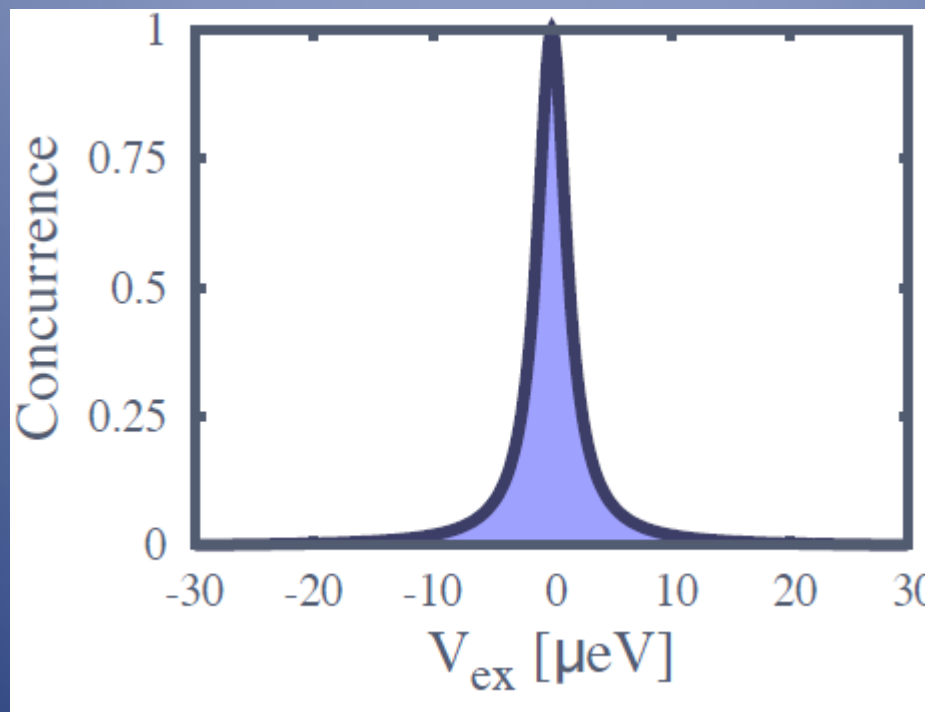
Quantum state tomography for fine structure splitting of  $10\mu\text{eV}$

Application: Source of polarization entangled photon pairs

Concurrence is an accepted measure for entanglement and defined via the off-diagonal element:

$$C = 2|\rho_{HV}|$$

$$\rho_{HV} \propto \langle c_H^\dagger c_H^\dagger c_V c_V \rangle$$



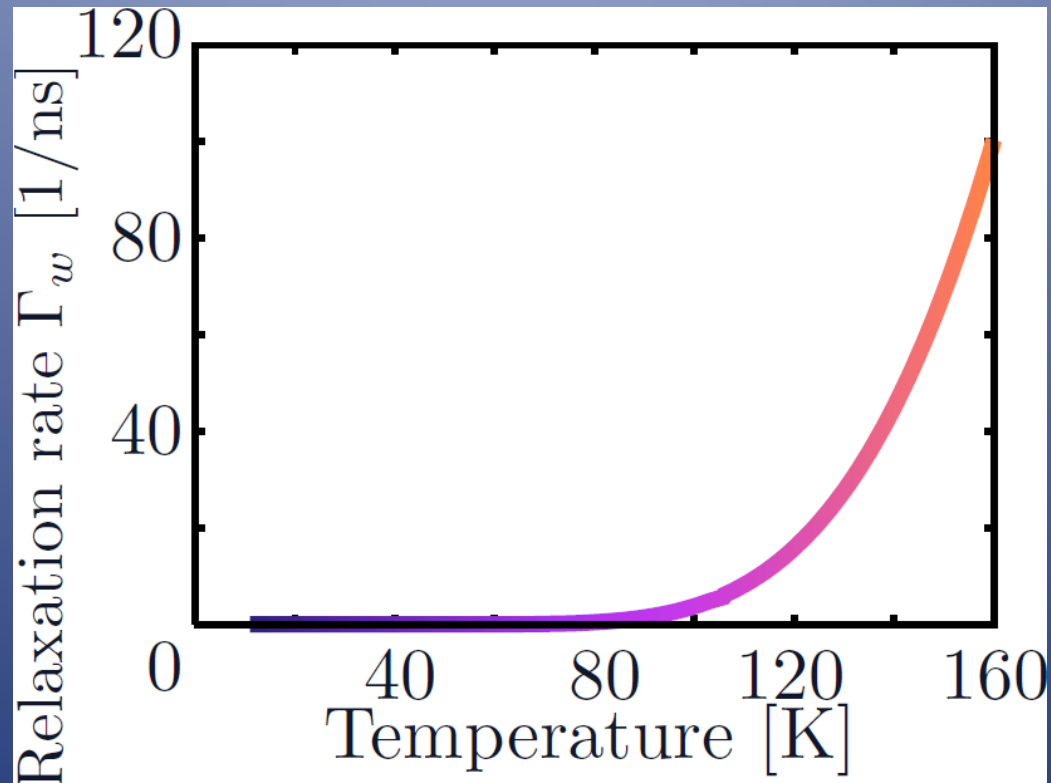
For a strong fine structure splitting, entanglement vanishes



Application: Source of polarization entangled photon  
pairs

Wetting layer contributions lead to an additional dephasing.

$$\Gamma = \Gamma_{rad} + \Gamma_{WL}$$

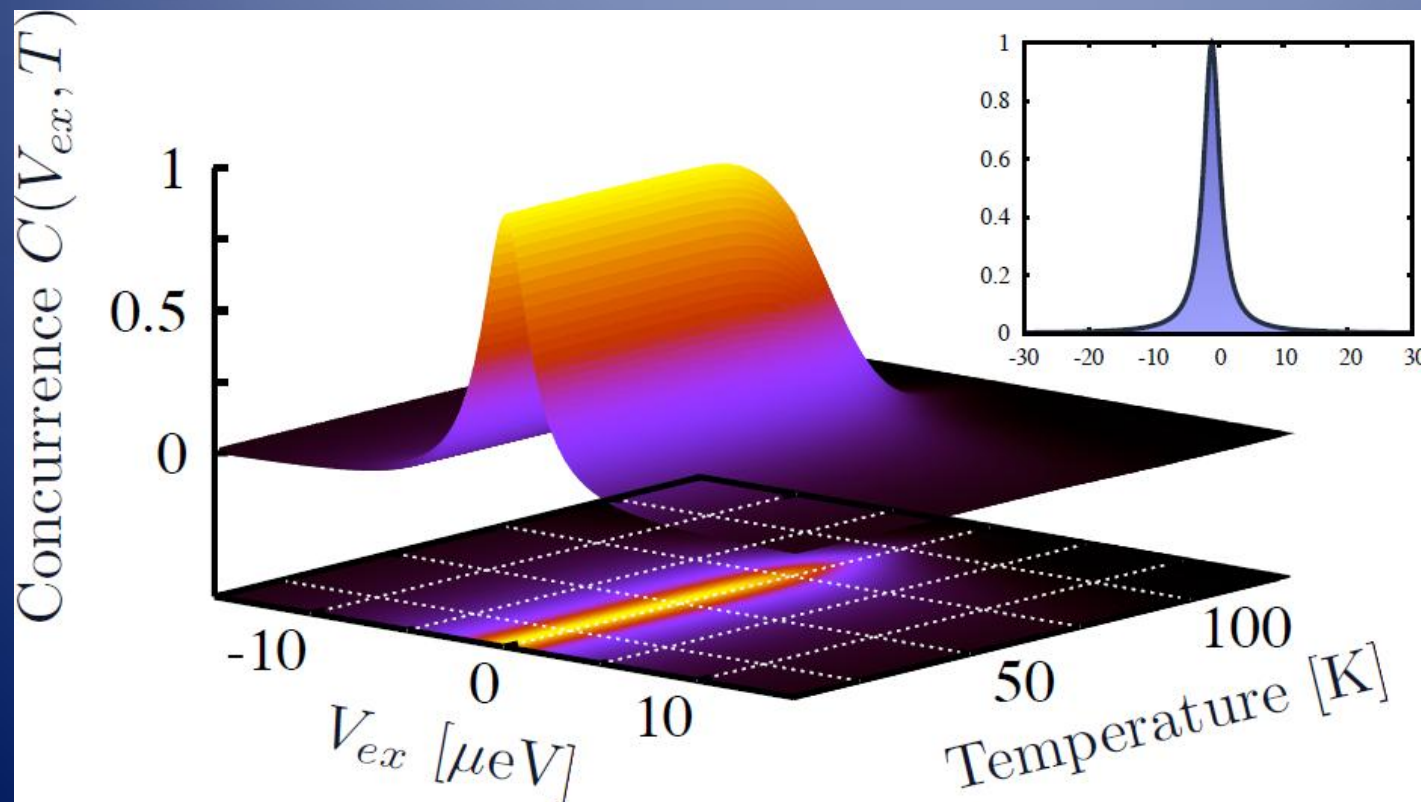


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Multiphonon processes attack the generation of polarization entangled photons



## Strong coupling – low carrier limit:

- Jaynes-Cummings model – **photon statistics**
- Decreased amplitude of the Rabi oscillations
- Electrically driven single-photon emitter – **photon statistics**

## Weak coupling – low carrier limit:

- Polarization entangled photon pairs: temperature dependent concurrence

**thank you!!**