FOPS 2015
Breckenridge, Colorado

Quantum Feedback Stabilized Solid-State Emitters

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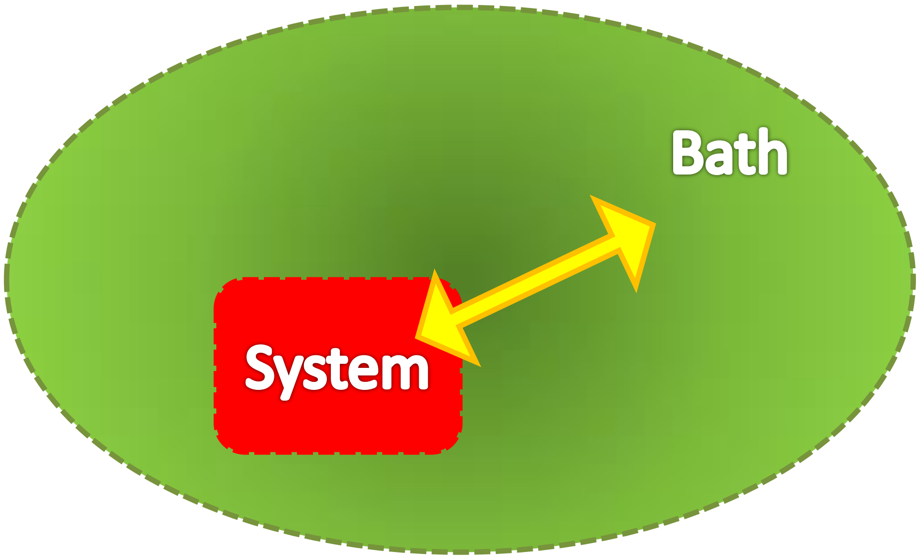
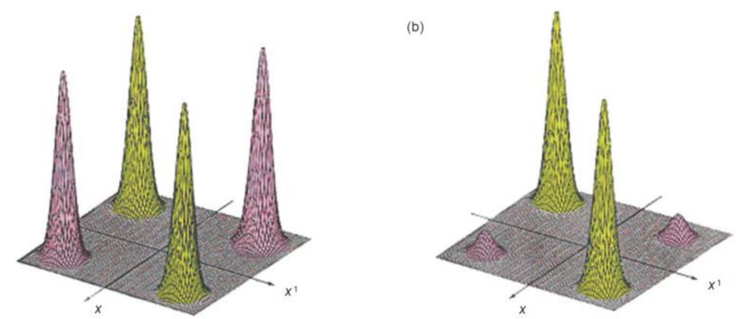
August 7, 2015

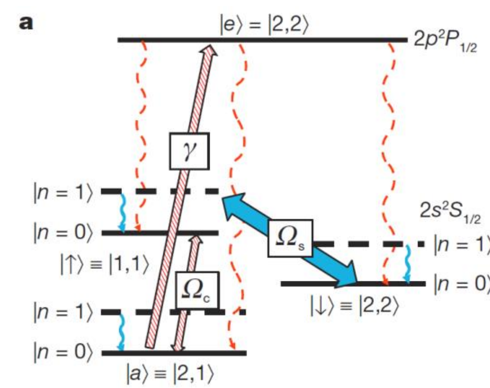
Unstructured Bath



$$\frac{\partial \hat{\rho}}{\partial t} = \frac{1}{i\hbar}\left[\hat{H},\hat{\rho}\right] + \hat{L}\hat{\rho},$$



Zurek, Phys. Today 44, 10 (1991)



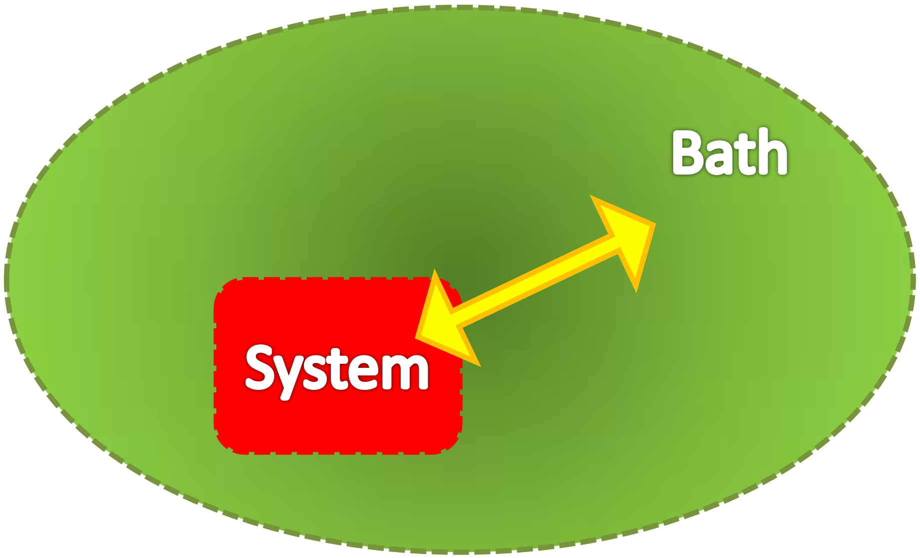
Ways to cope with environment induced losses

- accept time-scale before decoherence
- compensate losses with triggered feedback
- exploit dissipation as enabling feature

e.g. qubit entangling via radiative dephasing

Lin et al, Nature 504, 415 (2013)

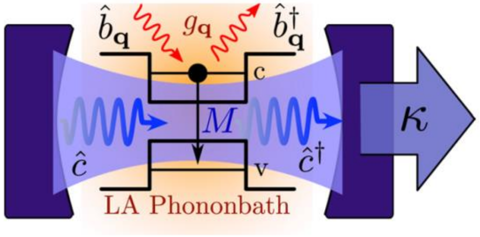
Structured Bath



$$\frac{\partial\hat{\rho}}{\partial t}=\frac{1}{i\hbar}\left[\hat{H},\hat{\rho}\right]+\hat{L}\hat{\rho},$$

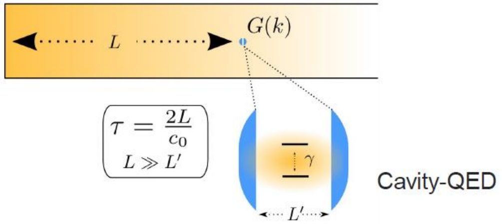
Lindblad formulation mostly impossible: non-Markovian effects, e.g. colored noise

$$\frac{dA}{dt}=\frac{i}{\hbar}[H,A]+\frac{\partial A}{\partial t}$$

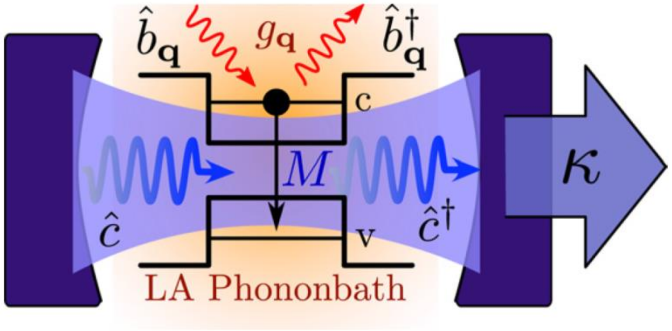


1. Electron-phonon interaction

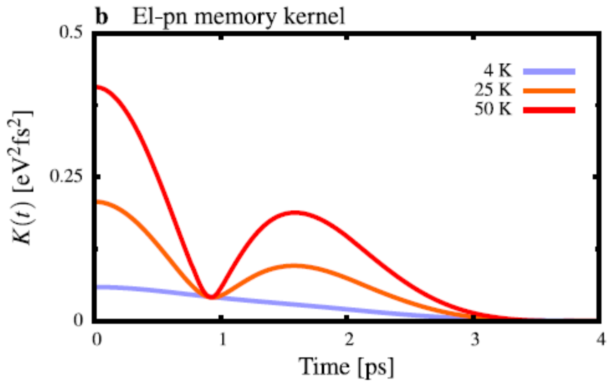
2. Optical self-feedback in half-cavities

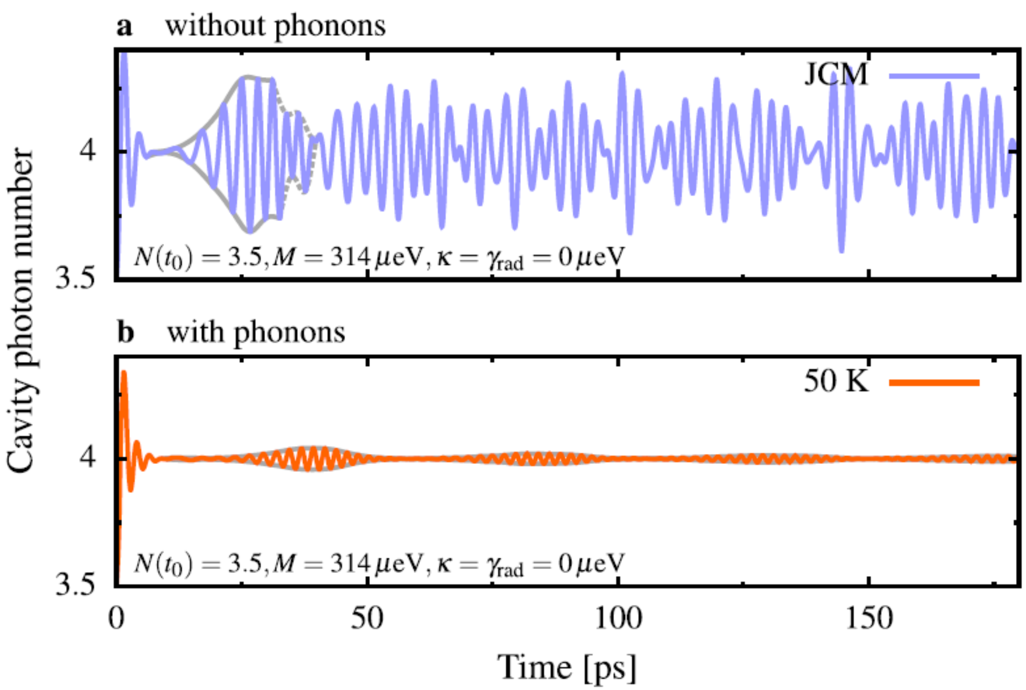


Stabilization of Quantum Coherence via Phonons



Collapse and revival phenomenon in cQED – with intrinsic revival times





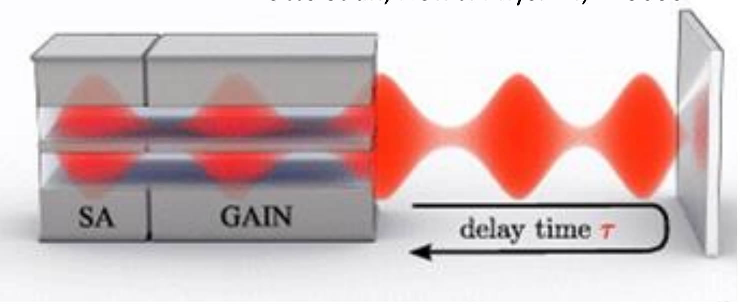
Semiconductor environment includes deformation potential coupling with intrinsic memory depth

Phonon bath with non-Markovian effects synchronizes collapse and revival events

Carmele et a. , New J. Phys. 15 105024 (2013)

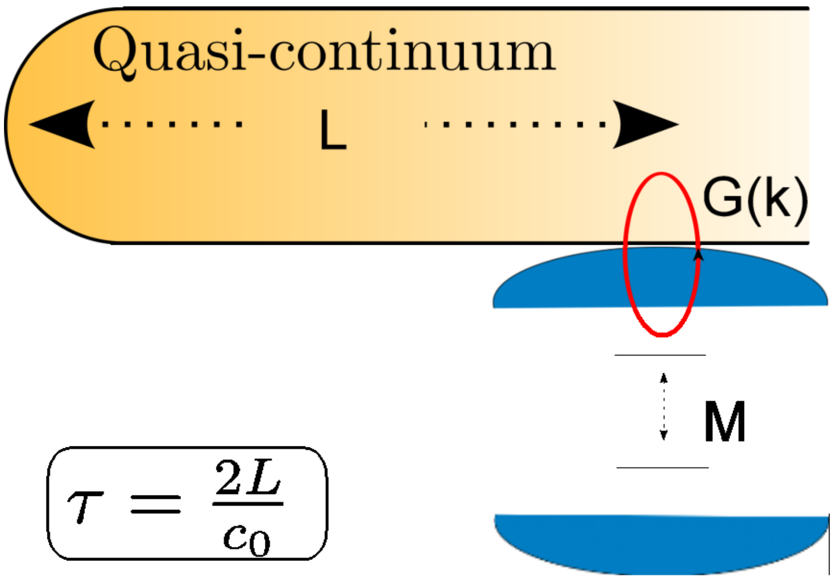
Optical Self-Feedback

Otto *et al.*, New J. Phys. **14**, 113033



Phonons have intrinsic memory Kernel – optical feedback allows for a design of memory effect: delay times

Non-invasive Feedback used in semiclassical limit (Lang-Kobayashi)



$$\tau = \frac{2L}{c_0}$$

Control of quantum state by shaping the environment with from mirror imposed boundary conditions

Carmele et al, PRL 110, 013601 (2013)

Outline

- **Quantum Feedback: Fixed Number of Excitations**
  - ➢ Stabilizing Rabi Oscillations
  - ➢ Entangling Cavities
  - ➢ Enhancing Photon Polarization Entanglement

Schrödinger
Picture

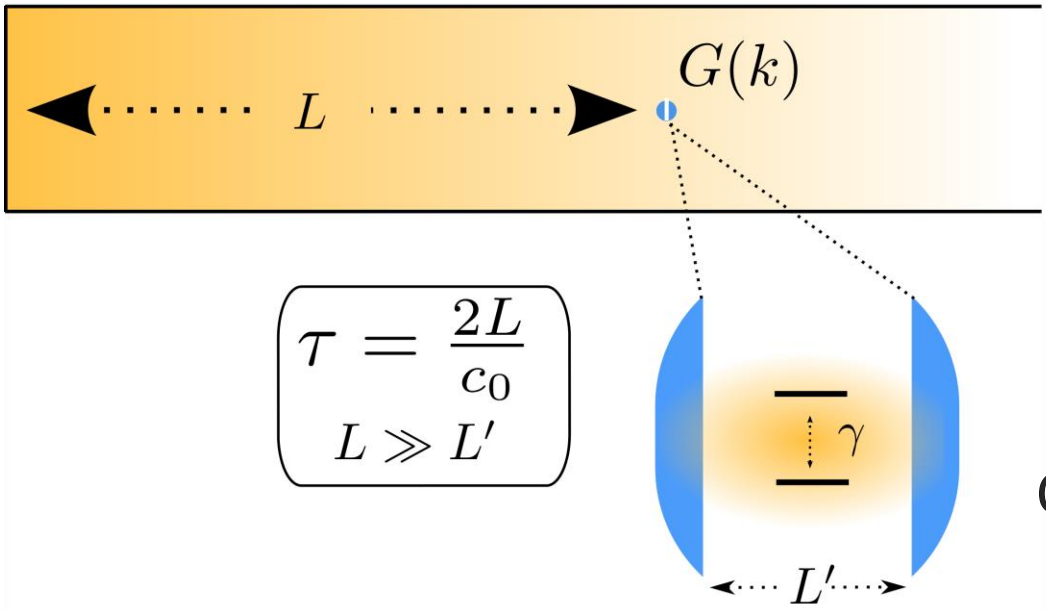
- **Feedback within Heisenberg Operator Language**
  - ➢ Unravelling the Delay Dynamics (1 EX)
  - ➢ Semi-Classical Factorization
  - ➢ Comparison with Super-Operator description

Heisenberg
Picture

System Hamiltonian

Reservoir

$$G(k) = G\sin(kL)$$



Cavity-QED

Exchange of cavity- with waveguide photons

$$\begin{aligned} H/\hbar &= -M\left(\sigma^- a^\dagger + \sigma^+ a\right) \\ &\quad - \int \mathrm{d}k\ G(k,t)\ a^\dagger d_k + G^*(k,t)\ d_k^\dagger a, \end{aligned}$$

Dorner *et al.*, Phys. Rev. A 66, 023816

Fixed Number of Excitations, e.g. N=1

Wave vector:

$$|\Psi\rangle \quad = \quad c_e\,|e,0,\{0\}\rangle + c_g\,|g,1,\{0\}\rangle \; + \int \mathrm{d}k \; c_{g,k}\,|g,0,\{k\}\rangle$$

Fixed numbers of excitations in the system, here N=1

$$\begin{aligned} \partial_t c_e &= i\; M\; c_g \\ \partial_t c_g &= i\; M\; c_e - \kappa\; c_g + \kappa\; c_g\; \Theta(t-\tau)\; e^{i\omega_0\tau} \end{aligned}$$

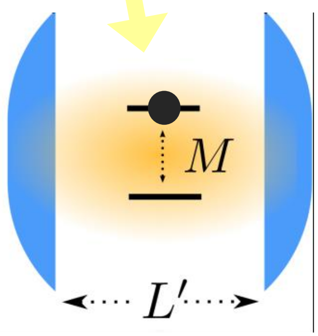
$$\kappa = \pi G^2/(2c_0)$$

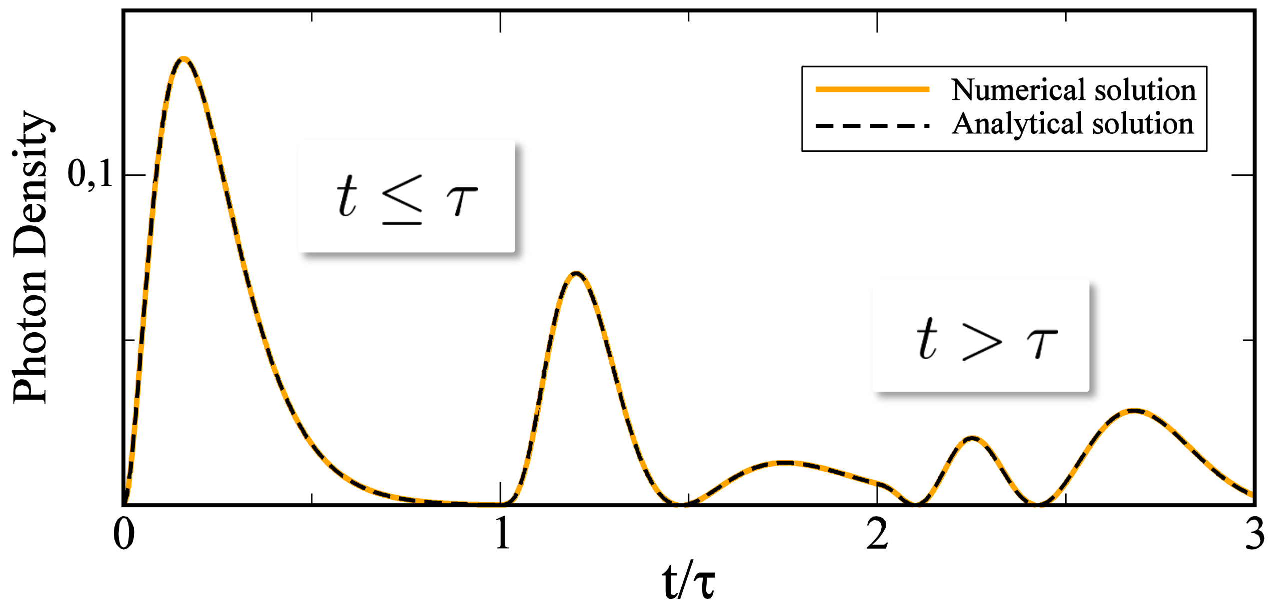
Feedback strength

Dynamics of cavity photons

Initial condition

$$|c_e(0)|^2 = 1$$





$$c_g(t) = i \frac{\sin\left[\sqrt{1-(\kappa/2M)^2}\; Mt\right]}{\sqrt{1-(\kappa/2M)^2}} e^{-\kappa/2\; t}$$

$t \leq \tau$ — Decay of photon number due to strong cavity leakage G

$t > \tau$ — Revival of photon number after τ and irregular oscillatory behavior

Kabuss et al, arXiv:1503.05722v1 (2015)

Long time solution

Find the singularities of ground state amplitude

$$c_g(s) \quad = \quad \frac{iM}{s^2 + M^2 + \kappa s - \kappa s e^{-(s-i\omega_0)\tau}}$$

$$\pm iM \quad \Rightarrow \quad e^{i(\omega_0 \mp M)\tau} \stackrel{!}{=} 1 \qquad \tau_{(i)} = 2n\pi/M \qquad \omega_0\tau = 2\pi m$$

$$c_g^{(i)}(t) \quad = \quad \frac{1}{2\pi i}\oint \mathrm{d}s\ c_g(s)\ e^{st} = \sum_{\text{Poles}} \mathrm{Res}\left[c_g(s)e^{st}\right]$$

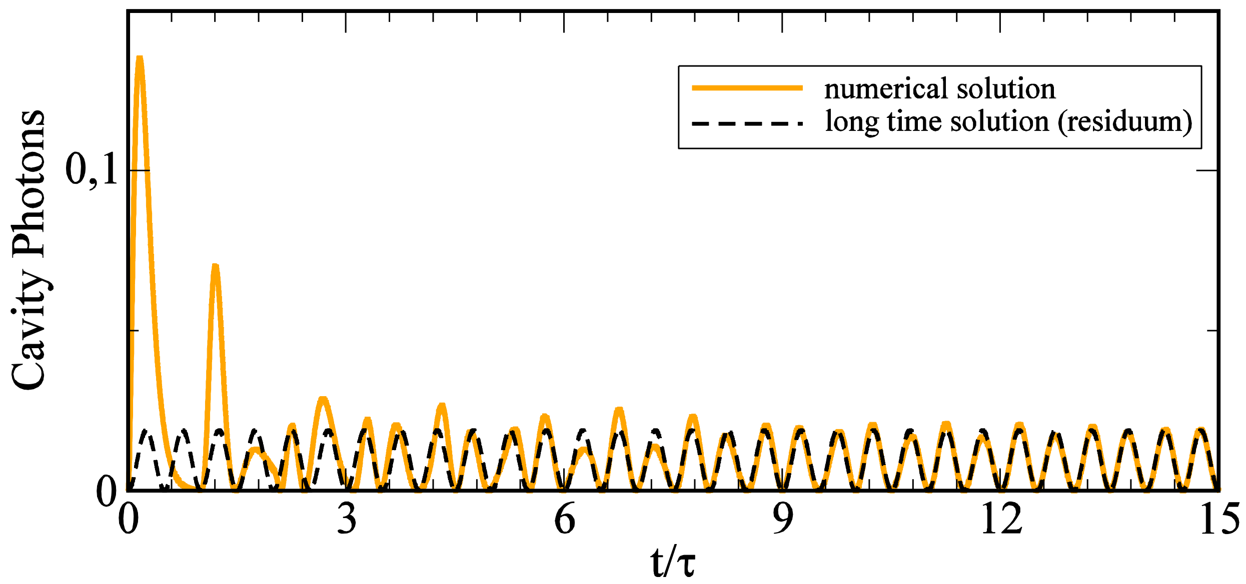
$$c_g^{(i)}(t) = \frac{i\sin[Mt]}{1 + \kappa n\pi/M}$$

- Rabi-frequency of JCM
- altered amplitude

Long time solution

Delay time

$$\tau_{(i)} = 2\pi/M$$



- Long time solution exactly reproduces numerics:
  - Same oscillatory frequency & amplitude
- Dynamics in the long time limes depend strongly on chosen delay time $\tau = 2n\pi/M$ or $\tau = (2n+1)\pi/M$

Kabuss et al, arXiv:1503.05722v1 (2015)

Interference of Photon Paths

System of equations in the Laplace domain:

$$\int d\omega \, \frac{G^2(\omega)}{s - i\omega}$$

$$\begin{pmatrix} c_e(0) \\ c_g(0) \end{pmatrix} = s\,[1 - \mathbb{L}] \begin{pmatrix} c_e(s) \\ c_g(s) \end{pmatrix} \qquad \mathbb{L} = \begin{pmatrix} 0 & i\frac{M}{s} \\ i\frac{M}{s} & -\frac{\kappa}{s}\left(e^{-\tau s} - 1\right) \end{pmatrix}$$

Alber *et al*., Phys. Rev. A 88, 023825 (2013)

$$\begin{pmatrix} c_e(s) \\ c_g(s) \end{pmatrix} = \frac{1}{s} \sum_{n=0}^{\infty} \mathbb{L}^n \begin{pmatrix} c_e(0) \\ c_g(0) \end{pmatrix}$$

$$= \sum_{n=0}^{\infty} \left[\frac{(iM)^n}{s^{n+1}} \begin{pmatrix} 0 & 1 \\ 1 & \frac{\kappa}{iM}\left(e^{-\tau s} - 1\right) \end{pmatrix}^n \right] \begin{pmatrix} c_e(0) \\ c_g(0) \end{pmatrix}$$

Kabuss et al, arXiv:1503.05722v1 (2015)

$n = 0 \quad n = 1 \quad n = 2 \quad n = 3$

$$\begin{pmatrix} c_e(s) \\ c_g(s) \end{pmatrix} = \frac{1}{s}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{s^2}\begin{pmatrix} 0 \\ iM \end{pmatrix} + \frac{1}{s^3}\begin{pmatrix} (-iM)^2 \\ -iM\kappa(e^{-s\tau} - 1) \end{pmatrix} + \frac{1}{s^4}\begin{pmatrix} (-iM)^2\kappa(e^{-s\tau} - 1) \\ (-iM)^2 - iM\kappa^2(e^{-s\tau} - 1)^2 \end{pmatrix} + \ldots$$

electron in excited state

swap of excitation

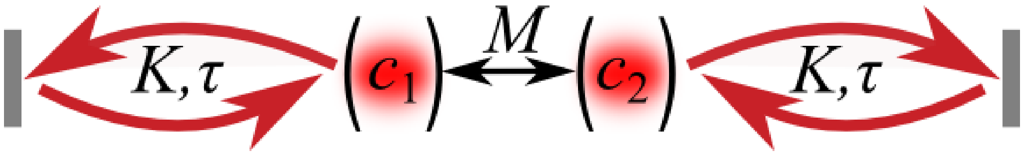
reabsorption after τ

spontaneous emission

interference with previous paths

Applications: (i)

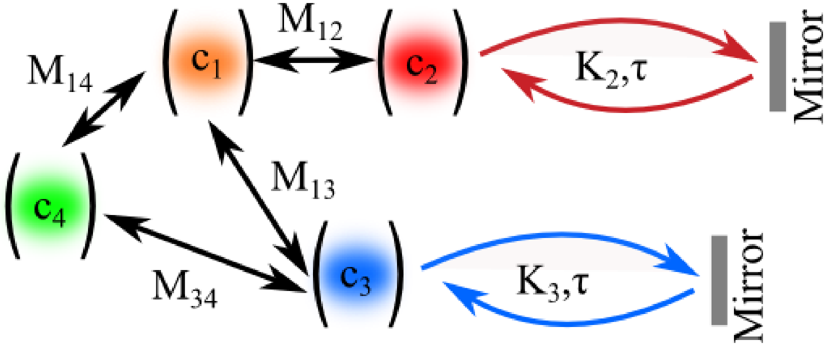
Entangling cavities via optical self-feedback:



- **Eigenmodes** with 1 photon:

$(|1,0\rangle \pm |0,1\rangle)/\sqrt{2}$ **entangled**

- **Eigenfrequencies** $\Omega = \omega \pm M$



- **Idea:** Stabilize **one** mode via Pyragas

control $\tau \cdot (\omega + M) = \mathbb{N} \cdot 2\pi$

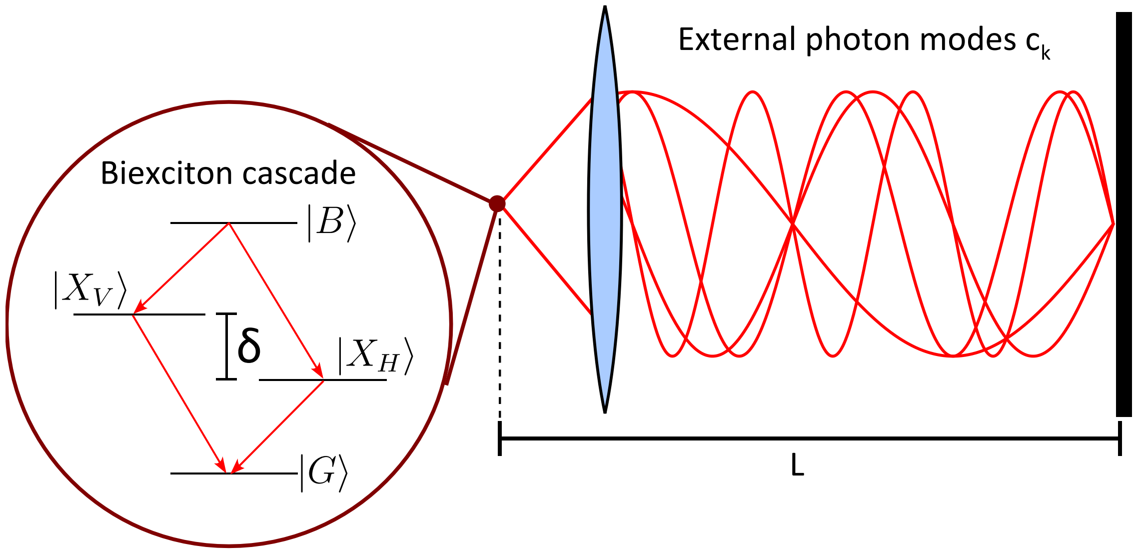
$\tau \cdot (\omega - M) \neq \mathbb{N} \cdot 2\pi$

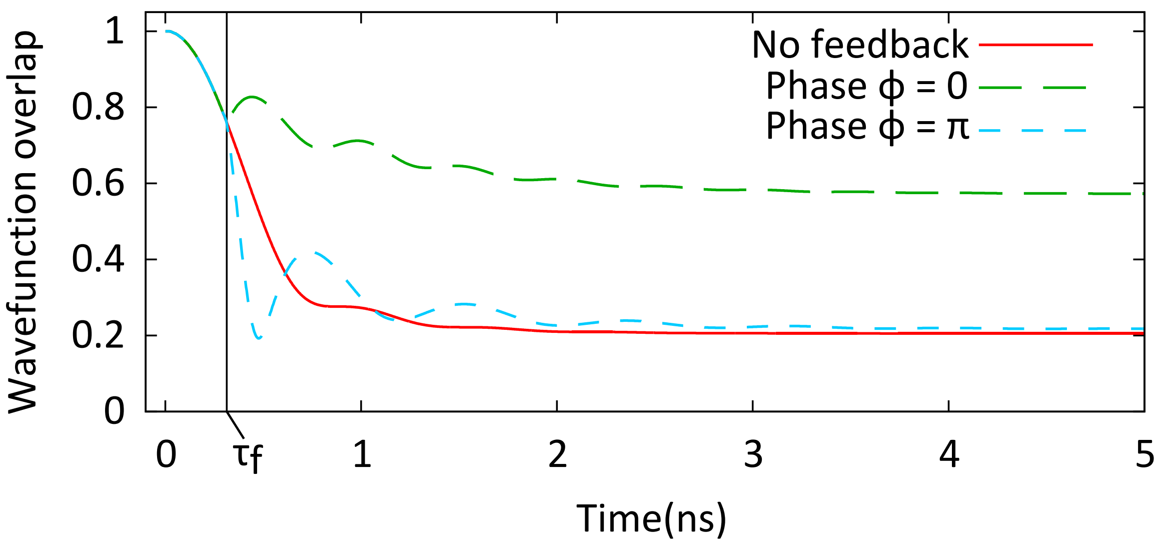
Entangle nodes on a cavity network on demand by selecting the respective Eigenmode to stabilize

Hein et al, PRA 91, 052321 (2015)

Applications: (ii)

Biexciton cascade generates polarization entangled photon pairs





Feedback enhances the degree of entanglement for finite fine structure splitting

Hein et al, PRL 113,027401 (2014)

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Schrödinger *Picture*

- **Feedback within Heisenberg Operator Language**
  - Unravelling the Delay Dynamics
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Heisenberg *Picture*

Time delayed Operator Equations

Heisenberg EOM:

$$d_\omega(t) = d_\omega(0) + G(\omega)\int_0^t dt' e^{i\omega t'} C_s,$$

$$\dot{C}_s(t) = i[H_{sys}, C_s] + \int d\omega\, g(\omega) e^{-i\omega t}$$

Eliminating reservoir

$$\dot{C}_s(t) = i[H_s, C_s] + \xi_s(t) + d_{in}(t)$$

$$d_{in}(t) \equiv \int d\omega\, G(\omega) e^{-i\omega t} d_\omega(0)$$

$$\xi_s(t) \equiv -\int_0^t dt' C_s(t') \int d\omega\, |G(\omega)|^2 e^{i\omega(t'-t)}$$

$$= -\kappa C_s(t) + \kappa C_s(t-\tau)\Theta(t-\tau) e^{-i\omega_0 \tau}$$

Markov

Structured reservoir (mirror)

JCM with Feedback

$$H_{JCM} = -\hbar g\, P^{\dagger} c + \hbar g\, c^{\dagger} P$$

$$P \equiv |v\rangle\langle c| \qquad O_j = O(t - j\tau)$$

$$\begin{aligned} \dot{c}_j &= -\kappa c_j + \tilde{\kappa} c_{j+1}\theta_{j+1} - igP_j + d_{in} \\ \dot{P}_j &= 2igP_j^{\dagger} P_j c_j - igc_j \end{aligned}$$

d_{in} can be omitted for negligible input noise

Calculating dynamics within time intervals: $t \in [i\tau, (i+1)\tau) \equiv I_i$

$I_0 = [0, \tau)$

$$\begin{aligned} \partial_t \langle c^{\dagger} c \rangle &= -2\kappa \langle c^{\dagger} c \rangle + 2g\mathrm{Im}\left[\langle P^{\dagger} c \rangle\right], \\ \partial_t \langle P^{\dagger} c \rangle &= -\kappa \langle P^{\dagger} c \rangle + ig\langle c^{\dagger} c \rangle - ig\langle P^{\dagger} P \rangle, \\ \partial_t \langle P^{\dagger} P \rangle &= 2\mathrm{Im}\left[\langle P^{\dagger} c \rangle\right], \end{aligned}$$

JCM with Feedback

$$I_i \equiv [i\tau, (i+1)\tau)$$

$$\begin{aligned}
\partial_t \langle c^\dagger c_j \rangle &= -2\kappa \langle c^\dagger c_j \rangle + \tilde{\kappa}^* \langle c_1^\dagger c_j \rangle + \tilde{\kappa} \langle c^\dagger c_{j+1} \rangle \theta_{j+1} \\
&+ ig \langle P^\dagger c_j \rangle - ig \langle c^\dagger P_j \rangle, \\
\partial_t \langle P^\dagger c_j \rangle &= -\kappa \langle P^\dagger c_j \rangle + \tilde{\kappa} \langle P^\dagger c_{j+1} \rangle \theta_{j+1} \\
&+ ig \langle c^\dagger c_j \rangle - ig \langle P^\dagger P_j \rangle, \\
\partial \langle c^\dagger P_j \rangle &= -\kappa \langle c^\dagger P_j \rangle + \tilde{\kappa}^* \langle c_1^\dagger P_j \rangle \\
&- ig \langle c^\dagger c_j \rangle + ig \langle P^\dagger P_j \rangle, \\
\partial_t \langle P^\dagger P_j \rangle &= ig \langle c^\dagger P_j \rangle - ig \langle P^\dagger c_j \rangle
\end{aligned}$$

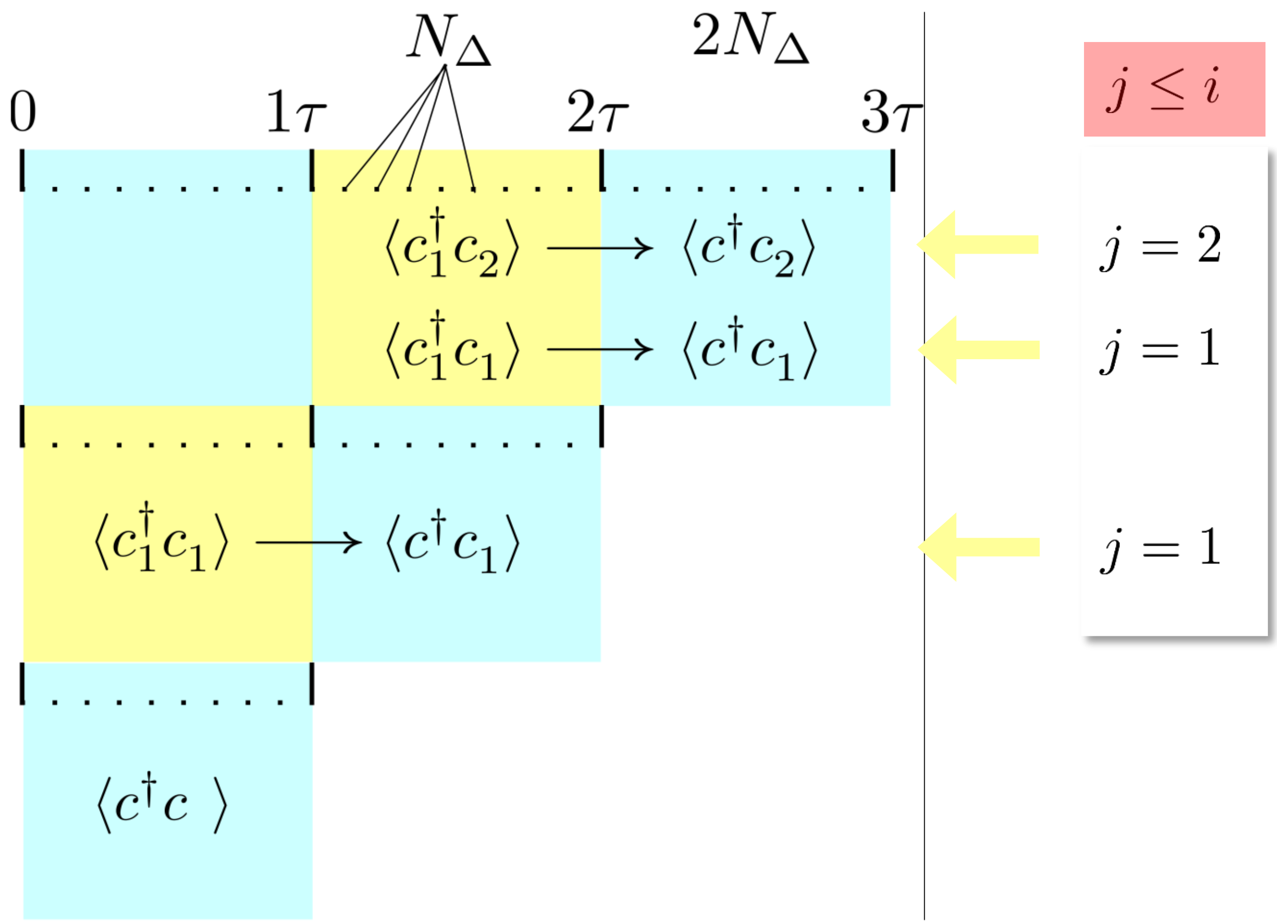
$$j \leq i \quad \longrightarrow \quad \{\langle c_1^\dagger c_j \rangle, \langle c_1^\dagger P_j \rangle\}_j$$

Only two expectation values from previous times have to be stored at each j

equation set grows linearly with index of the τ-intervals

In preparation

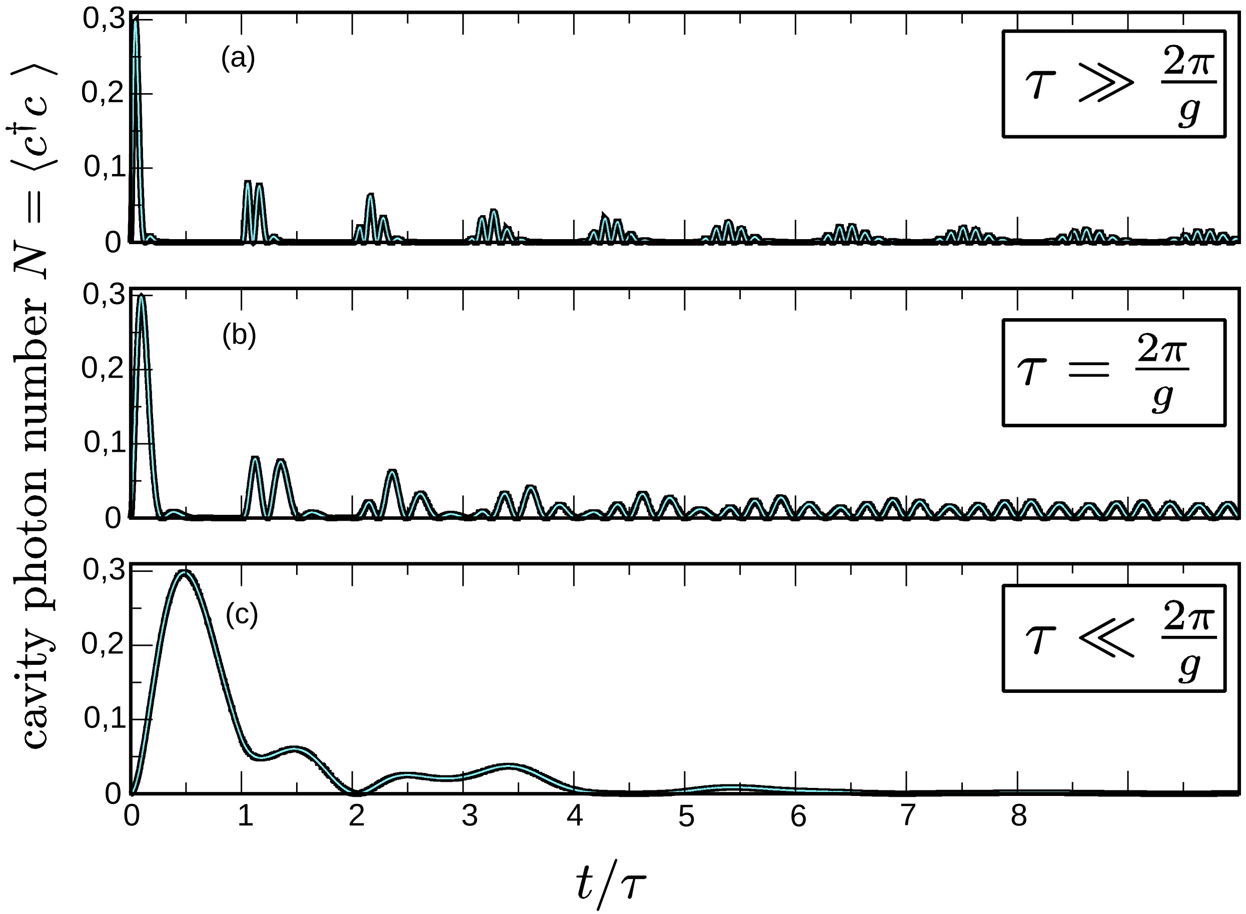
Feedback memory



- equation set grows linearly with index of the τ-intervals
- initial conditions at corners of the intervals are calculated on the fly

In preparation

Feedback times



long:
no overlap between in- and outgoing excitation

Intermediate:
Oscillation stabilization due to interference

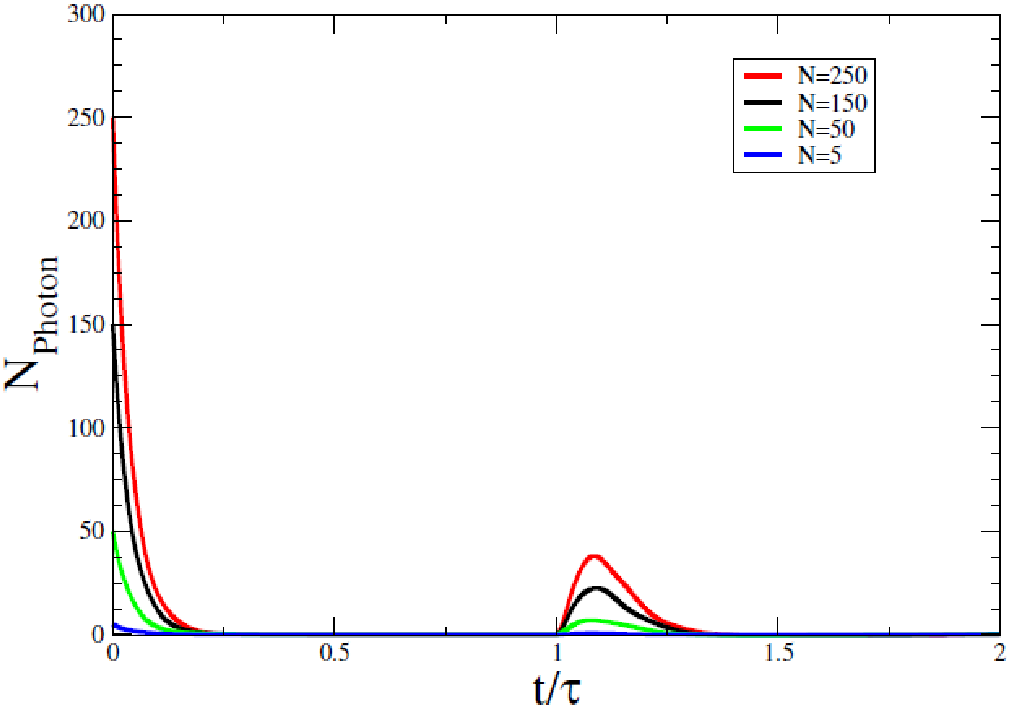
short:
Strong overlap between in- and outgoing population

Applications: (i)

In the Heisenberg picture, controlled factorization approaches become possible, e.g. Born approximation or cluster expansion:

$$\langle c^\dagger P^\dagger P c \rangle \approx \langle c^\dagger c \rangle \langle P^\dagger P \rangle \qquad \langle c_\tau^\dagger P_\tau^\dagger P_\tau P \rangle \approx \langle c_\tau^\dagger P \rangle \langle P_\tau^\dagger P_\tau \rangle$$

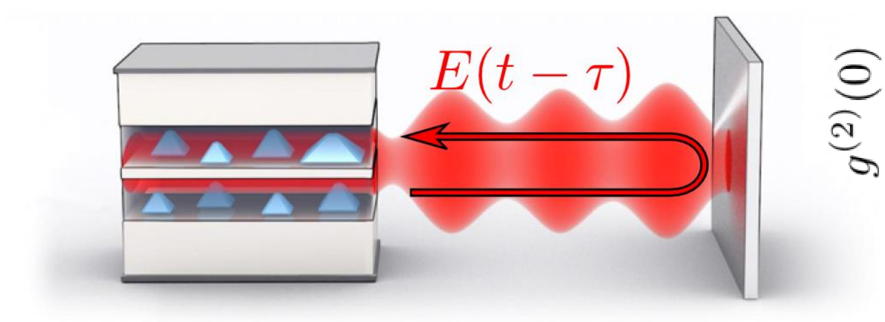
The more photons, the better the factorization: Incoherent pumping in the steady-state limit computable.



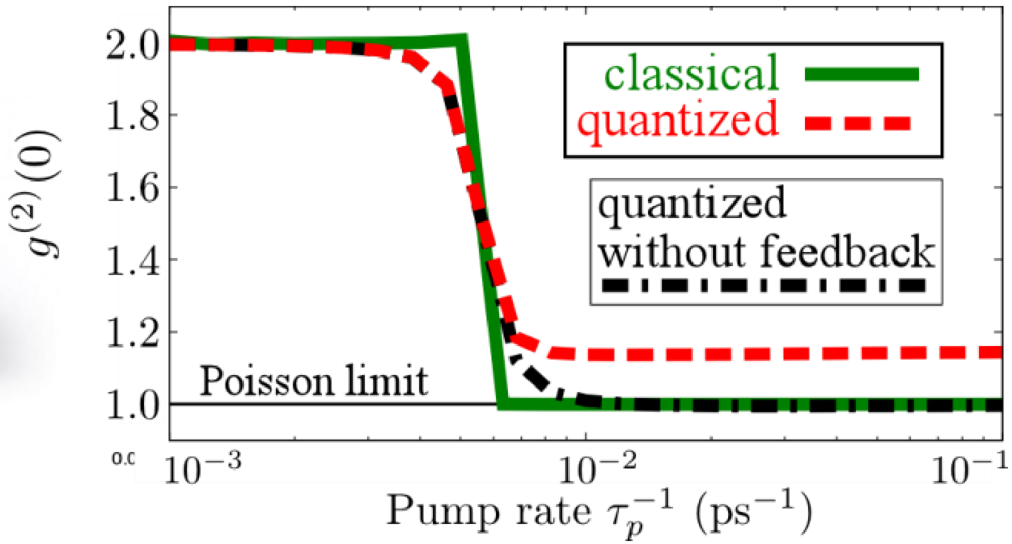
In preparation

Applications: (ii)

Laser dynamics tractable in the cluster-expansion approach:



Lingnau et al., NJP 15, 093031 (2013)



- Change in the photon-photon correlation:
  $g^{(2)}(0)>1$ with feedback
- Matches to experiment: Albert et al., Nat. Comm. 2, 355 (2011)

Schulze et al, Phys. Rev. A. 89, 041801(R) (2014)

Conclusion

- Feedback is an interesting example for a simple structured bath, not tractable with Master equations and Lindblad approaches
- Feedback can be used for enhancing entanglement, for quantum eraser experiment, for Rabi oscillations stabilization, and for laser point optimization
- Challenging from the theoretical point of view is the highly temporal entanglement: even for fixed number of excitation huge numerical effort
- Solution: Operator technique in the Heisenberg picture
- … still much to do!!

Thank you for your attention!

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