Wittenberg 2017
SFB 910: Meeting

Quantum Feedback and Noise

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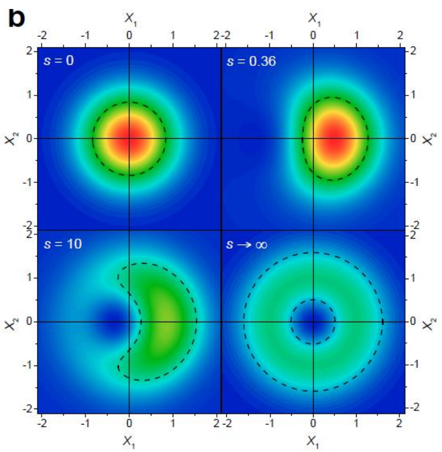
August 30, 2017

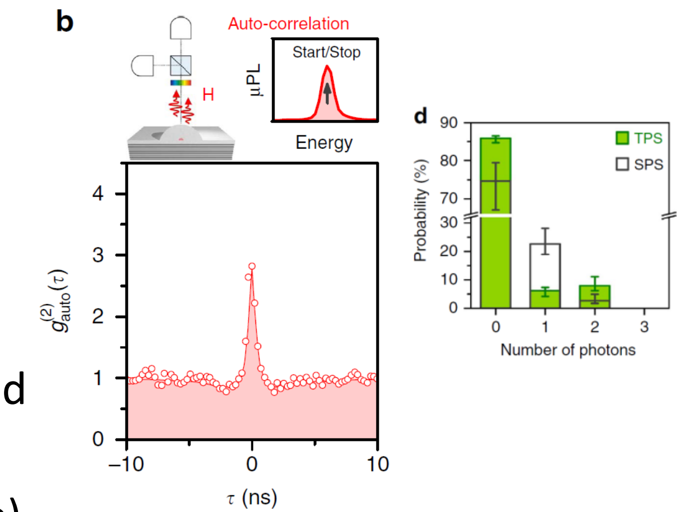
Recent successes in semiconductor quantum optics

Schulte et al, Nature 525, 222 (2015)



Photon squeezing with semiconductor quantum dot

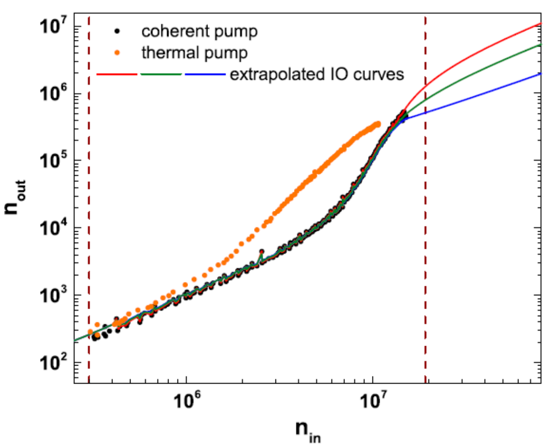
(Quantum Metrology)



Twin Photon Source triggered in Biexciton cascade

(bright quantum light source)

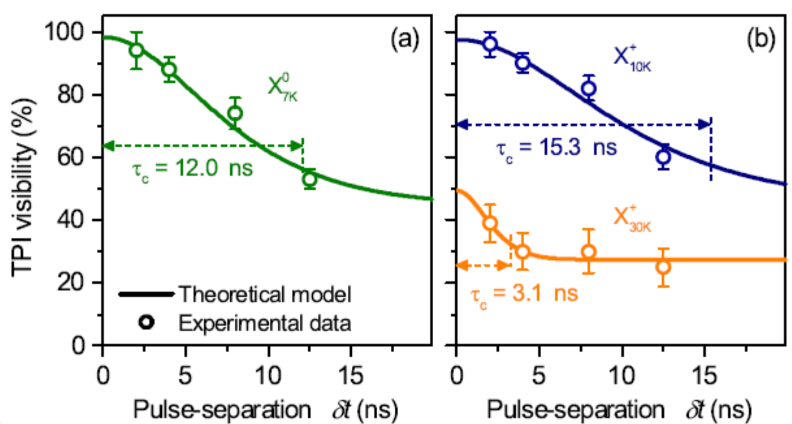
Heindel et al, Nat. Comm. 8, 14870 (2017)



Höfling et al, PRL 115, 027401 (2015)

Strauß et al, PRB 93, 241306 (2016)

Control of quantum emission (tailored photon statistics)



Thoma et al, PRL 116, 033601 (2016)

Dephasing time between single photon events

(indistinguishability)

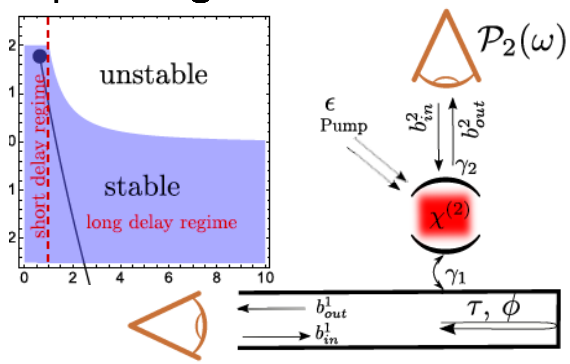
Ding et al, PRL 116, 020401 (2016)

Kim et al, Optica 3, 577 (2016)

Wang et al, PRL 116, 213601 (2016)

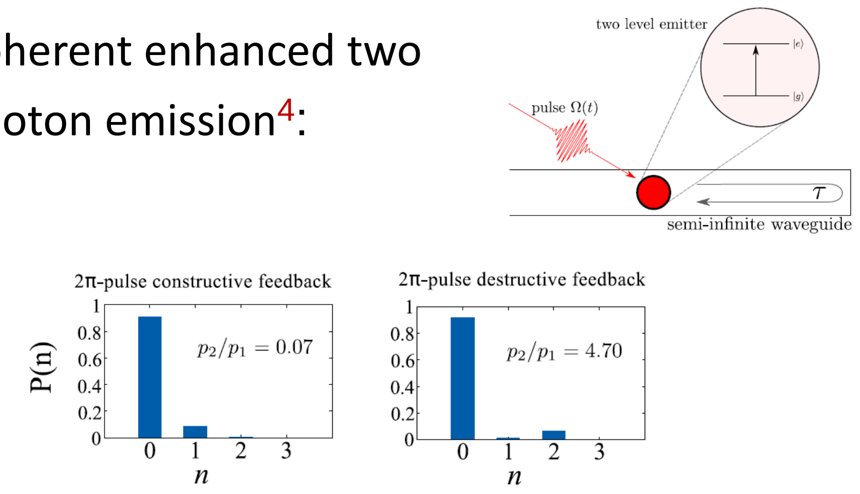
Feedback motivation

Squeezing with feedback[1,2]:



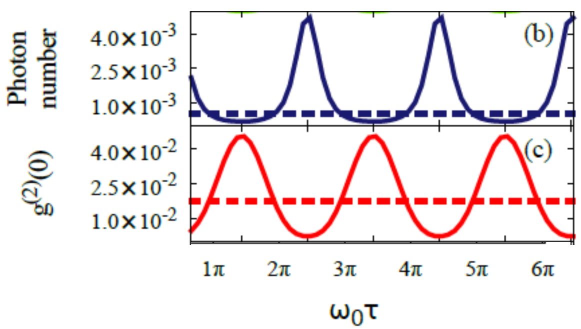
[1]M. Kraft et al, Phys. Rev. A **94**, 023806 (2016)
[2]N. Nemet et al, Phys. Rev. A **94**, 023809 (2016)

Coherent enhanced two photon emission[4]:



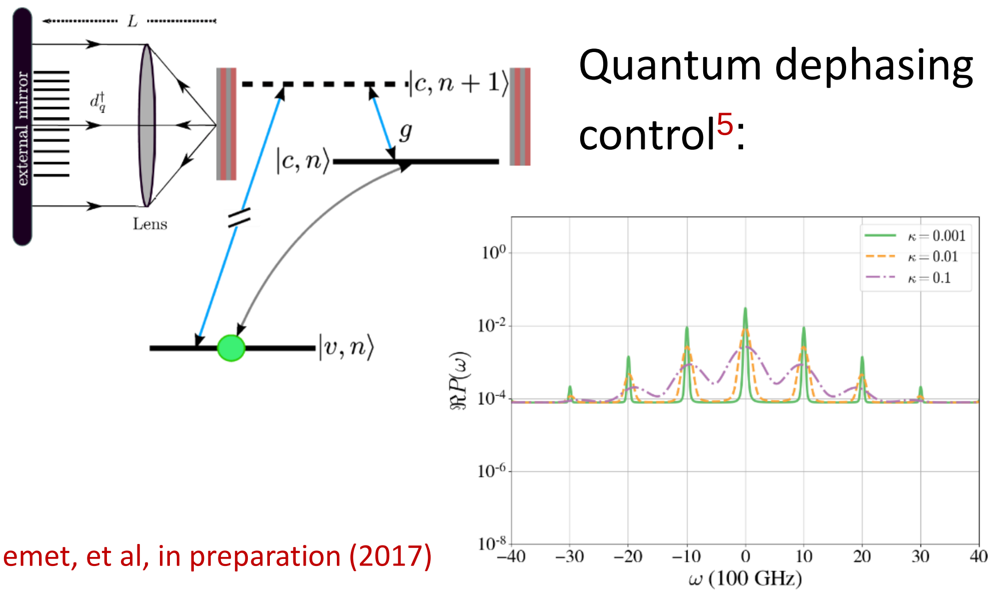
[4]N. Naumann, in preparation (2017)

Quantum control of photon statistics with feedback[3]:



[3]Y. Lu et al, Phys. Rev. A **95**, 063840 (2017)

Quantum dephasing control[5]:



[5]N. Nemet, et al, in preparation (2017)

Outline

- **Quantum description of a feedback mechanism**
  - How is a feedback introduced in the dynamics?
  - Find an effective model for quantum feedback
  - Experimental benchmark and application

Full model
-numerical solvable-
(state vector size)

- **Quantum Feedback via time-ordered operator method**
  - Derivation of observable dynamics
  - Numerically tractable and linear scaling
  - Quantum noise and commutation relations

Observable
-time ordered hierarchy-
(quantum noise)

- **Quantum Feedback via matrix product state approach**
  - Quantum stochastic Schrödinger equation
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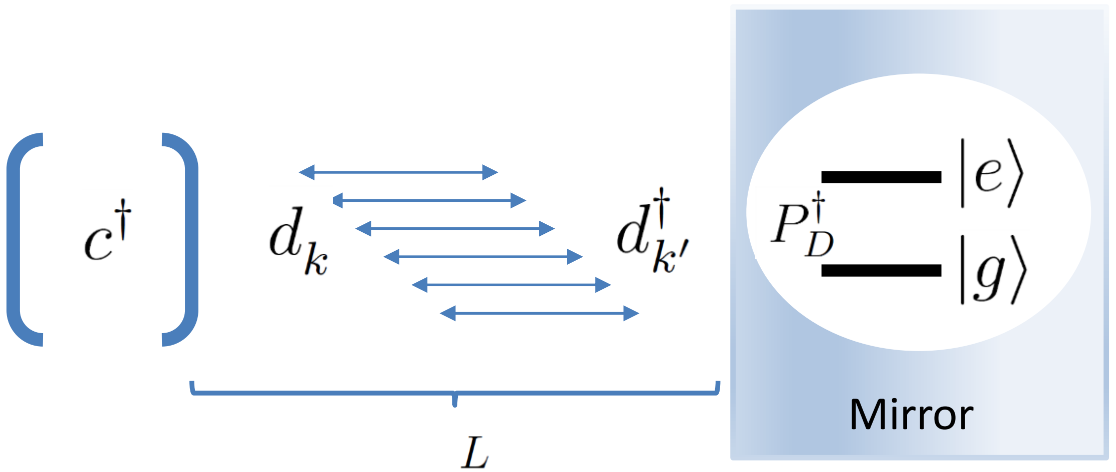
  - Full quantum entanglement addressable

QSSE
-Effective state vector-
(entanglement)

Feedback model

Modelling feedback in the quantum regime[1]:



$$[c, c^\dagger] = 1$$
$$[d_k, d^\dagger_{k'}] = \delta(k - k')$$
$$P^\dagger_D \equiv |e\rangle\langle g|$$

- Continuum between system (emitter/cavity) and dielectric medium (mirror)
- Mirror consists of near-resonant two-level systems
- Excitation from emitter is transmitted to dielectric and re-emitted back (feedback)

$$H/\hbar = \omega_D P^\dagger_D P_D + \omega_0 c^\dagger c + \int dk\ \omega_k\ d^\dagger_k d_k + \int dk\ \tilde{g}_k (d^\dagger_k c + c^\dagger d_k) + \int dk\ \tilde{M}_k (d^\dagger_k P_D + P^\dagger_D d_k)$$

[1]F. Faulstich et. al, J. Mod. Opt (2017); arXiv:1703.05928.

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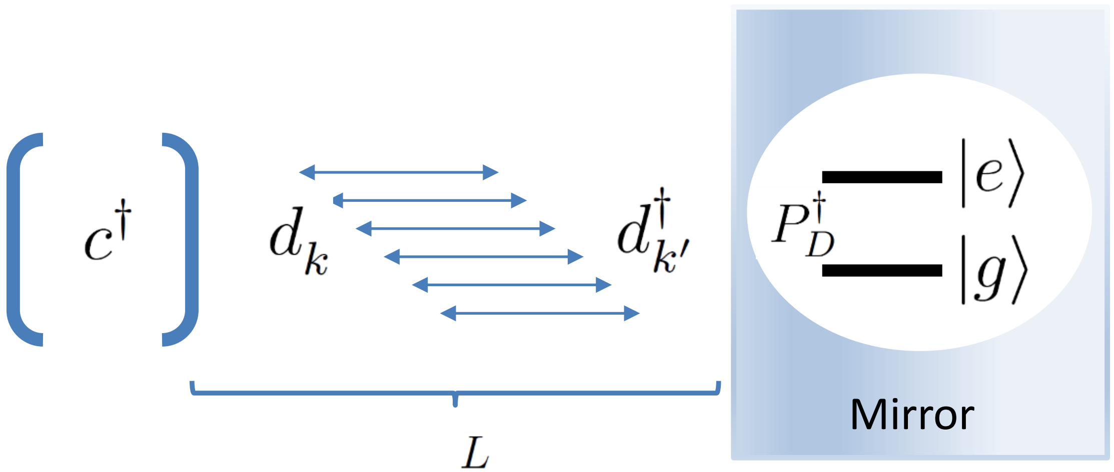


$$[c, c^\dagger] = 1$$
$$[d_k, d^\dagger_{k'}] = \delta(k - k')$$
$$P^\dagger_D \equiv |e\rangle \langle g|$$

- Calculate Heisenberg equation of motion of the reservoir operators[1]
- Trace out medium and reservoir modes

$$\tau = \frac{2L}{c_0}$$

$$\frac{\mathrm{d}}{\mathrm{dt}} c = i[H, c]$$
$$\frac{\mathrm{d}}{\mathrm{dt}} d_k = i[H, d_k]$$
$$\frac{\mathrm{d}}{\mathrm{dt}} P_D = i[H, P_D]$$

$$\Rightarrow$$

$$\frac{\mathrm{d}}{\mathrm{dt}} c^\dagger(t) \approx -\Gamma c^\dagger(t) + \Gamma \chi_D(\omega_0) c^\dagger(t - \tau) \Theta(t - \tau)$$

[1]F. Faulstich et. al, J. Mod. Opt (2017); arXiv:1703.05928.

Feedback coupling

- Include refraction index definition from classical electrodynamics[1]

$$n^2(\omega_e) - 1 \approx \mathrm{Re}(\chi(\omega_e)) \qquad \chi_D(\omega_0) \approx e^{i\omega_0\tau}$$

- Leads to Pyragas[2] type control equation

$$\frac{\mathrm{d}}{\mathrm{dt}} c^\dagger(t) = -\Gamma\left[c^\dagger(t) - c^\dagger(t-\tau)e^{i\omega_0\tau}\right]$$

- Effective Hamiltonian[3,4]

$$H/\hbar = \omega_0 c^\dagger c + \int dk\ \omega_k\ d_k^\dagger d_k + \int dk\ g_k \sin(kL)(d_k^\dagger c + c^\dagger d_k)$$

- However, not exact – dynamics only valid for passive and perfect mirror

Coupling mechanism agrees well with experiment!

[1]F. Faulstich et. al, J. Mod. Opt (2017); arXiv:1703.05928.

[2]E. Schöll et al, ed., Control of Self-Organizing Nonlinear Systems (2016)

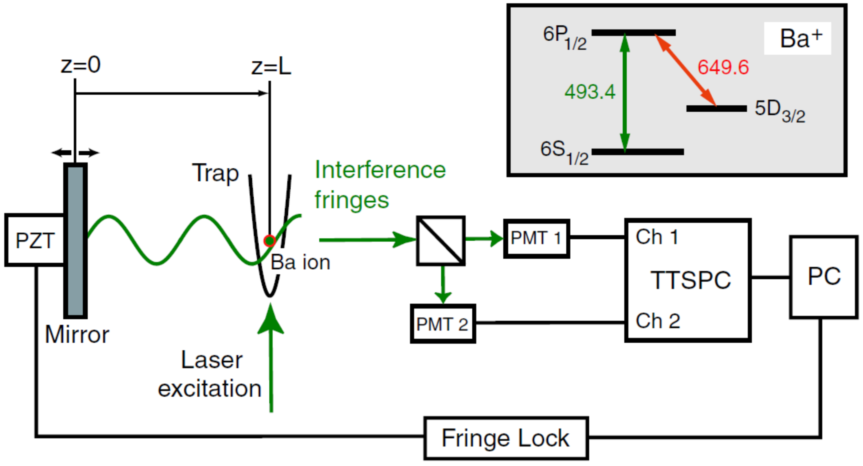
[3]R. J. Cook et al, Phys. Rev. A **35**, 5081 (1987) .

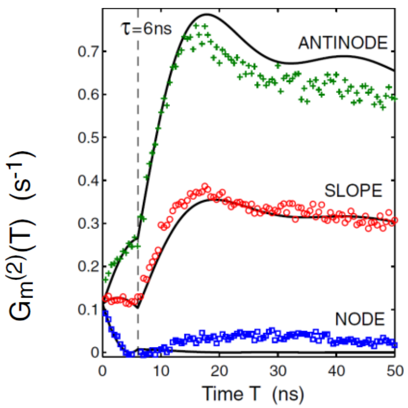
[4]A. Carmele et. al, Phys. Rev. Lett. **110**, 013601 (2013).

Quantum Optics

Experimental confirmation of feedback coupling:

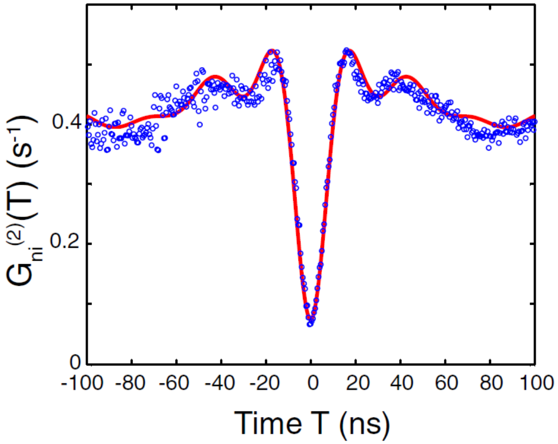
- Experiments with cold atoms[1,2]
- Control and probe laser
- Emitted light field corresponds to aforementioned derivation





- Dissipative dynamics of emitter, position dependent
- Note kink in signal
- Photon-photon correlation exhibit interference signature

➔ Quantum model confirmed[3]



[1]J. Eschner et al., Nature (London) **413**, 495 (2001).

[2]F. Dubin et al, Phys. Rev. Lett. **98**, 183003 (2007).

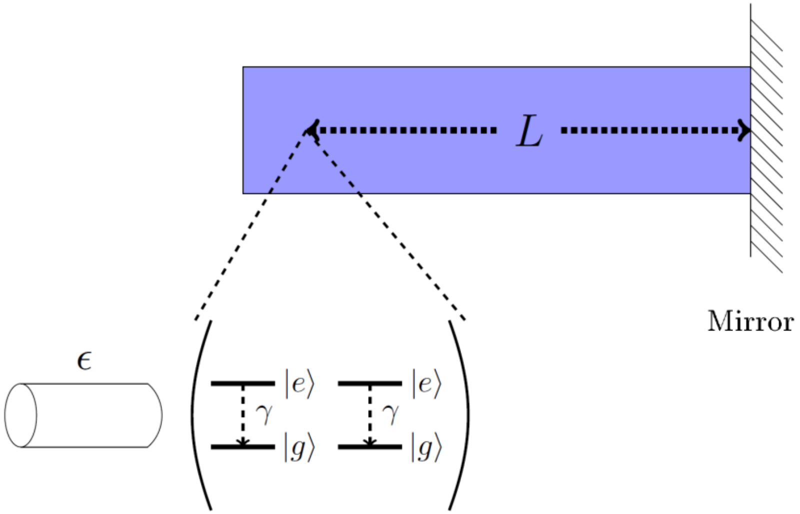
[3]U. Dorner et al, Phys. Rev. A **66**, 023816 (2002).

Example: driven QED

General model of quantum self-feedback[1]:

$$H/\hbar = \omega_0 c^\dagger c + \int dk\ \omega_k\ d_k^\dagger d_k + \int dk\ g_k \sin(kL)(d_k^\dagger c + c^\dagger d_k)$$

- In principle solvable, mode dependent coupling includes delay
- But state vector includes infinite amount of modes!!
- Example: Weakly driven to two emitter cavity-QED in the Schrödinger picture[2]:



$$i\hbar \frac{\mathrm{d}}{\mathrm{dt}} |\varphi\rangle = H\ |\varphi\rangle$$

$$\begin{aligned}|\varphi\rangle = &\int dk \int dk' C_{gg0kk'} |g,g,0,\{k\},\{k'\}\rangle \\ &+ \int dk C_{eg0k} |e,g,0,\{k\}\rangle + \int dk C_{gg0k} |g,g,0,\{k\}\rangle \\ &+ \int dk C_{gg1k} |g,g,1,\{k\}\rangle + \int dk C_{ge0k} |g,e,0,\{k\}\rangle \\ &+ C_{ge10} |g,e,1,\{0\}\rangle + C_{eg10} |e,g,1,\{0\}\rangle \\ &+ C_{gg10} |g,g,1,\{0\}\rangle + C_{gg20} |g,g,2,\{0\}\rangle \\ &+ C_{ee00} |e,e,0,\{0\}\rangle + C_{eg00} |e,g,0,\{0\}\rangle \\ &+ C_{ge00} |g,e,0,\{0\}\rangle + C_{gg00} |g,g,0,\{0\}\rangle.\end{aligned}$$

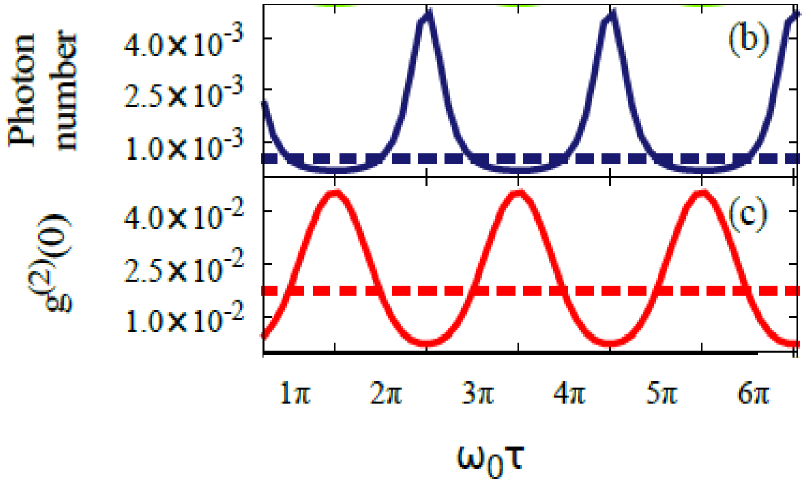
[1] J. Kabuss et al, JOSA B, **33**(7), C10-C16 (2016).

[2] Y. Lu et al, Phys. Rev. A **95**, 063840 (2017)

Photon statistics

Schrödinger dynamics with fixed state vector[1]:

- Increased antibunching in two emitter cQED - counterintuitive
- True quantum interference effect (only occurring in two photon limit)
- Dependent on phase and delay time



- But, extremely expensive numerical evaluation
- Very susceptible to discretization errors
- Not tractable, effectively, for driven systems

➔ Heisenberg picture

[1]Y. Lu et al, Phys. Rev. A **95**, 063840 (2017)

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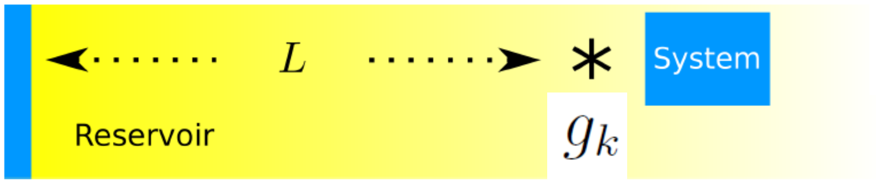
Operator dynamics

Heisenberg equation of motions with feedback contributions[1]:



$$H/\hbar = \omega_e P^\dagger P + \omega_0 c^\dagger c + M(P^\dagger c + c^\dagger P) + \int dk\ \omega_k\ d_k^\dagger d_k$$

$$\int dk\ g_k \sin(kL)(d_k^\dagger c + c^\dagger d_k)$$

$$-i\dot{A} = [H, A] \qquad \dot{d}_k = -i\omega_k d_k - i g_k^*\, c$$

$$d_k(t) = d_k(0)\, e^{-i\omega_k t} - i \int_0^t \mathrm{d}t'\, g_k^* e^{-i\omega_k (t-t')}\, c(t')$$

- Use linear coupling to integrate out the reservoir mode
- But take into account the full solution with the initial conditions
- Full dynamics are encoded in the solution → in principle tractable

[1]J. Kabuss et al, JOSA B, **33**(7), C10-C16 (2016).

Normal-ordering

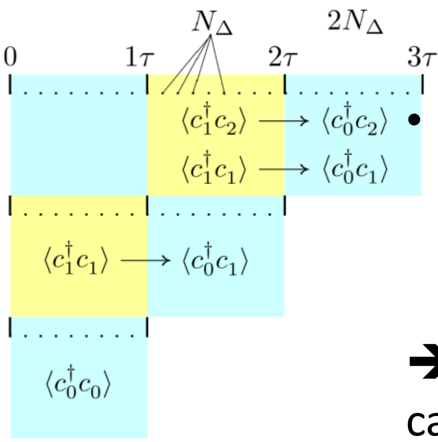
The Heisenberg operator set of differential equations is in principle closed[1]:

$$\dot{c} = -i\omega_c\, c - iM\, P - i\int \mathrm{d}k\; d_k(0)e^{-i\omega_k t} g_k - \int_0^t \mathrm{d}t'\, c(t')f(t,t')$$

- The system of interest is decoupled from feedback inducing reservoir dynamics
- For example: Single excitation limit allows to keep normal-ordering, and noise contributions vanish identically (no memory effects in case of single excitation).

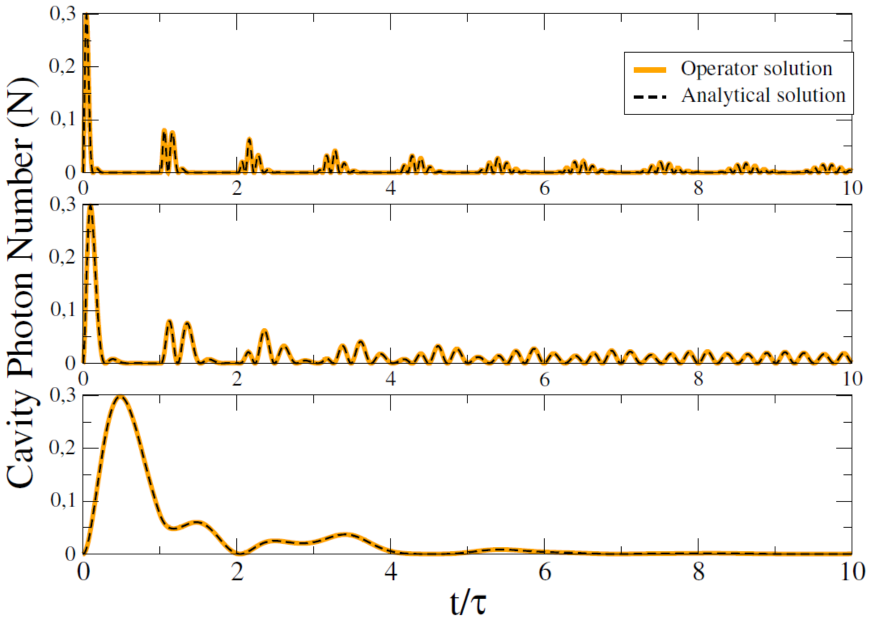
$$f(t,t') := \int \mathrm{d}k\, |g_k|^2 e^{-i\omega_k(t-t')}$$

$$f(t,t') = \Gamma\left(2\delta(t-t') - \delta(t-t'-\tau) - \delta(t-t'+\tau)\right)$$



- Inductive equation of motion in principle possible if quantum noise can be neglected

➔ But quantum noise cannot be neglect



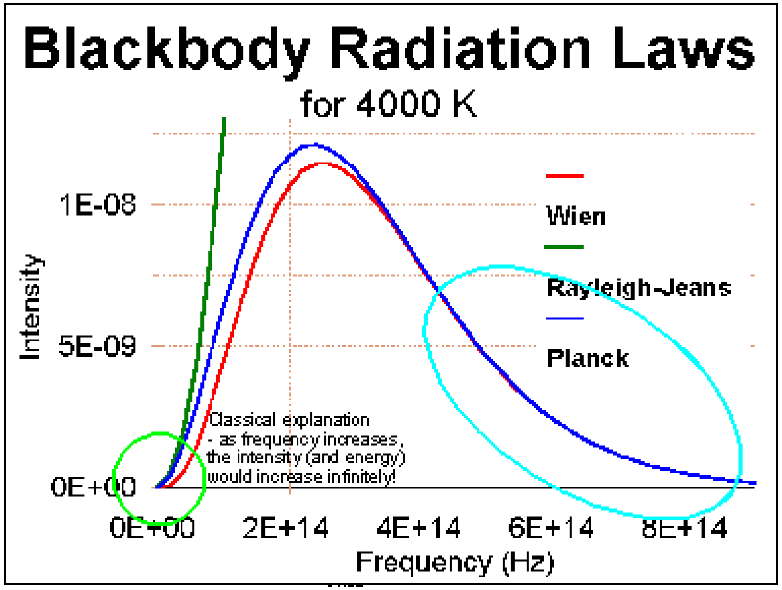
[1]J. Kabuss et al, JOSA B, **33**(7), C10-C16 (2016).

Quantization matters

Quantum correlations and quantum noise: $[c, c^{\dagger}] = 1$

$$\dot{c} = -i\omega_c\, c - iM\, P - i\int \mathrm{d}k\;\; d_k(0)e^{-i\omega_k t} g_k - \int_0^t \mathrm{d}t\,'\; c(t')f(t,t')$$

- Noise cannot only be neglected with distributed feedback



Kinoshita, Toronto (2004)

- Commutation relations essential for the quantum regime and its vacuum fluctuation
- Wien's Radiation law exhibits a constant discrepancy exactly due to the commutation relations (quantum noise)

➔ Quantum noise renders inductive equation of motion approach effectively intractable

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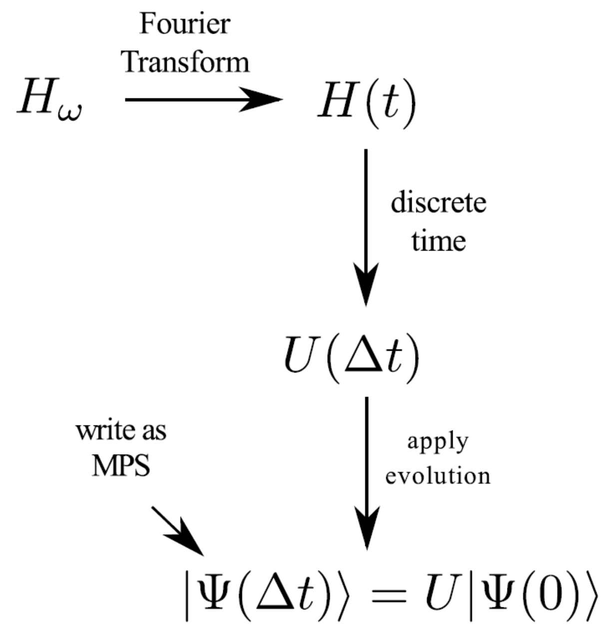
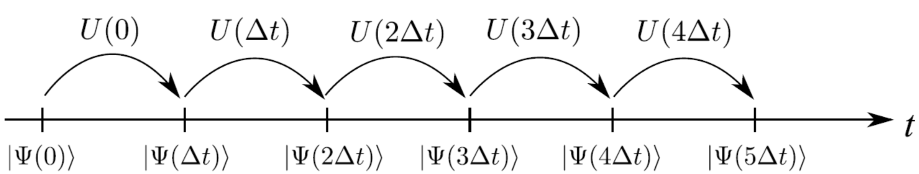
Stroboscopic dynamics

Quantum noise introduces a correlated system-bath dynamics[1,2]:

- Idea behind the matrix product state approach: Control the correlation depth of reservoir contribution to the system's dynamics
- Stroboscopic evolution
- No approximation, numerically exact





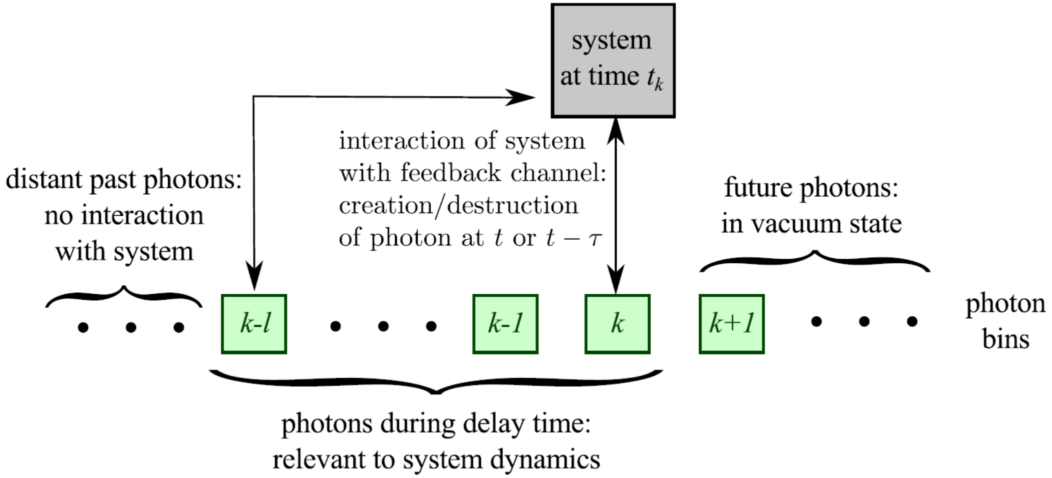
$$U(k\Delta t) = \exp\left[\frac{i}{\hbar} H_S \Delta t + \left(\underbrace{\sqrt{\gamma_L}\Delta B^\dagger(k\Delta t)c}_{\text{Emission of photon from cavity to bath}} + \underbrace{\sqrt{\gamma_R}\Delta B(k\Delta t - \tau)e^{i\phi}c^\dagger}_{\text{Absorbtion of photon emitted previously}} - h.c.\right)\right]$$

$$\Delta B^\dagger(\Delta t) = \int_0^{\Delta t} \left[\int_{-\infty}^{\infty} d_k^\dagger(t) e^{i(\omega_0 - \omega_k)t}\, dk\right] dt$$

[1]Y. Lu et al, Phys. Rev. A **95**, 063840 (2017)

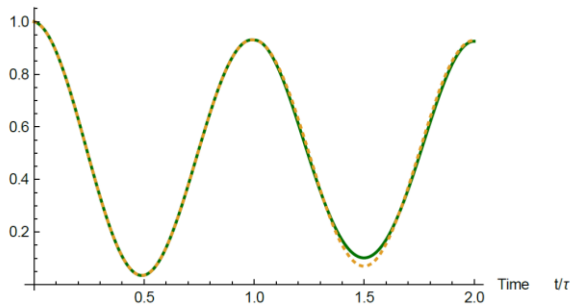
[2]H. Pichler, Phys. Rev. Lett. **116**, 093601 (2016).

Singular value decomposition

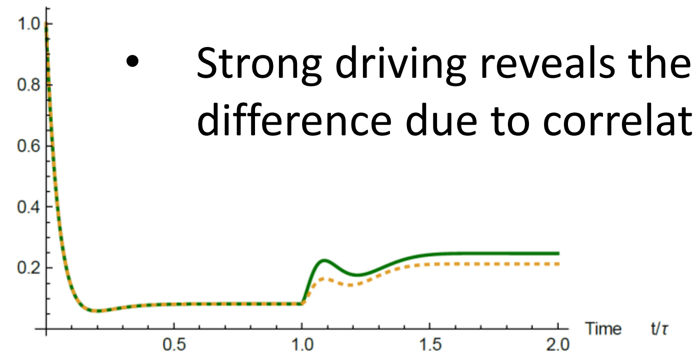


- Evolution is modelled by a step to step evaluation of the system – reservoir dynamics
- Only strongly correlated contributions are considered
- Numerical effort efficient and tractable

- Singular value decomposition reveals the important parts of the dynamics
- Measure is quantum mechanical relevant: Entanglement.
- Cut-off dynamical and numerical exact evaluation guaranteed

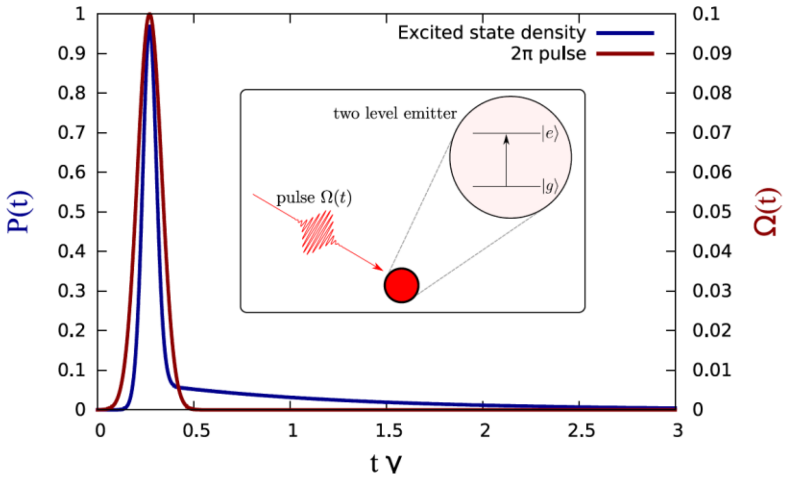


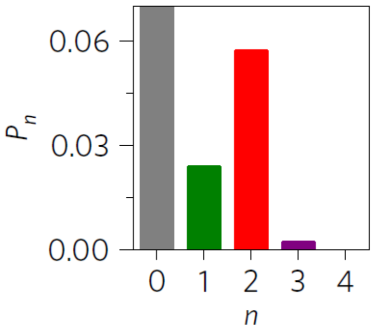
Weak driving agrees in case of factorization



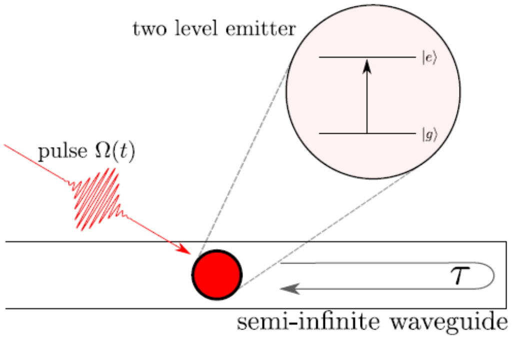
- Strong driving reveals the difference due to correlations

Two-Photon Sensing

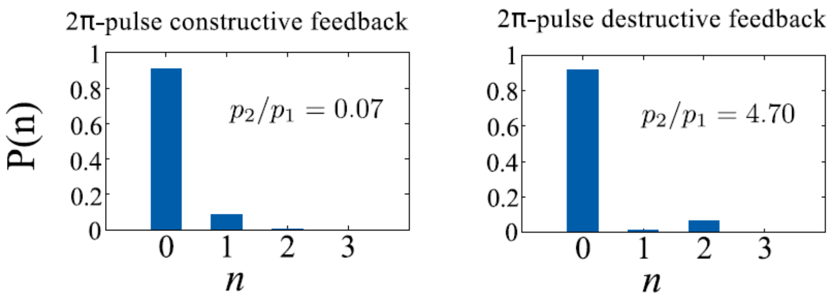




- Feedback dynamics in the non-linear Mollow regime now possible (MPS)
- Stimulated emission for large pulse areas leads to stimulated emission and two photon pulses[1]



- Feedback coupling testable in the nonlinear regime
- Feedback enhances effect by 200%
- Test of derived effective Hamiltonian
- And genuine two-photon quantum interference[2]



[1]K. Fischer et al., Nat. Phys. **13**, 649 (2017)

[2]N. Naumann, in preparation, (2017).

Conclusion

- Quantum feedback is an interesting example for a simple structured reservoir, not tractable with Master equations and Lindblad approaches
- Quantum feedback includes in the many-excitation limits inevitable quantum noise contributions and strongly correlated dynamics
- Matrix product states allow for numerical exact solutions
- Outlook: Apply quantum feedback dynamics to strongly correlated systems such as spin chains and exotic quantum statistics
- Goal: Feedback control of quantum many body states

Thanks for your attention!

SFB 910