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# Non-Markovian quantum feedback control of photon statistics and quantum many body dynamics

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**Alexander Carmele**

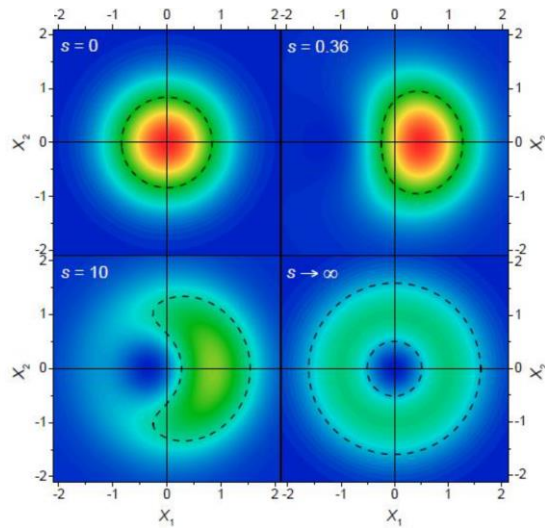
Technische Universität Berlin, Institut für Theoretische Physik, Germany

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# Semiconductor quantum optics

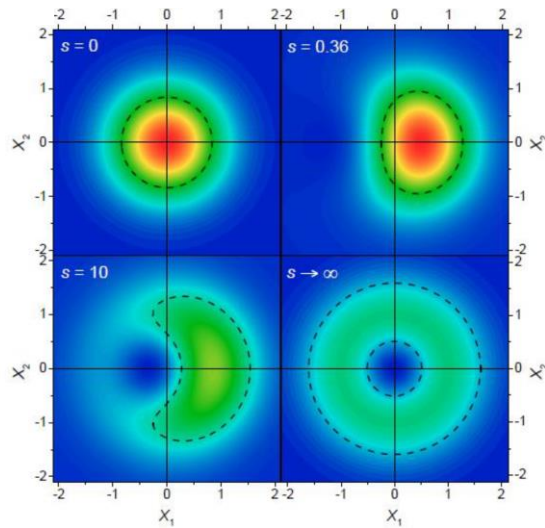
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Walls and Zoller, PRL 47, 709 (1981)  
Schulte et al, Nature 525, 222 (2015)



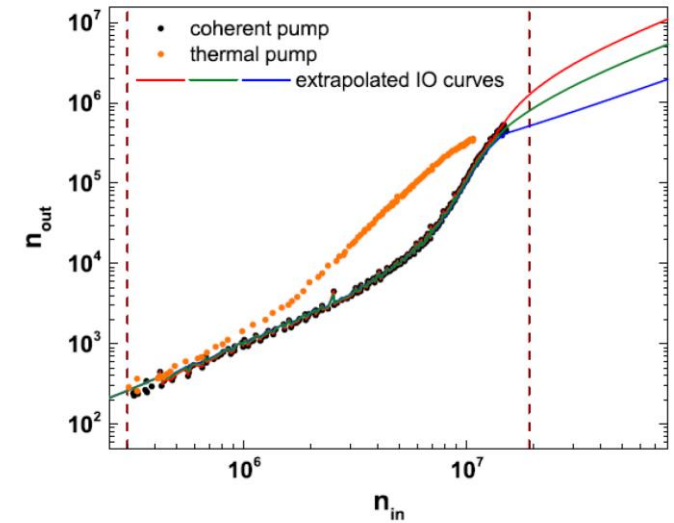
Laser-driven quantum dot exhibits squeezing in resonance fluorescence

Walls and Zoller, PRL 47, 709 (1981)  
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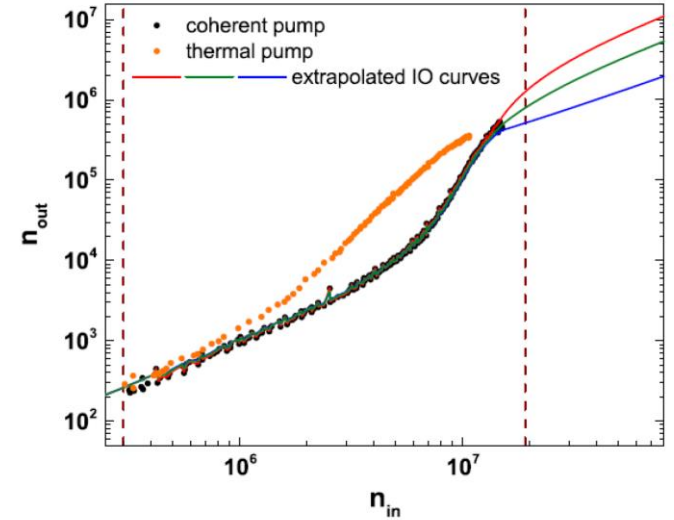
Quantum cascaded-driven laser systems exhibit qualitatively different threshold behavior and input-output curve



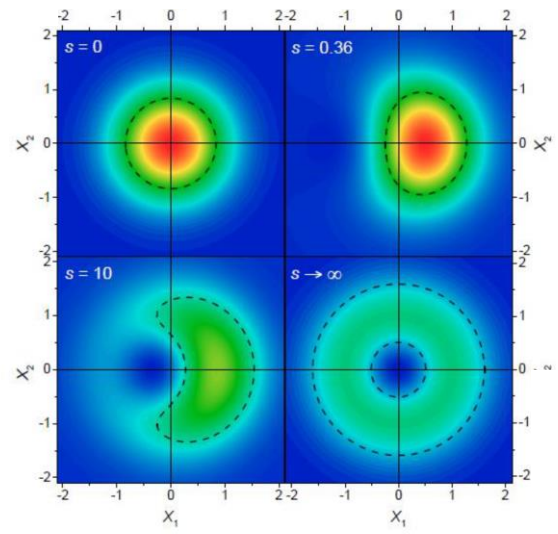
Höfling et al, PRL 115, 027401 (2015)

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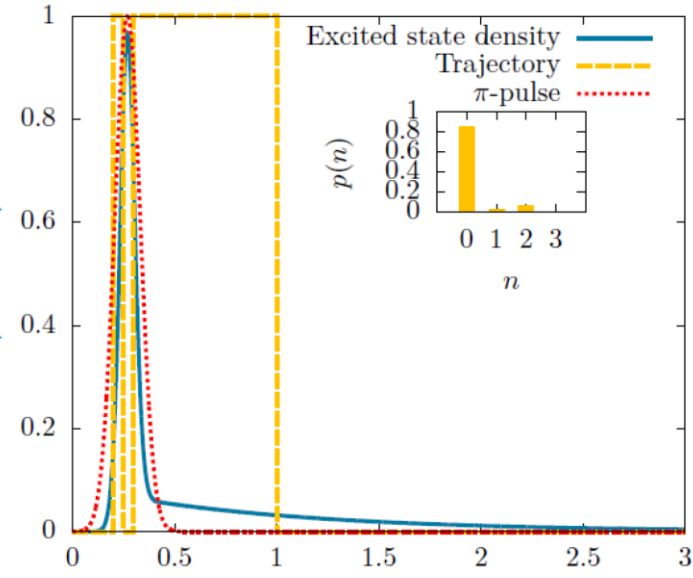


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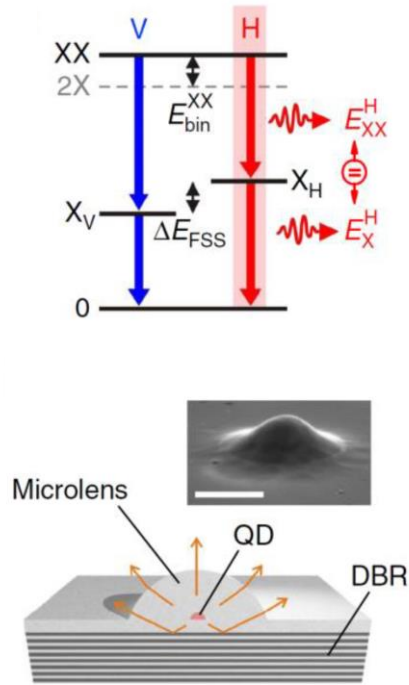
$$\langle \sigma + \sigma \rangle$$



K. Fischer et al., Nat. Phys. 13, 649 (2017)

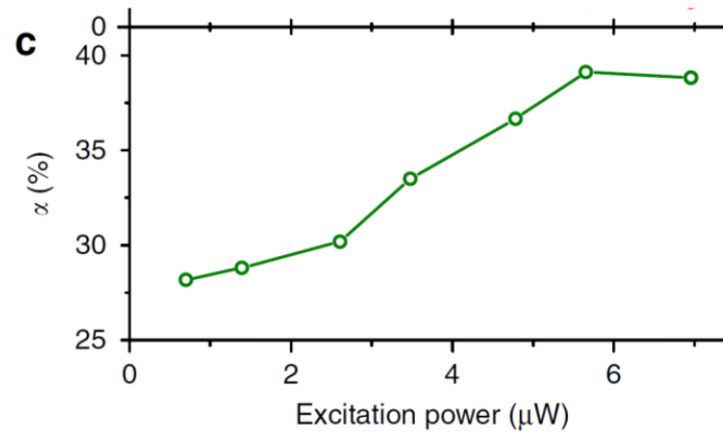
Single laser-pulsed quantum performs as a dynamical tunable two-photon source

# Example of Markovian quantum optical signals

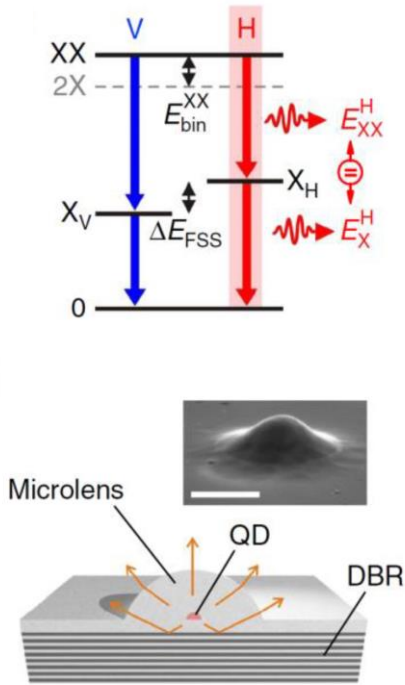


Ideal two-photon source of a quantum dot microlens with high brightness and fast repetition rate

Heindel, AC et al, Nat. Comm. 8, 14870 (2017)

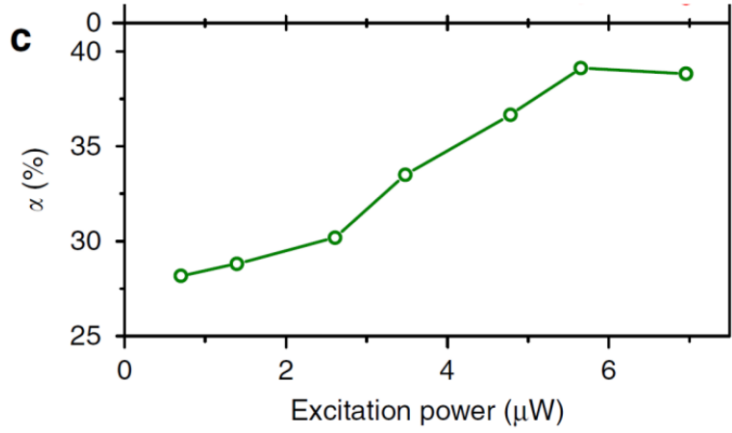


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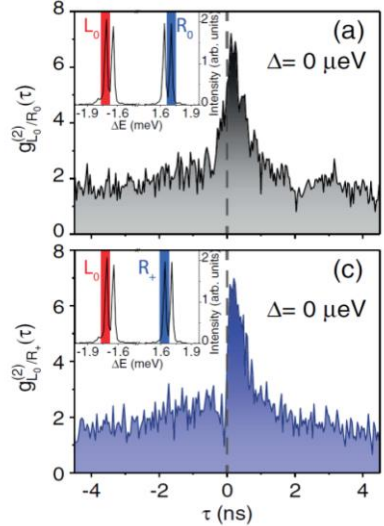
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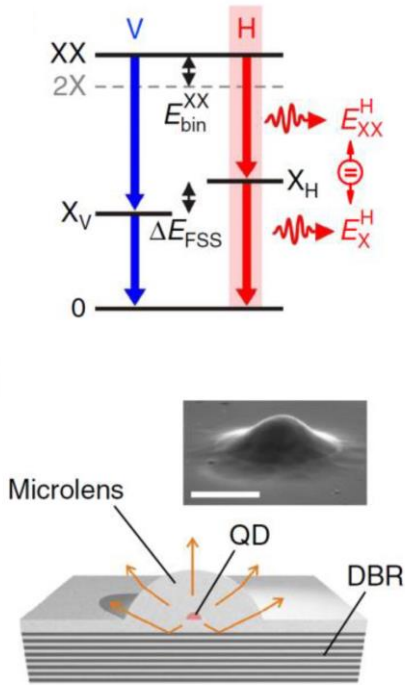


Time-reordered photon pairs of a biexciton driven at two-photon resonance

Bounouar, AC et al, PRL 118, 233601 (2017)

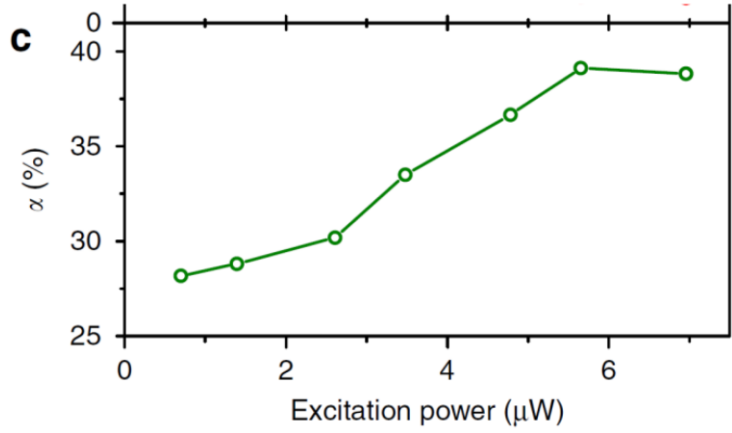


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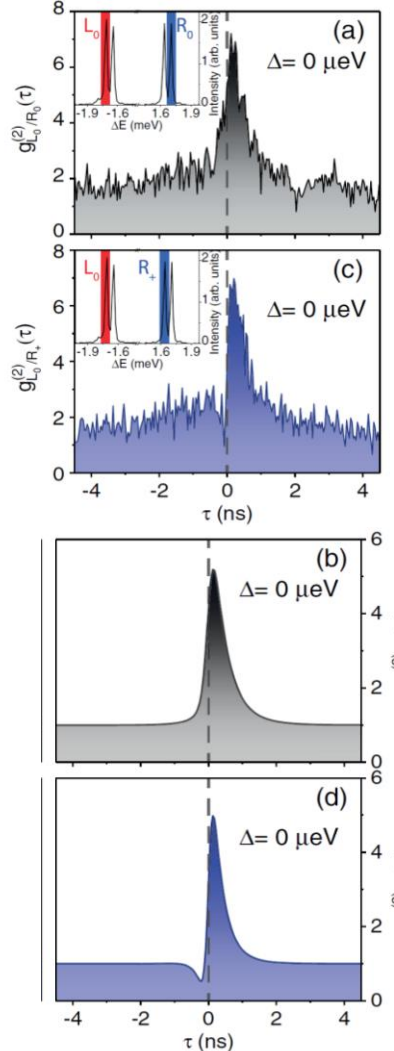
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$$\partial_t \rho(t) = -\frac{i}{\hbar} [H(t), \rho(t)] + \sum_{\alpha} L[C_{\alpha}] \rho(t)$$

$$L[C] = C \rho C^{\dagger} - \frac{1}{2} C^{\dagger} C \rho + \frac{1}{2} \rho C^{\dagger} C$$

Experimental data satisfactorily and analytically explained by Markovian, Lindblad master equation

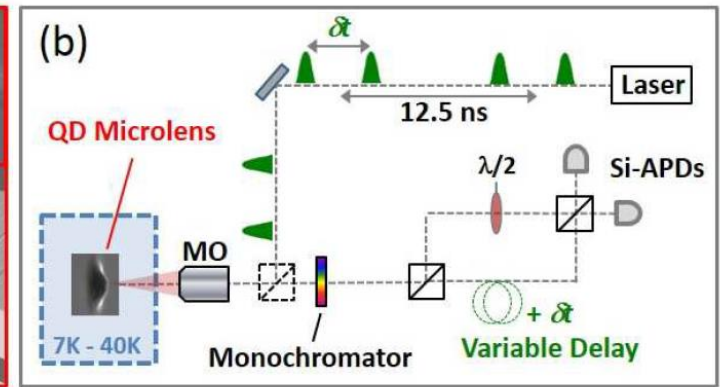
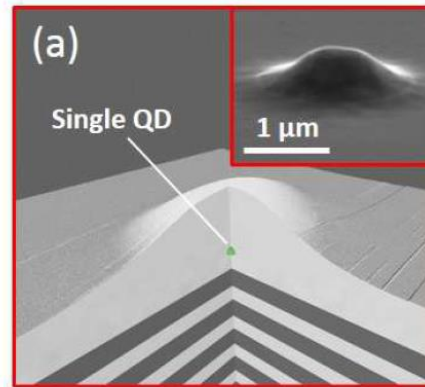
Schleibner, AC et al, arXiv:1710.03031



# Example of non-Markovian quantum optical signals

Pulsed Hong-Ou-Mandel experiments on single quantum dots allow to monitor the memory depth of environment fluctuations

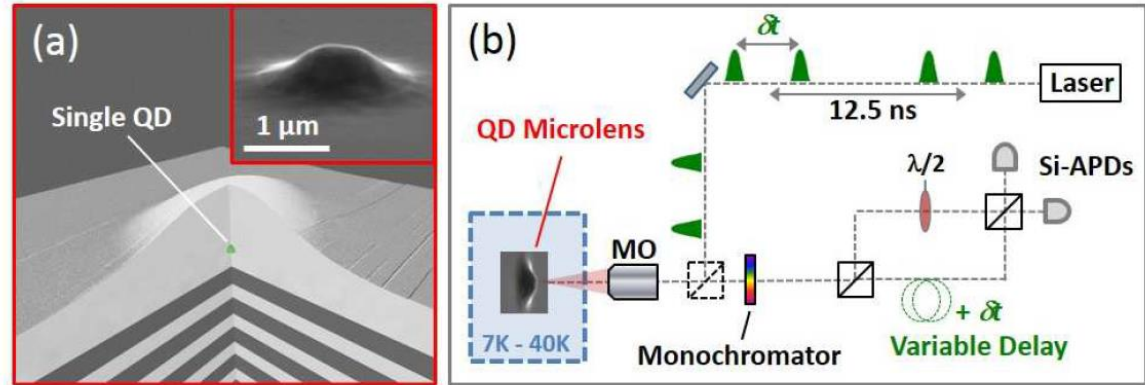
Thoma, AC et al, PRL 116, 033601 (2016)



$$\mathcal{H}_I = \Omega(t)(\sigma_{eg} + \sigma_{ge}) + g \int d\omega e^{i(\omega - \omega_e)t - i\phi_0(t)} c_\omega \sigma_{ge} + e^{-i(\omega - \omega_e)t + i\phi_0(t)} \sigma_{eg} c_\omega$$

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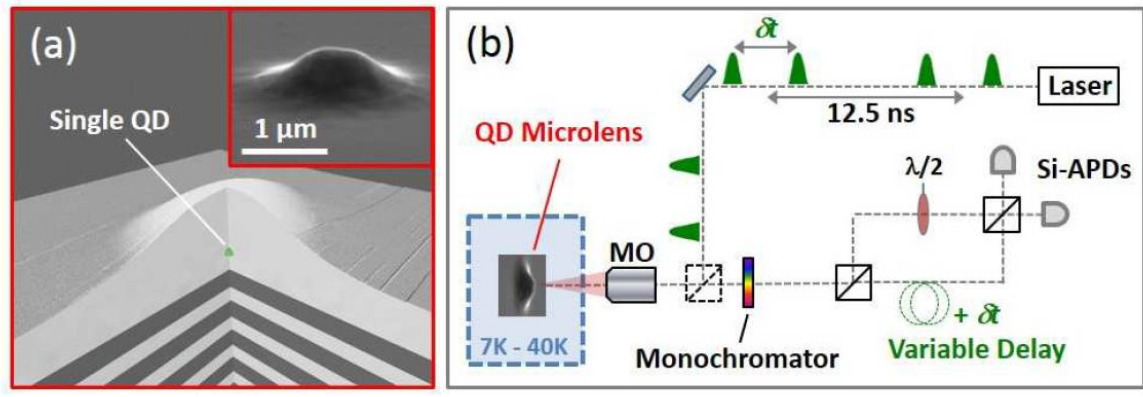
Figure of merit: two-times correlation function (for perfect indistinguishability  $\rightarrow 0$ )

$$G^{(2)}(t_D, \tau) = g^4 \pi^4 e^{-\Gamma(2t_D + \tau)} \cdot \left[ \mathcal{T}^2 + \mathcal{R}^2 - 2\mathcal{RT} \operatorname{Re} \left[ \left\langle e^{-i\phi(t_D + \tau) - i\phi_{\delta t}(t_D) + i\phi_{\delta t}(t_D + \tau) + i\phi(t_D)} \right\rangle \right] \right]$$

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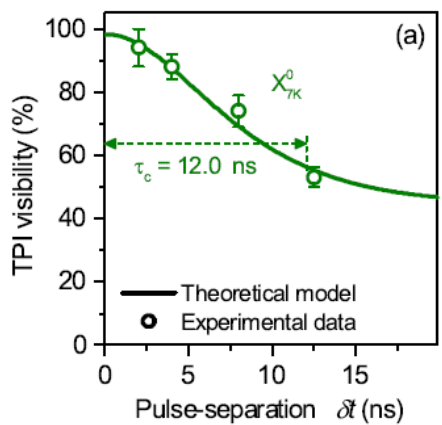
Thoma, AC et al, PRL 116, 033601 (2016)



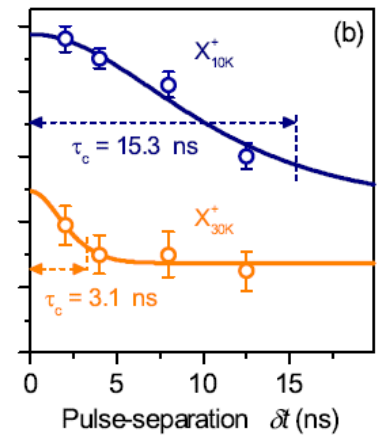
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However, for non-Markovian environment noise-induced dephasing depends on the pulse separation  $\rightarrow$  pulse separation allows to read-out material memory kernel

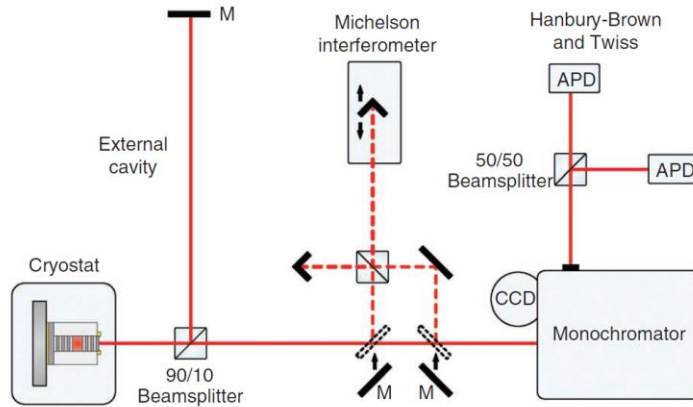


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# Non-Markovian quantum control

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## External mirror serves as control device

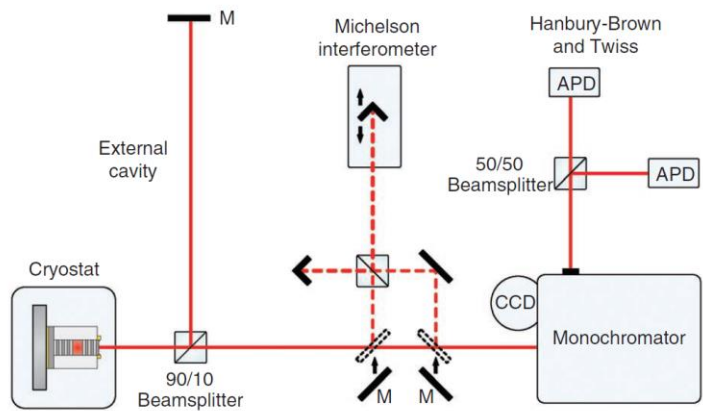


In principle (many photon, many emitter limit), to be modelled in a Lang-Kobayashi approach for the field amplitude

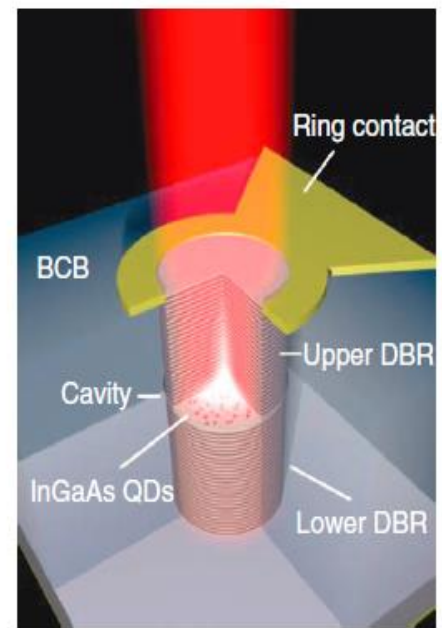
$$\mathcal{E} = \sqrt{N_{\text{ph}}(t)} e^{-i\phi(t)}$$

# Classical feedback control of semiconductor-based laser devices

External mirror serves as control device

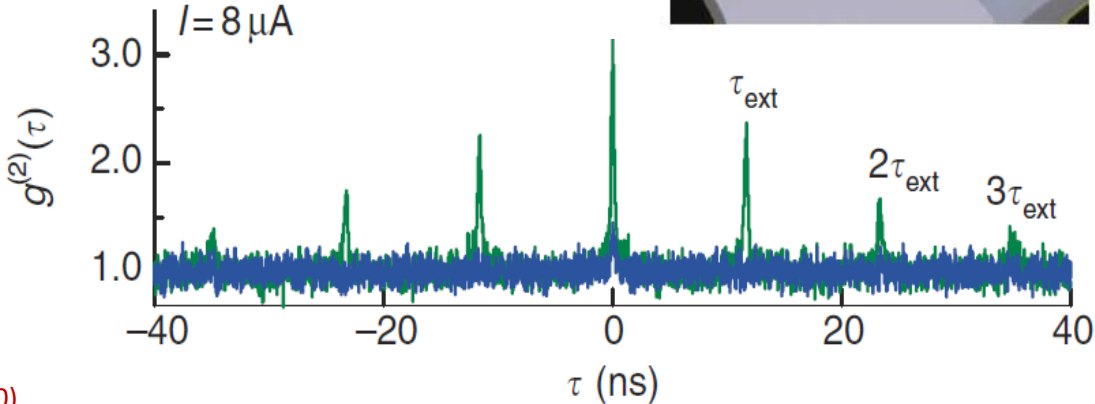


Feedback-induced nonlinear behavior in the output statistics of micropillar lasers



In principle (many photon, many emitter limit), to be modelled in a Lang-Kobayashi approach for the field amplitude

$$\mathcal{E} = \sqrt{N_{ph}(t)} e^{-i\phi(t)}$$



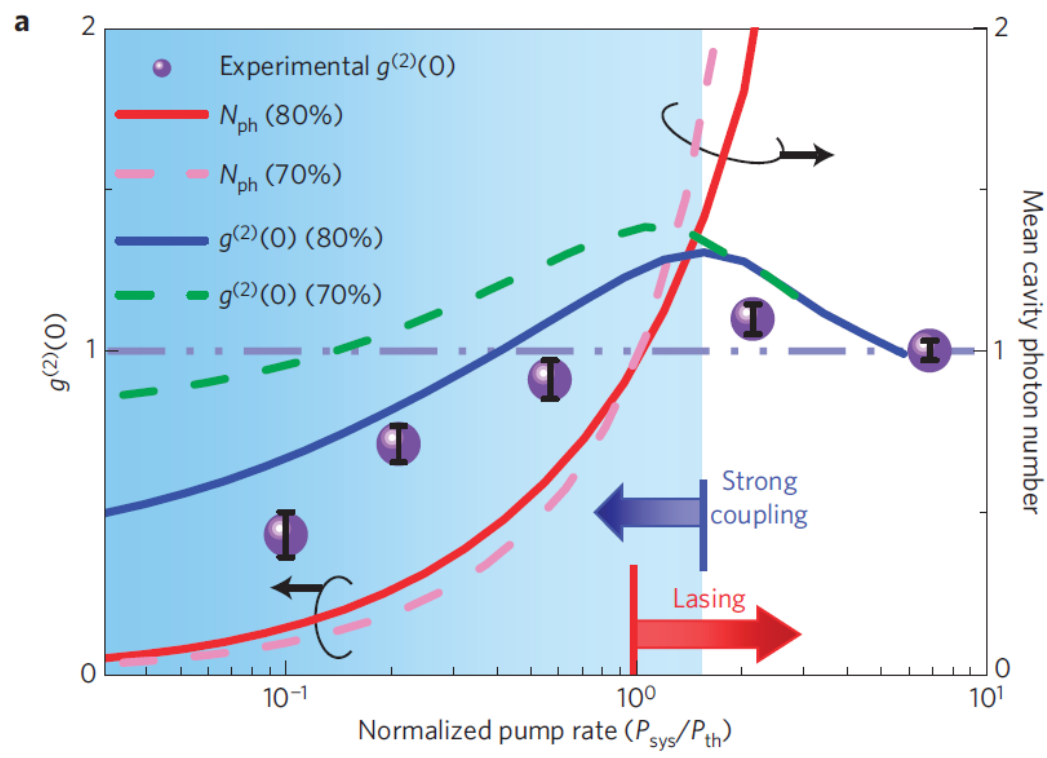
Lang, Kobayashi, IEEE J. Quantum Electron. 16, 347 (1980)

Albert et al, Nat. Comm. 2, 366 (2011)



Lasing in the non-classical regime  
(time-local correlation function):

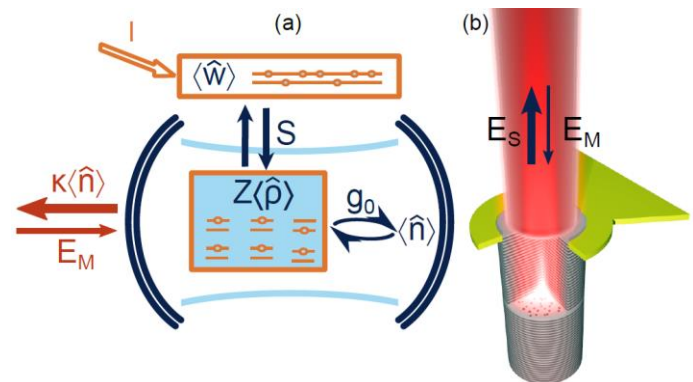
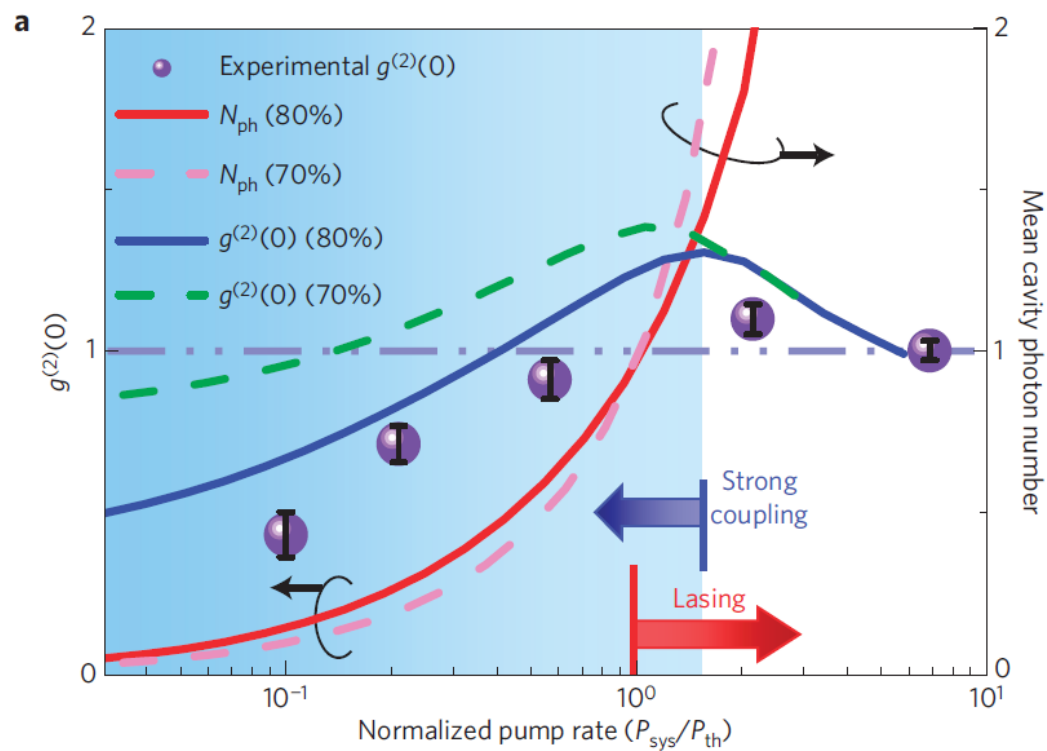
Nomura et al, Nat. Phys. 6, 279 (2010)



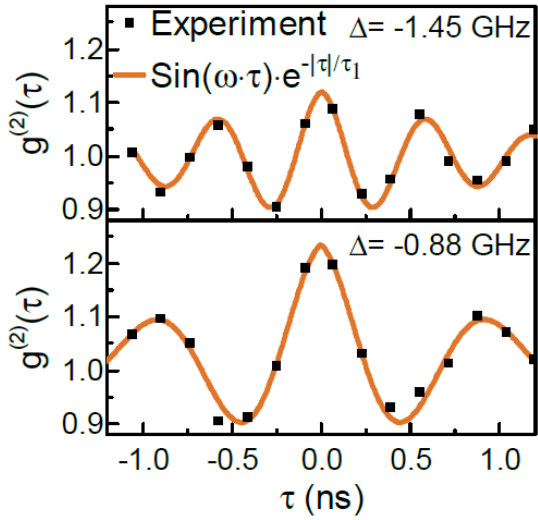
An incoherently-pumped single quantum dot exhibits a strong non-classical output

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Nomura et al, Nat. Phys. 6, 279 (2010)



Schlottmann et al, Phys. Rev. Appl. 6, 44023 (2016)



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An incoherently-pumped single quantum dot exhibits a strong non-classical output

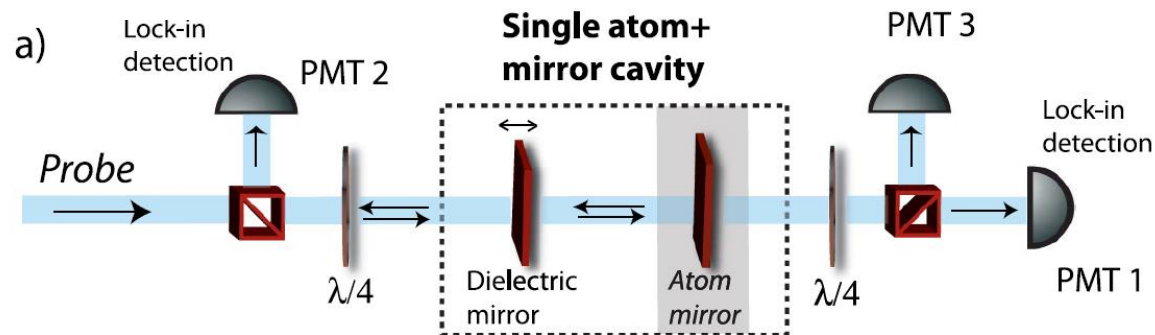
Few-emitter dynamics, even driven with classical field amplitudes leads quantum



Experiments on the single quanta level feedback coupling:

- Experiments with cold atoms

Single atom-mirror:

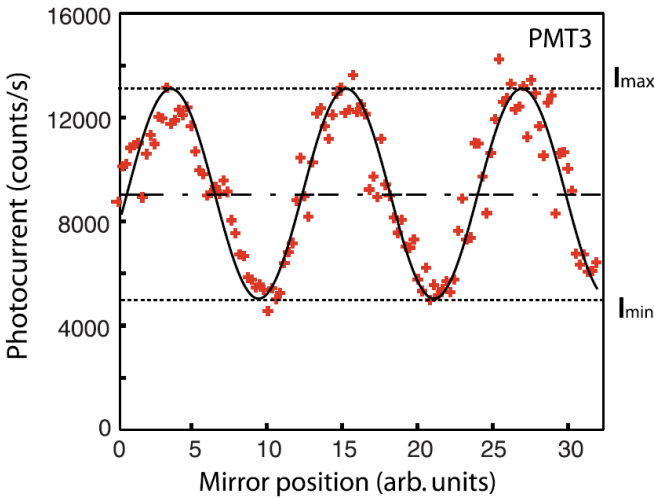
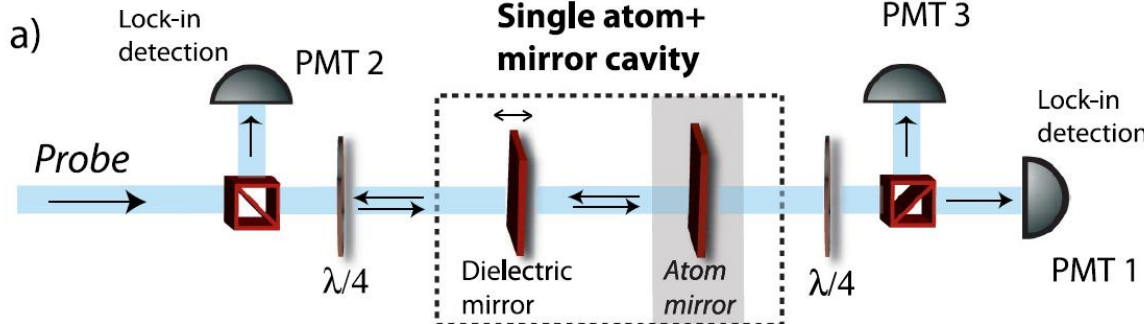


## Experiments on the single quanta level feedback coupling:

- Experiments with cold atoms

- Transmission controlled by the atom's position at length  $L$
- Sinusoidal dependence

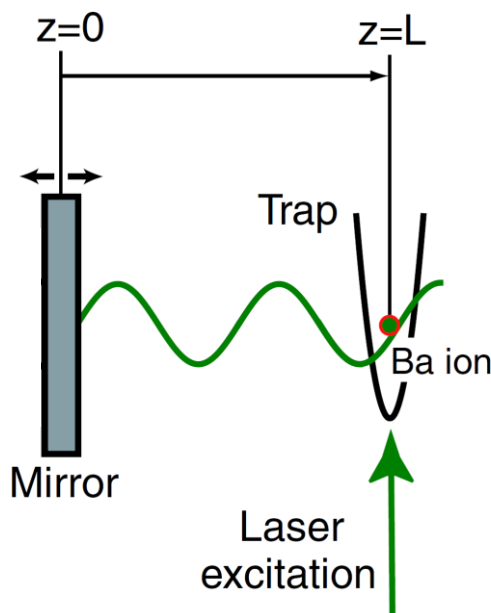
### Single atom-mirror:



G. Hetet et al, Phys. Rev. Lett. 107, 133002 (2011).

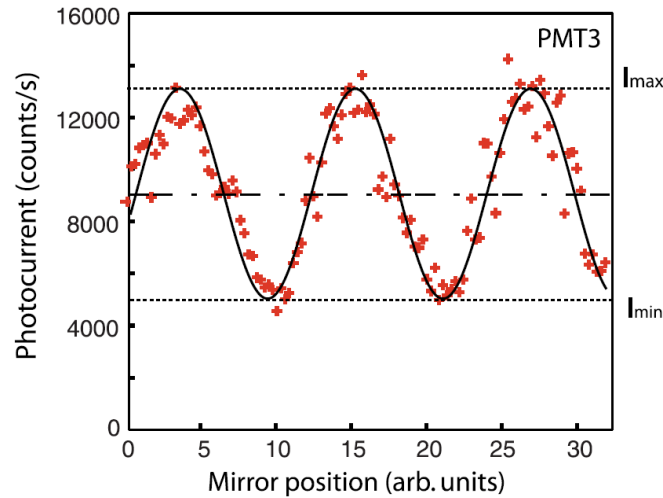
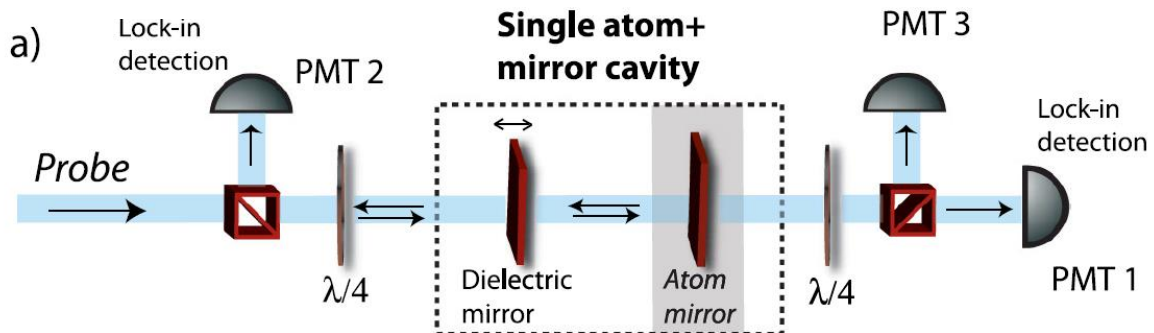
## Experiments on the single quanta level feedback coupling:

- Experiments with cold atoms
  - Dissipative dynamics of a laser-driven emitter, position dependent
  - Transmission controlled by the atom's position at length L
  - Sinusoidal dependence



F. Dubin et al, Phys. Rev. Lett. 98, 183003 (2007).

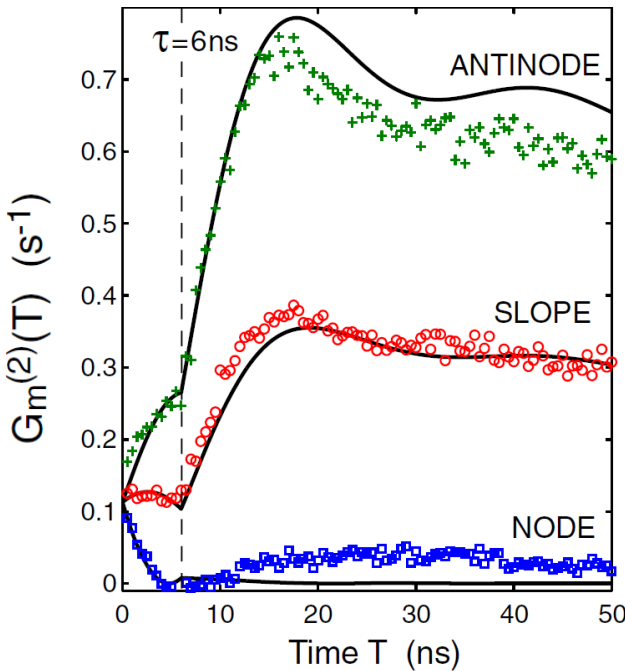
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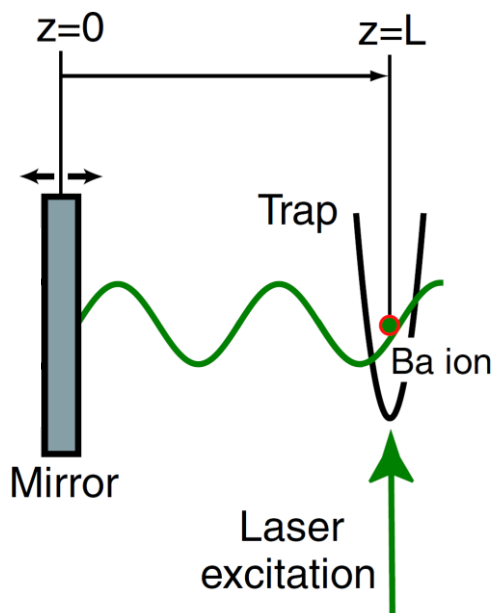
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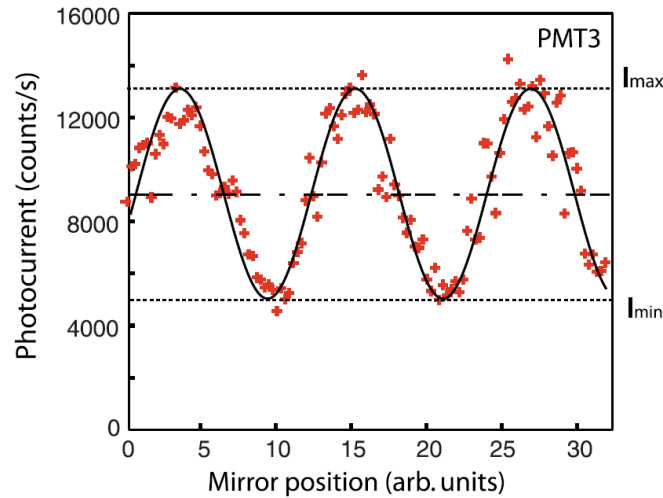
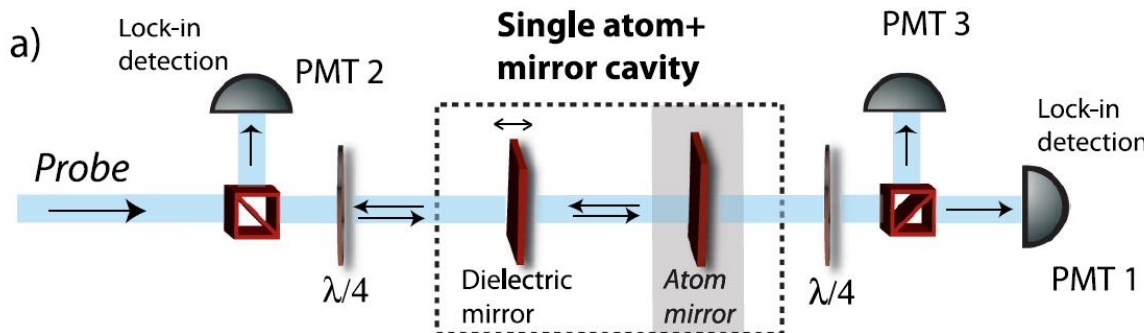


- Dissipative dynamics of a laser-driven emitter, position dependent
- Note kink in signal
- Transmission controlled by the atom's position at length  $L$
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F. Dubin et al, Phys. Rev. Lett. 98, 183003 (2007).

## Single atom-mirror:

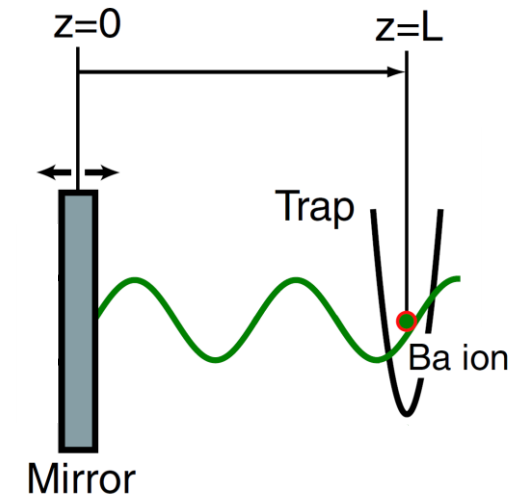


G. Hetet et al, Phys. Rev. Lett. 107, 133002 (2011).

Theoretical modelling of quantum feedback:

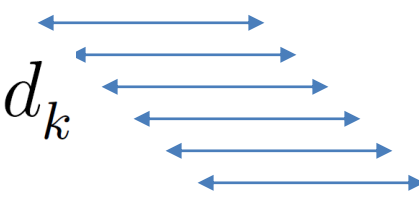
Assume a system which couples to an ensemble of two-level emitters via a structured reservoir

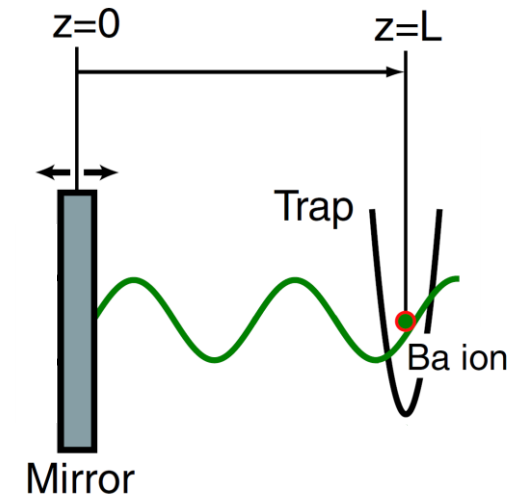
$$\left( \begin{array}{c} \text{---} \\ \text{---} \end{array} c^\dagger \right)$$



Theoretical modelling of quantum feedback:

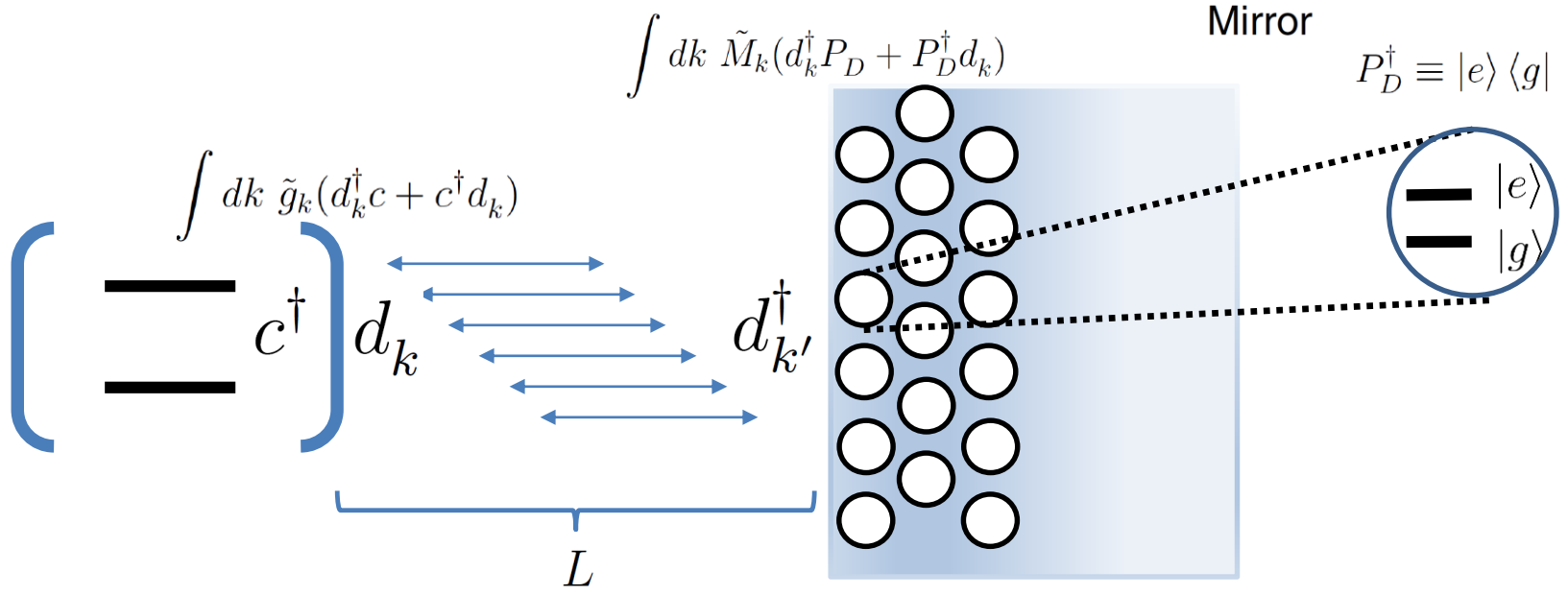
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$$\left( \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} c^\dagger \right) \int dk \tilde{g}_k (d_k^\dagger c + c^\dagger d_k)$$




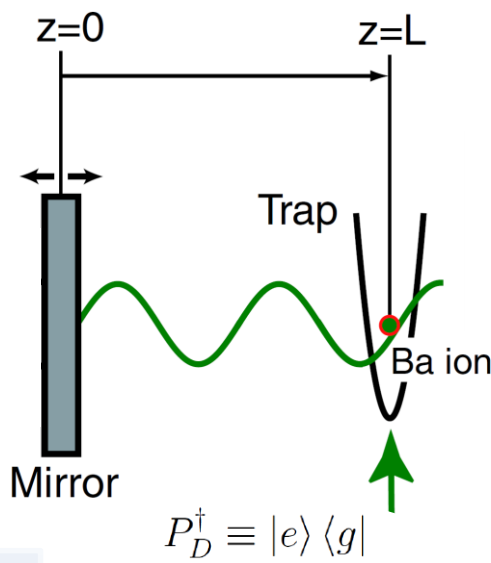
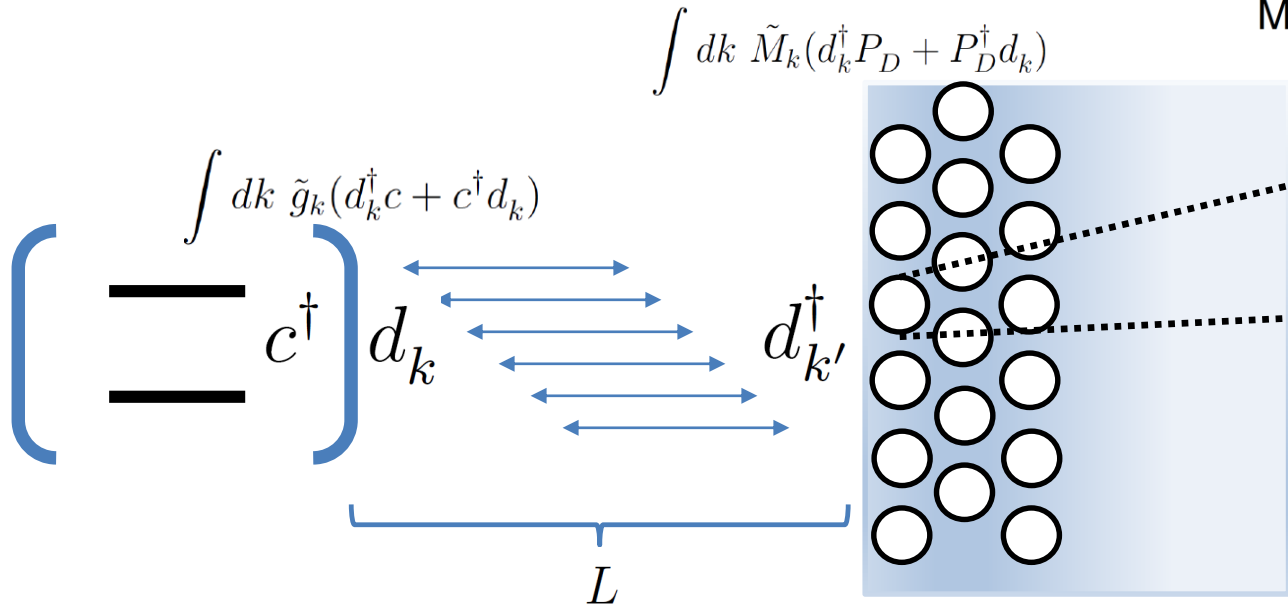
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Theoretical modelling of quantum feedback:

Assume a system which couples to an ensemble of two-level emitters via a structured reservoir



Eliminate the two-level systems and the reservoir to yield an effective equation of motion of Pyragas type

$$\dot{c} = -(i\omega_c + \Gamma) c(t) - iM P + \Gamma_\tau c(t - \tau) \Theta(t - \tau) - i\Delta B(t)$$

→ Effective Hamiltonian



Equation of motion reproduced via:

$$H/\hbar = \omega_0 c^\dagger c + \int dk \omega_k d_k^\dagger d_k + \int dk g_k \sin(kL) (d_k^\dagger c + c^\dagger d_k)$$

and employing Heisenberg equation of motion  $-i\hbar \frac{dO}{dt} = [H, O]$

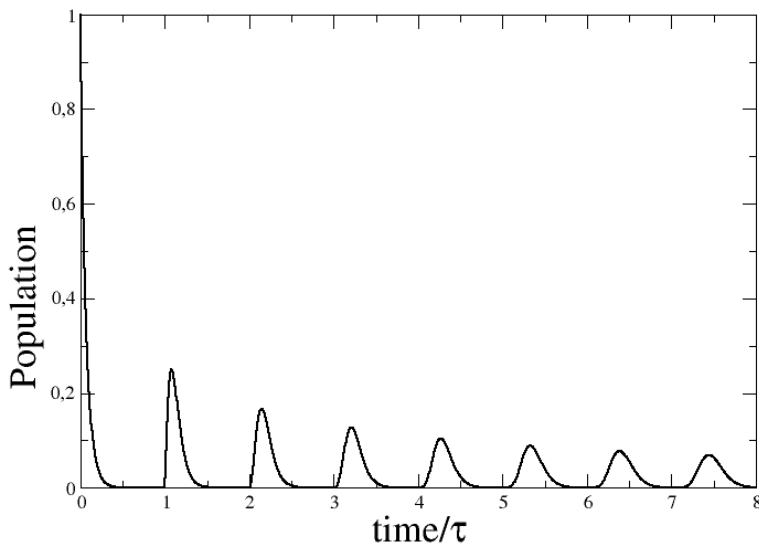
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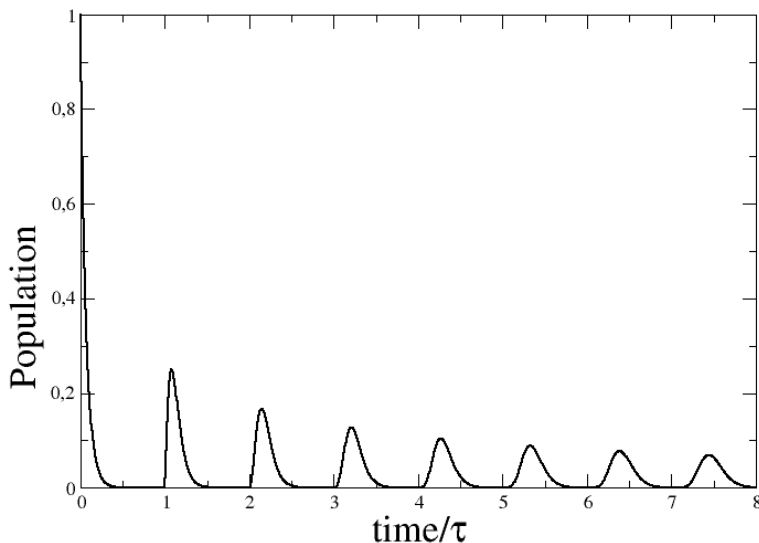
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$$\Gamma = \pi g_0^2 / c.$$



Feedback strength (depends on the interaction element system-bath coupling in the Hamiltonian)

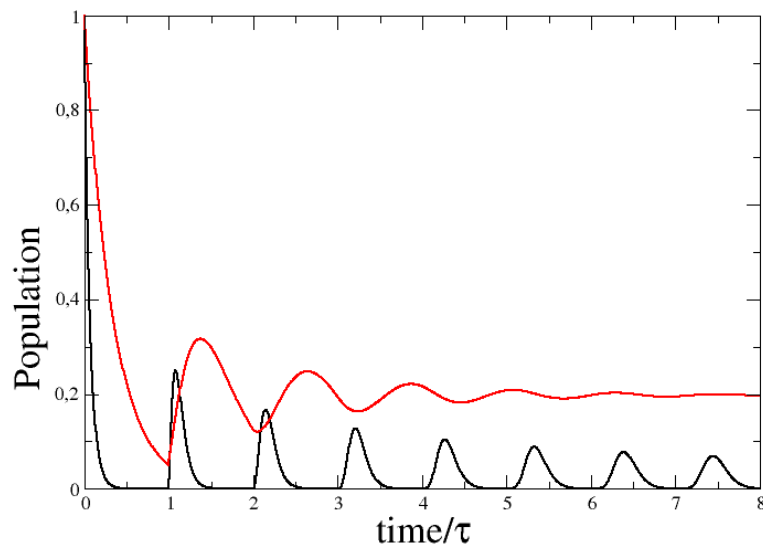
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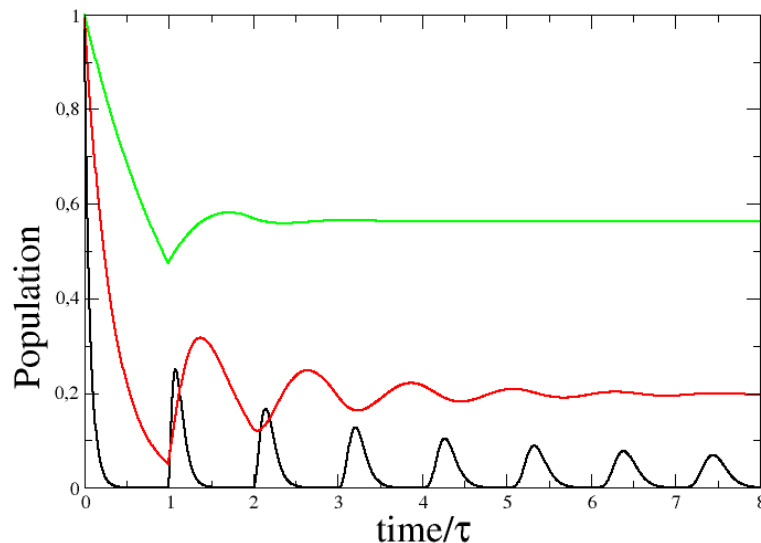
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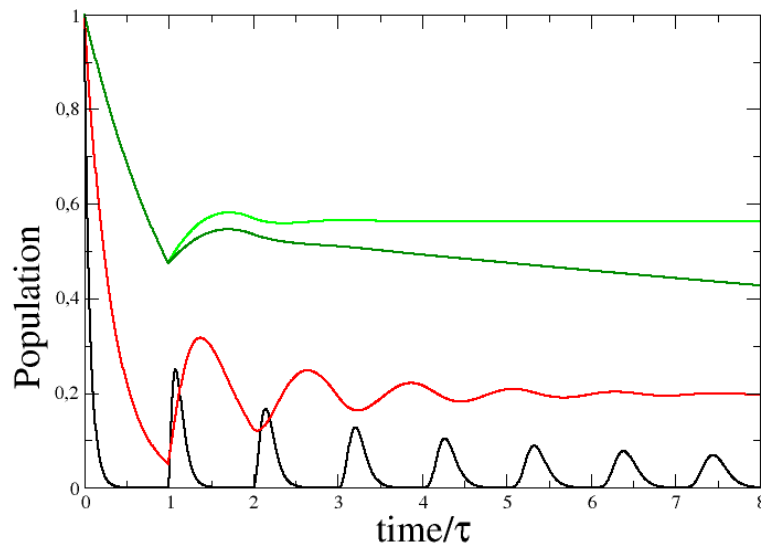
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$$\Gamma = \pi g_0^2 / c, \quad \Gamma_\tau = \Gamma \exp[i\omega_0 \tau]$$



Feedback strength (depends on the interaction element system-bath coupling in the Hamiltonian)

Phase of feedback signal (depends on the round trip time and the transition frequency of the system operator)

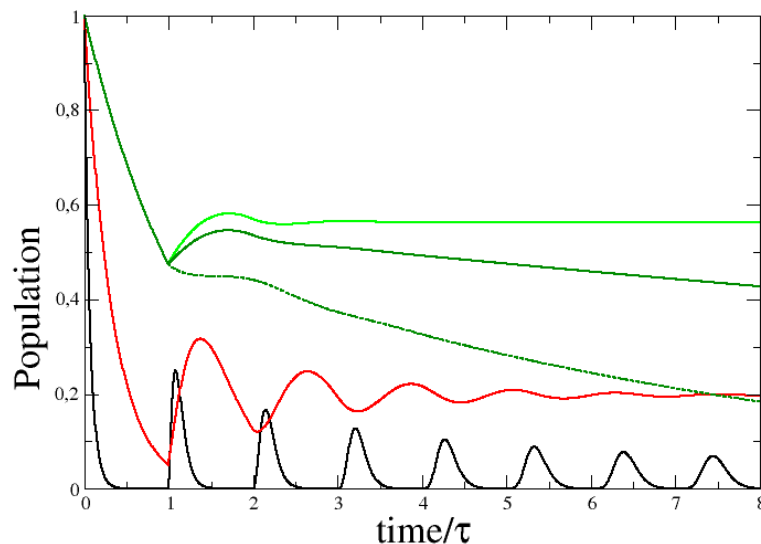
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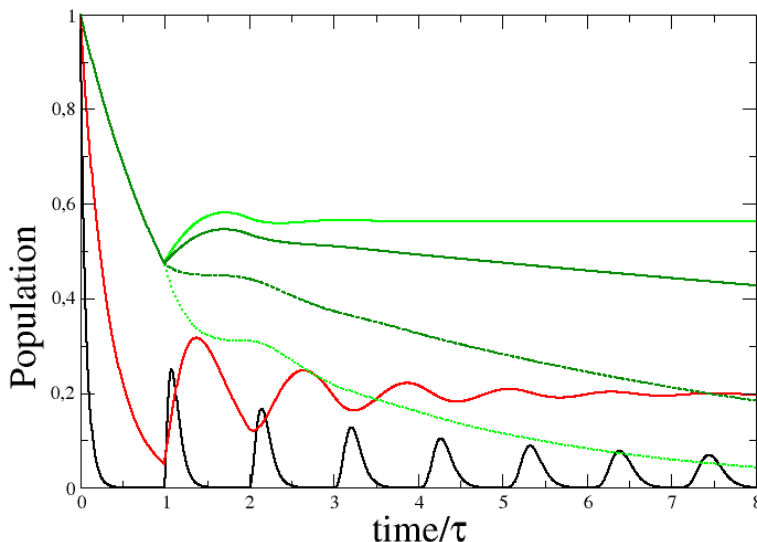
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and employing Heisenberg equation of motion  $-i\hbar \frac{dO}{dt} = [H, O]$

$$\dot{c} = -(i\omega_0 + \Gamma) c(t) + \Gamma_\tau c(t - \tau) \Theta(t - \tau) - i\Delta B(t)$$

$$\Gamma = \pi g_0^2 / c, \quad \Gamma_\tau = \Gamma \exp[i\omega_0 \tau]$$



Feedback strength (depends on the interaction element system-bath coupling in the Hamiltonian)

Phase of feedback signal (depends on the round trip time and the transition frequency of the system operator)



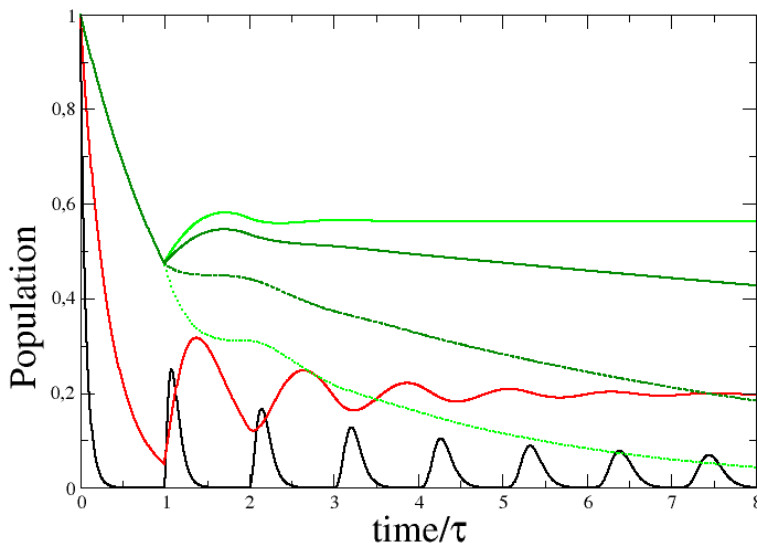
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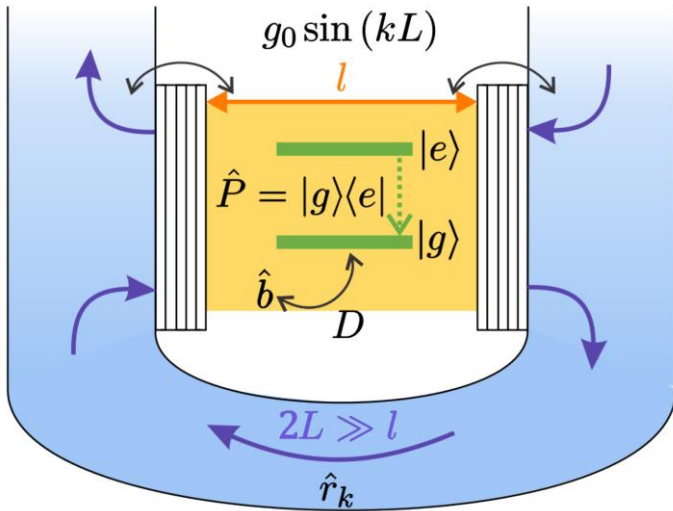
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# Suppression of decoherence via quantum feedback-stabilized acoustic cavities

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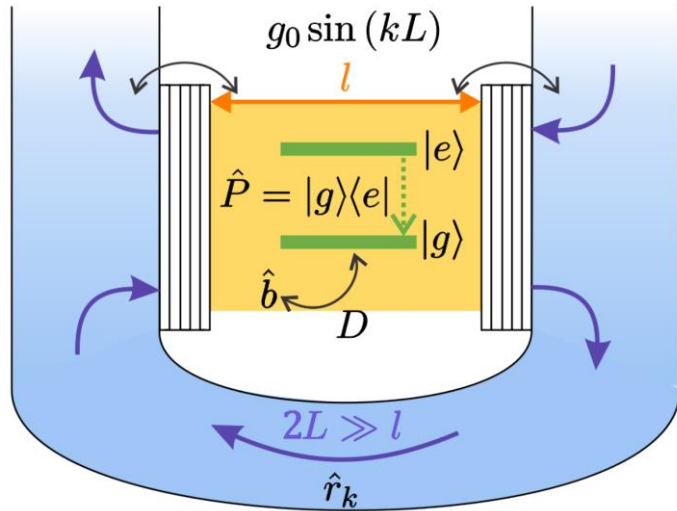
# Motivation (ii): Good model to investigate non-Markovian dynamics



Goal: Stabilize an initial given coherence even in the presence of a reservoir at finite temperature

$$\hat{H} = \hat{H}_S + \hat{H}_R + \hat{H}_{LB}(\hat{b}, \hat{b}^\dagger, \hat{P}_i, \hat{P}_i^\dagger)$$

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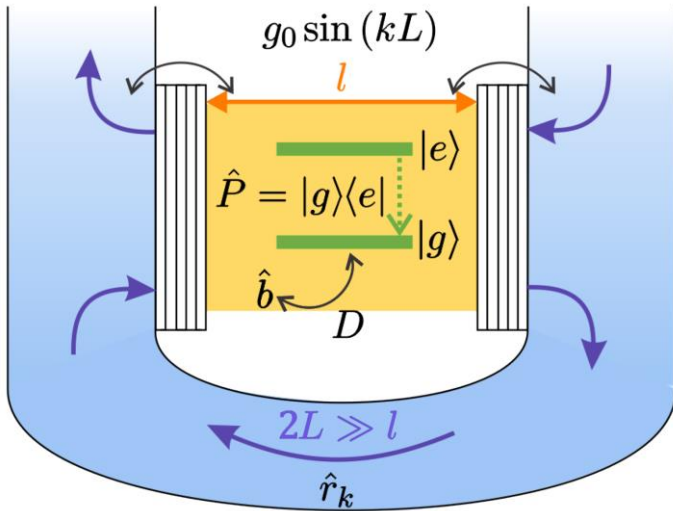
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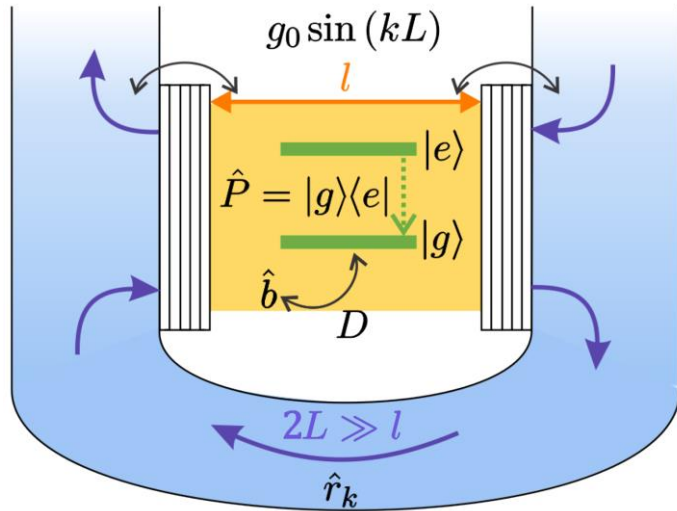
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We assume a reservoir at  $T > 0$  with non-Ohmic spectral density with delay

$$J(\omega_k) = \sin^2 \left( \frac{\omega_k \tau}{2} \right) e^{-i\omega_k(t-t')}$$

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$$J(\omega_k) = \sin^2 \left( \frac{\omega_k \tau}{2} \right) e^{-i\omega_k(t-t')}$$

Due to the linear coupling between the acoustic cavity mode and the reservoir, an exact solution exist

$$\hat{b}(t) = F(t) \hat{b}(0) + \int G_k(t) \hat{r}_k(0) dk$$

In the linear regime, the system dynamics can be exactly evaluated via a Feynman-Vernon influence functional or Suzuki-Trotta expansion

With given initial conditions, the dynamics can be evaluated

$$\hat{\rho}_P(t) = \exp \left\{ \left( -i \int_0^t \hat{\mathcal{B}}(t_1) dt_1 - \frac{1}{2} \int_0^t \int_0^{t_1} [\hat{\mathcal{B}}(t_1), \hat{\mathcal{B}}(t_2)] dt_2 dt_1 \right) \hat{P}^\dagger(0) \hat{P}(0) \right\} \hat{\rho}_P(0)$$

Our figure of merit is the survival time of an initial introduced coherence, e.g. via an delta pulse

$$\eta(t) = \frac{|\langle \hat{P}(t) \rangle|^2}{|\langle \hat{P}(0) \rangle|^2}$$

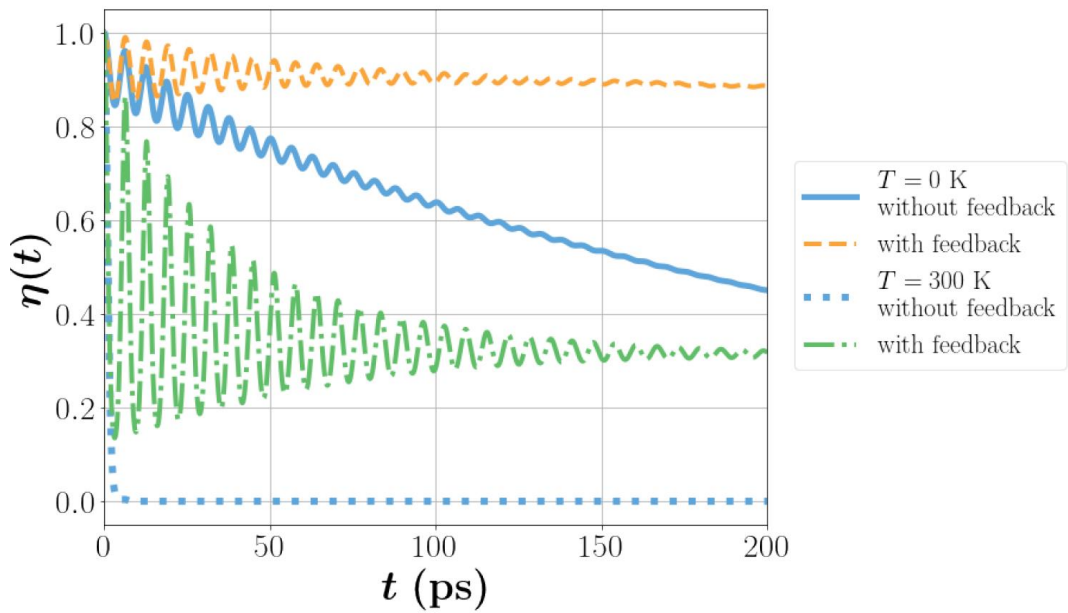


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Feedback stops via quantum interference the decoherence process – a synchronisation between the oscillators take place

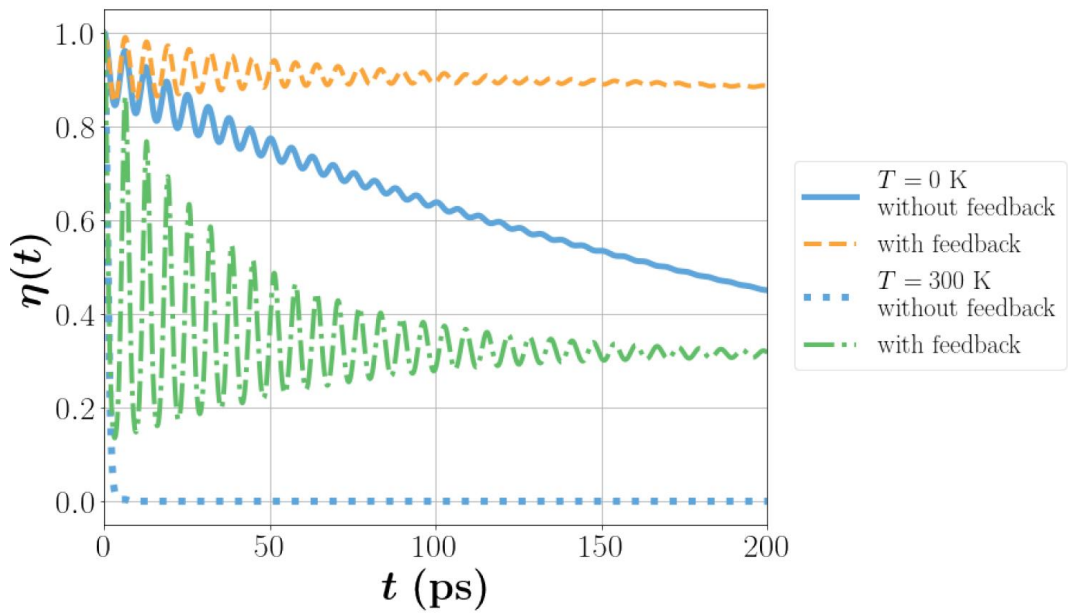


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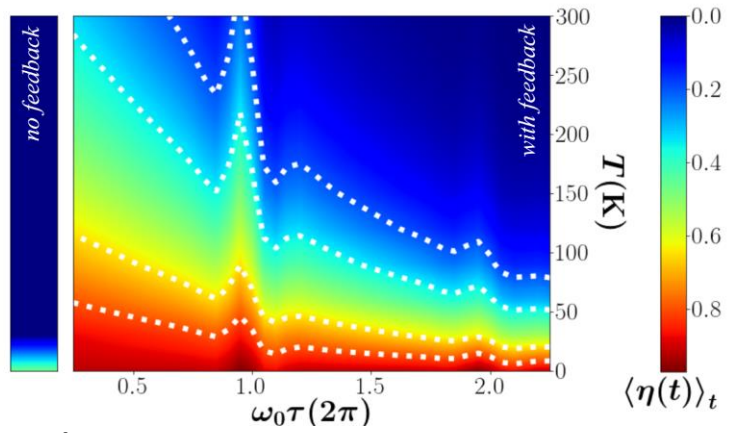
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Delay time and phase-matching allow very long coherence times  
 initial coherence at room temperature up to 200ps

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# Quantum Pyragas control – Two-photon purification of quantum light emission

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However, for open quantum system case dynamics, the model is too detailed in the bath description:

$$H/\hbar = \omega_0 c^\dagger c + \int dk \omega_k d_k^\dagger d_k + \int dk g_k \sin(kL) (d_k^\dagger c + c^\dagger d_k)$$

within the interaction picture  $H_I(t) = -i\hbar g_0 \left( c^\dagger \left[ \int dk (1 - e^{i2kL}) d_k e^{-i(\omega_k - \omega_0)t} \right] - \text{h.c.} \right)$

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and solve stroboscopically

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# Circumvent the limitations of the model

However, for open quantum system case dynamics, the model is too detailed in the bath description:

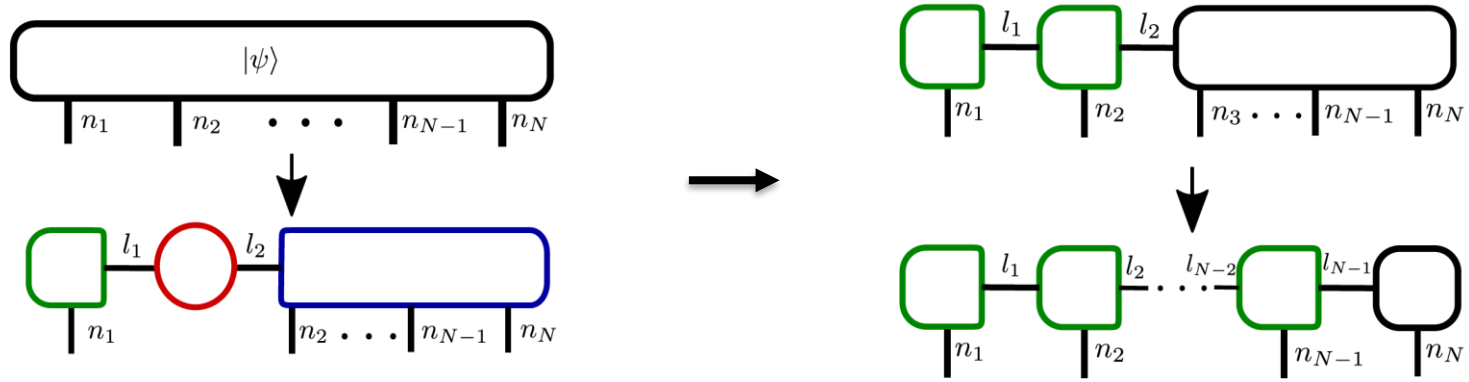
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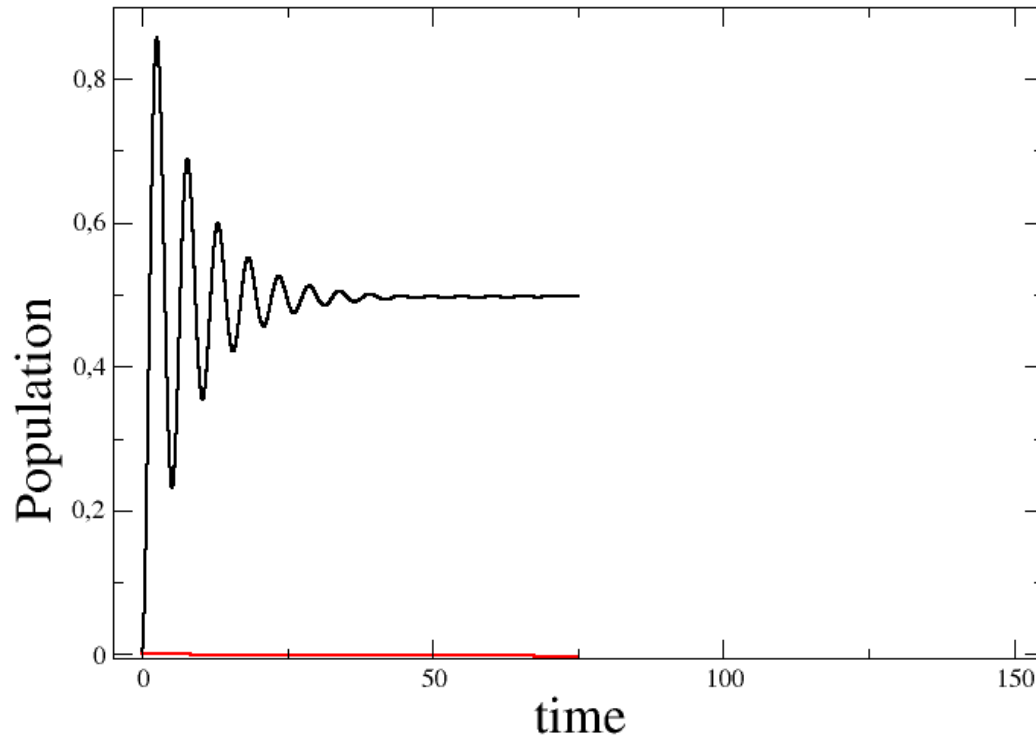


after SVD, yielding an MPS form

$$|\Psi\rangle = \sum_{i_1 \dots i_N} A_{i_1}^{[1]} \dots A_{i_N}^{[N]} |i_1\rangle \dots |i_N\rangle = \sum_{\mathbf{i}} A_{\mathbf{i}} |\mathbf{i}\rangle$$

Schrödinger equation yields reversible dynamics.  
Example: Driven and decaying two-level system.

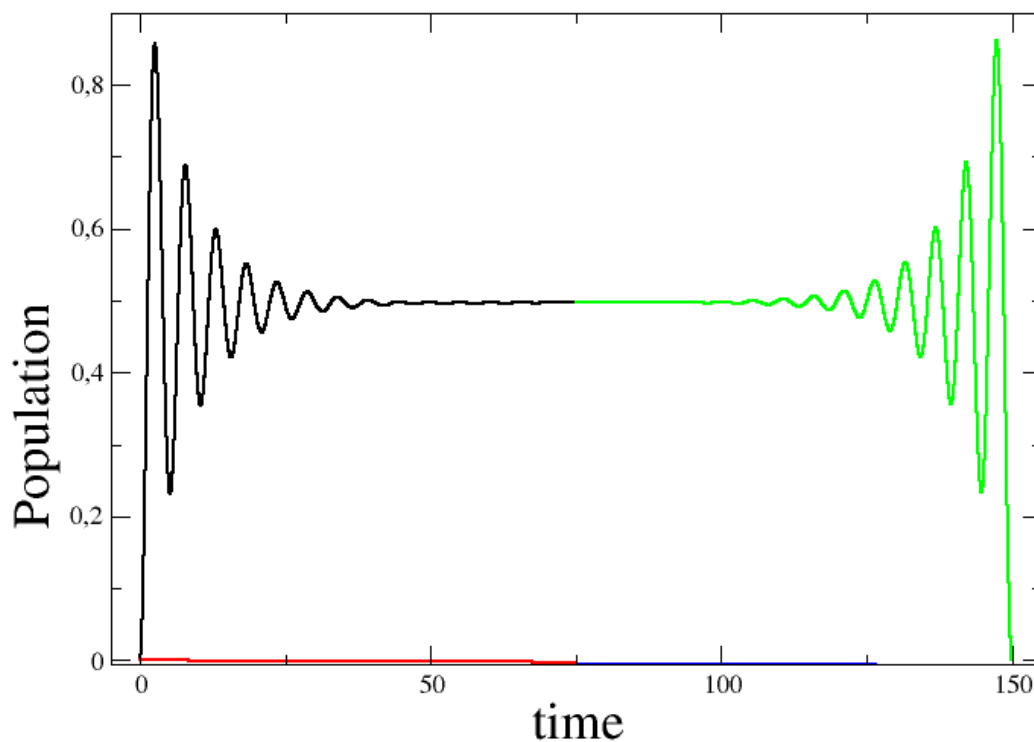
$$|\psi(n+1)\rangle = \exp \left[ -i\Delta t\Omega_L (\sigma^+ + \sigma^-) - \sqrt{\Gamma\Delta t}\sigma_- \Delta R^\dagger(n) \right] |\psi(n)\rangle$$





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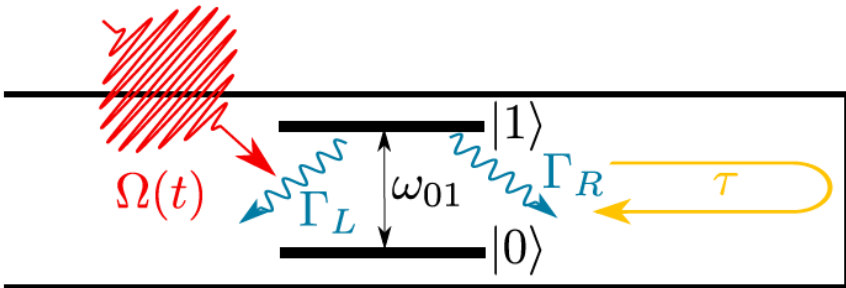
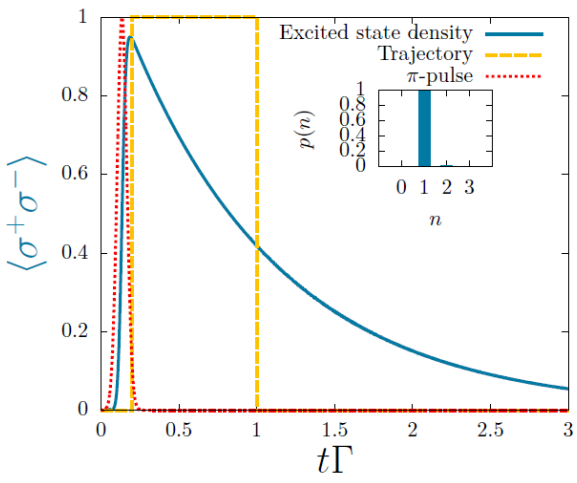


Time-reversal yields initial state. Full information of the reservoir in state. Numerical exact solution and dissipatively driven-correlation included.

$$\langle\psi(n-1)| = \langle\psi(n)| \exp \left[ i\Delta t\Omega_L (\sigma^+ + \sigma^-) - \sqrt{\Gamma\Delta t}\sigma_- \Delta R(n) \right]$$

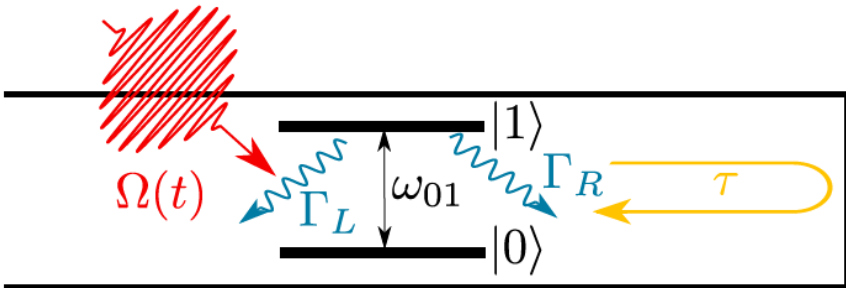
Pulsed and decaying two-level system.

Nearly perfect single photon emission for  $\pi$ -pulse

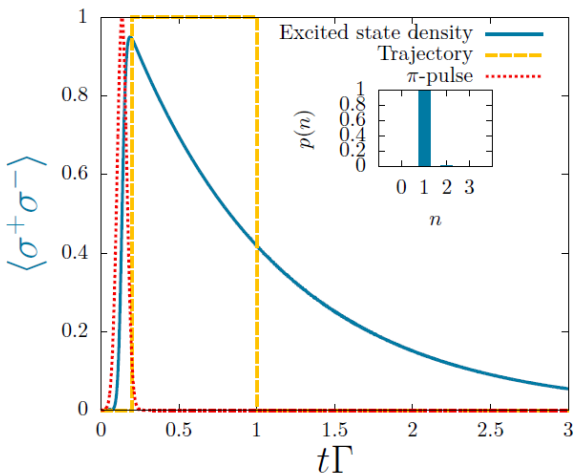


# Examples: (i) Two-photon purification

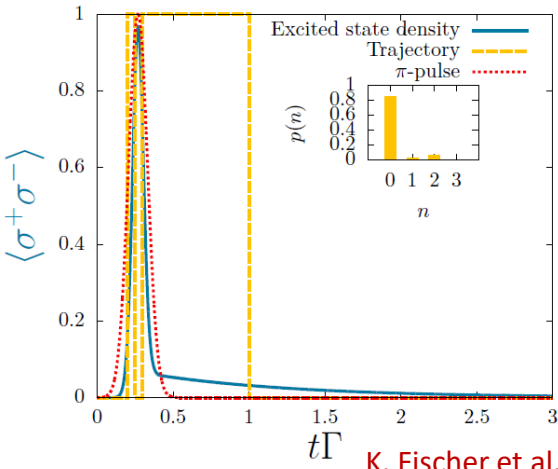
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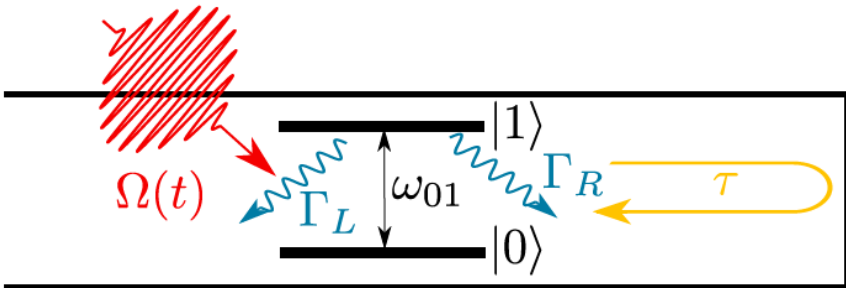


Two-photon emission events are favored for  $2\pi$ -pulses.

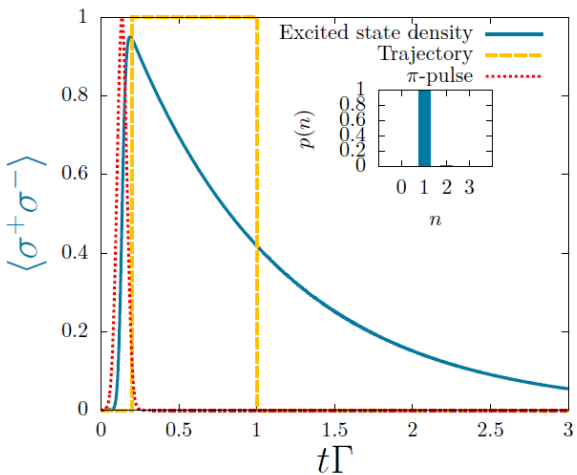


K. Fischer et al., Nat. Phys. 13, 649 (2017)

Pulsed and decaying two-level system.

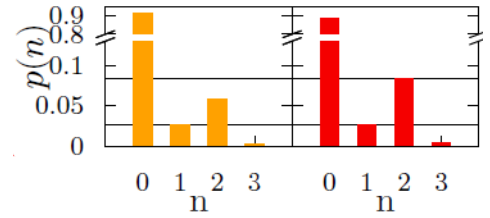
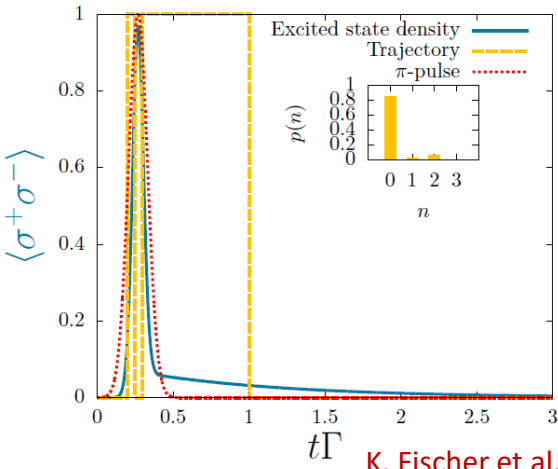


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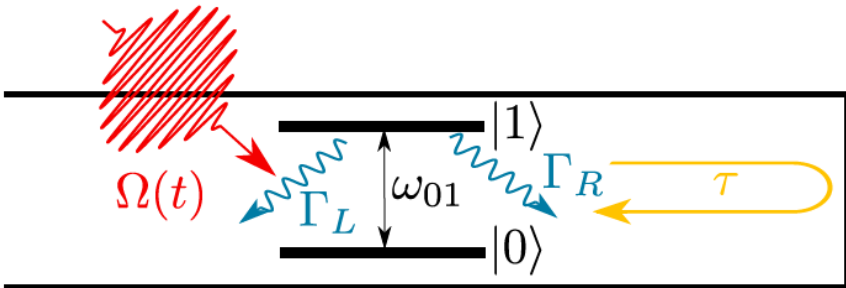


Quantum feedback allows to steer between purified two-photon and single photon emission, selectively.

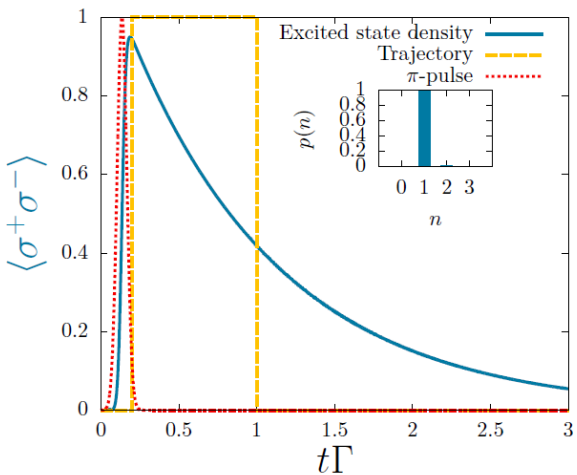
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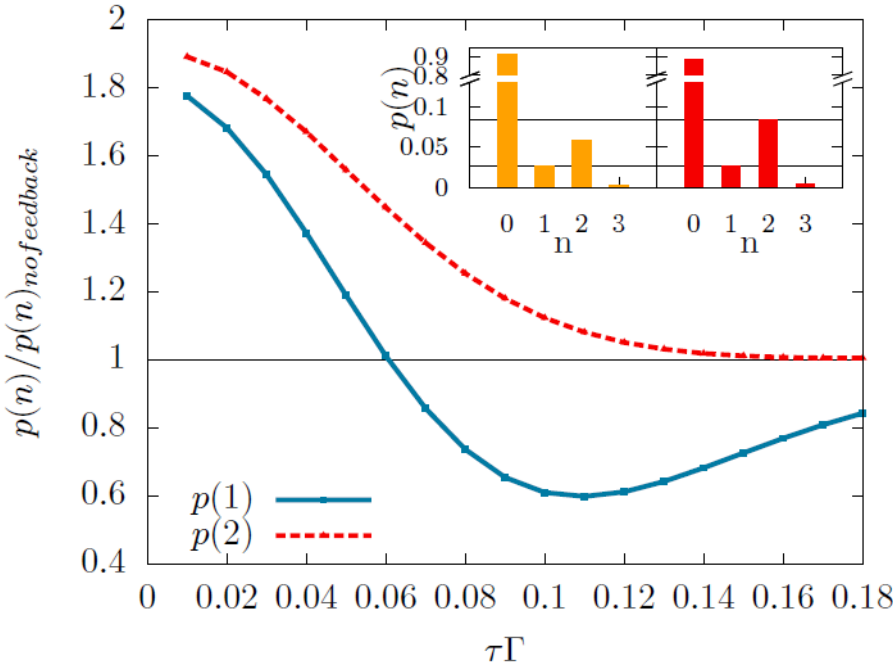
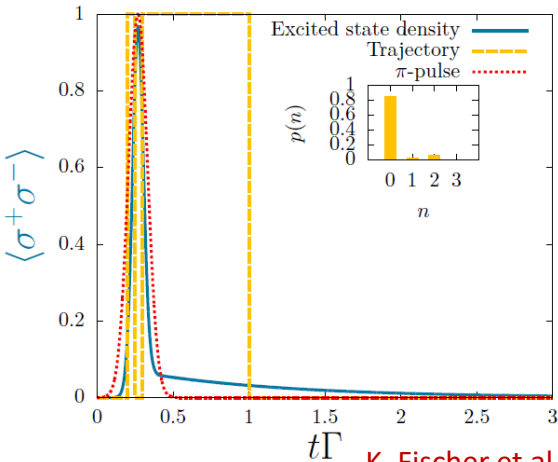


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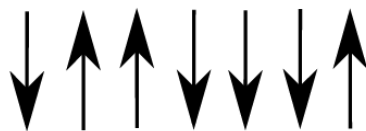
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# Feedback-stabilized discrete time crystal dynamics

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Illustration of a discrete time-crystal



$$\mathcal{H}_F = (\Omega \quad ) \sum_{i=1}^N \sigma_i^x$$

$$\langle M \rangle = \frac{1}{N} \sum_{i=1}^N (-1)^i \frac{\langle \sigma_i^z \rangle}{2}$$

If driving is perfect  $\epsilon=0$ , the magnetization shows a single peak in the Fourier spectra. Perfect periodicity.

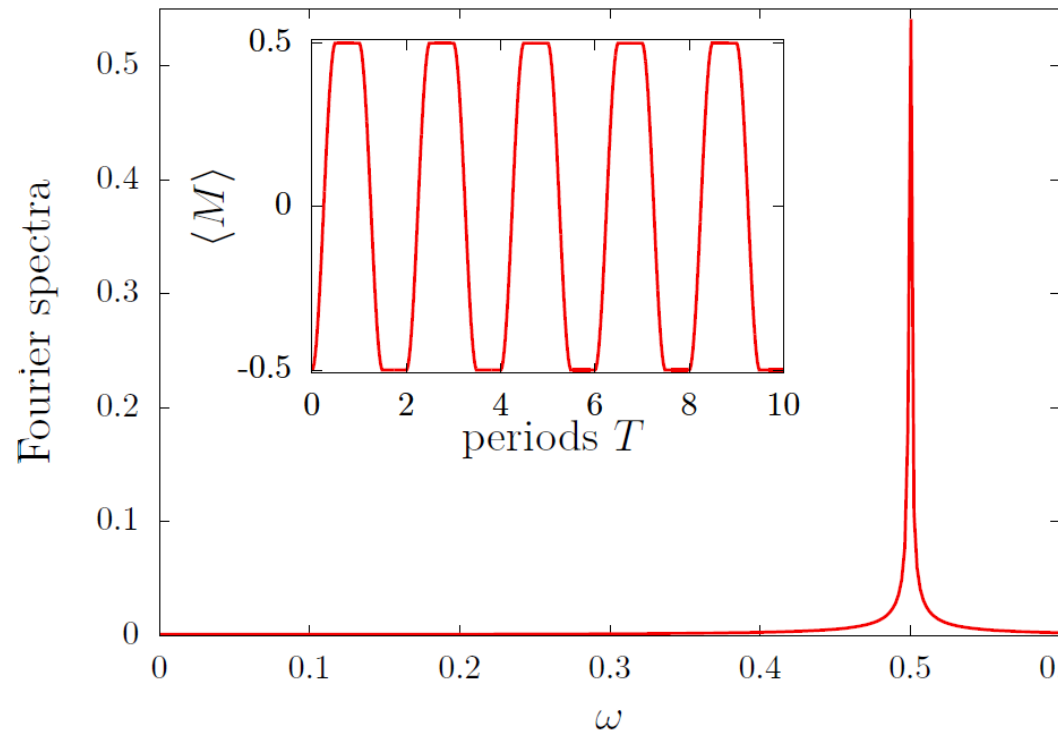
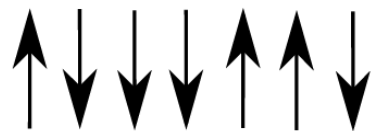




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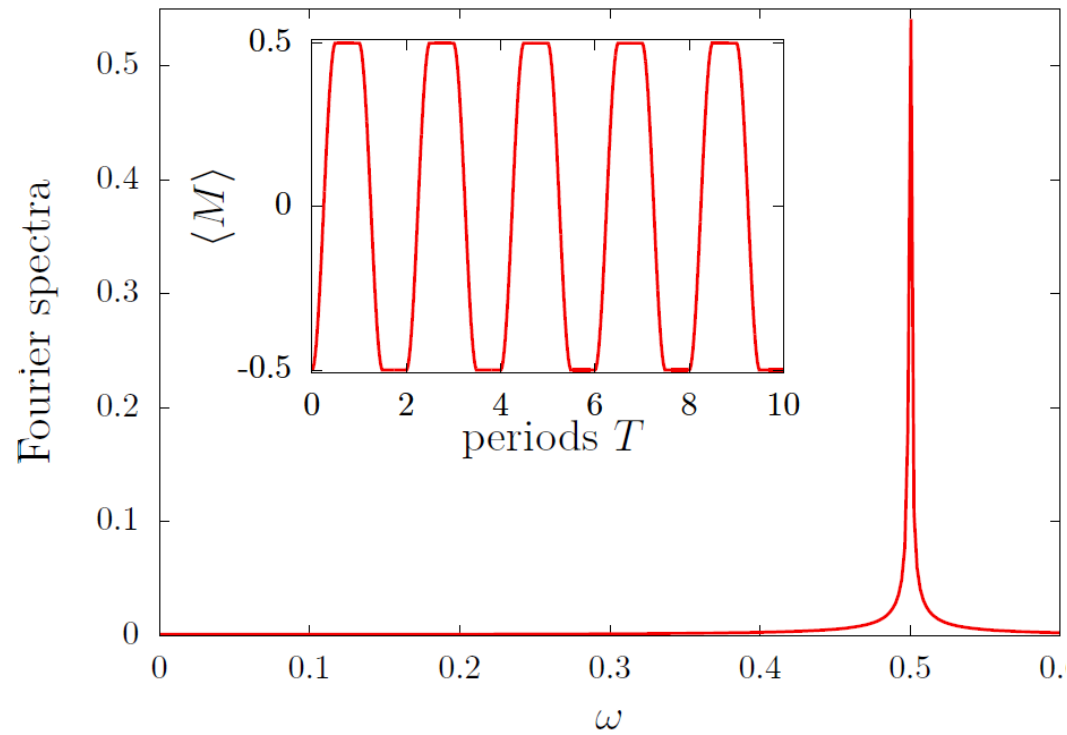
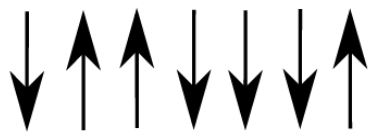






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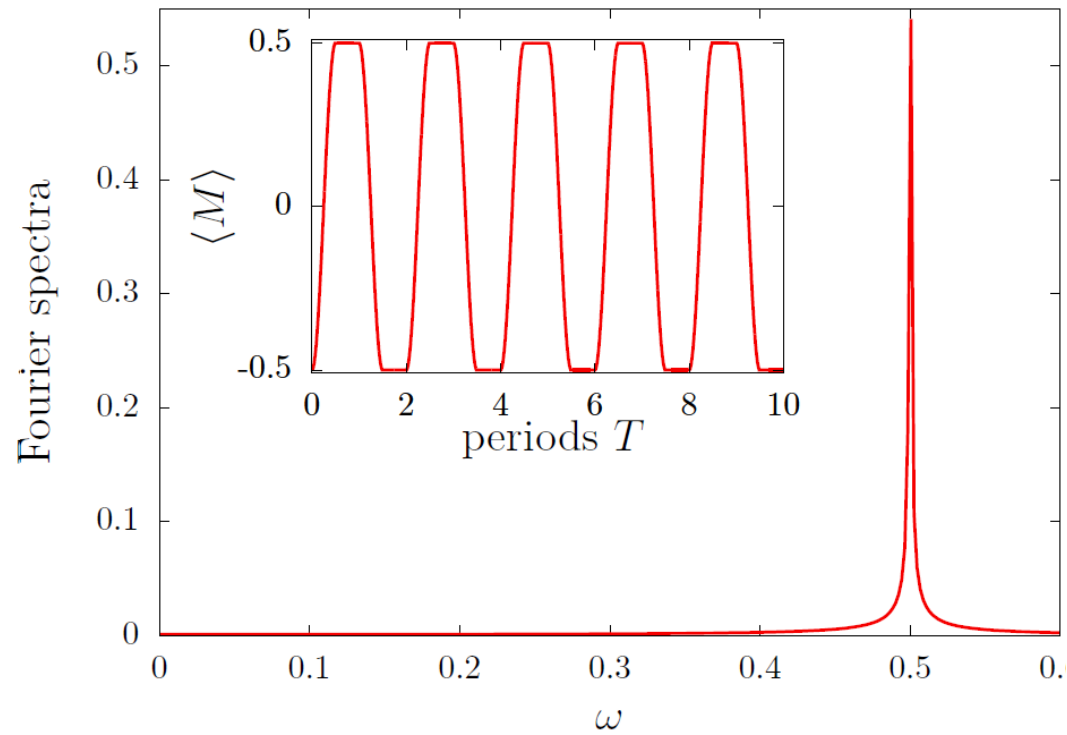
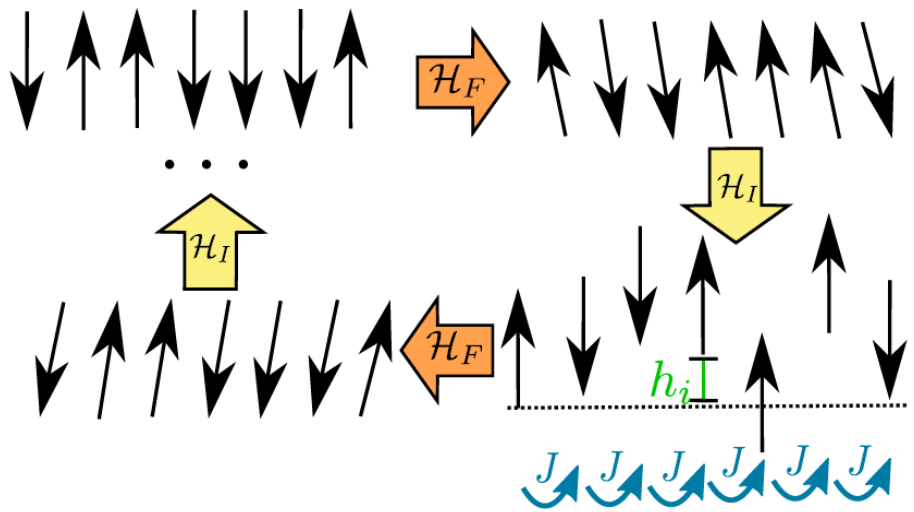


Illustration of a discrete time-crystal

$$\mathcal{H}_F = (\Omega - \epsilon) \sum_{i=1}^N \sigma_i^x$$



The spin chain of N spins returns despite **imperfect** rotation back to its initial state. Figure of merit and observable (staggered magnetization):

Yao et al, Phys. Rev. Lett. 118, 030401 (2017)

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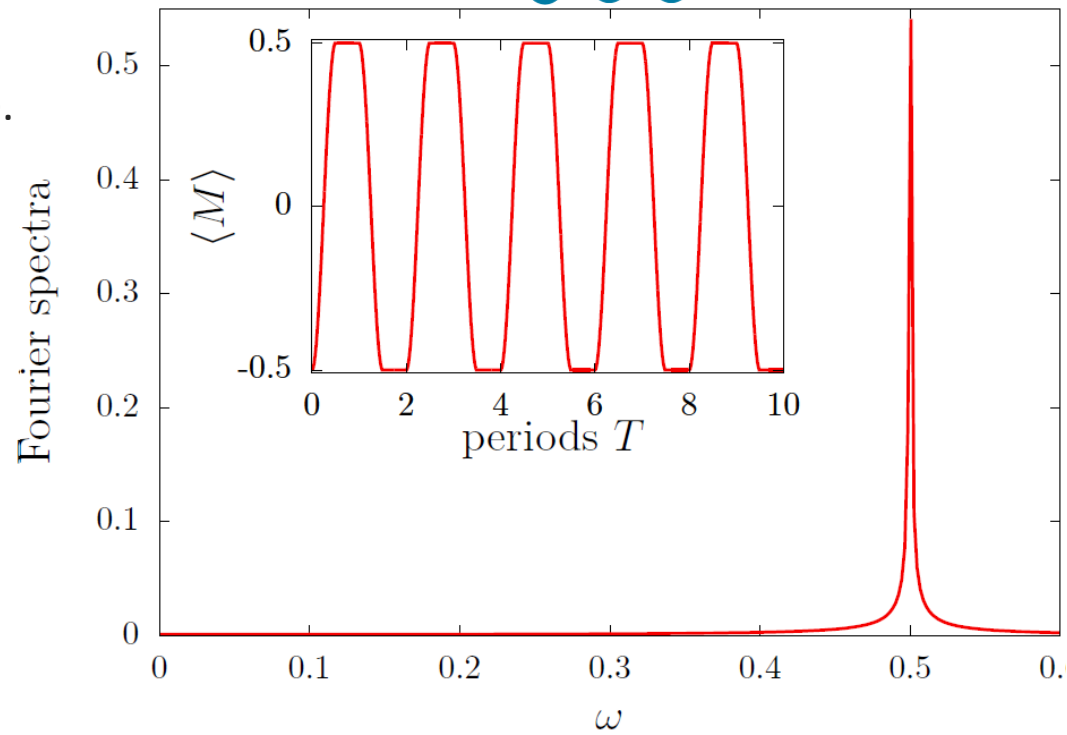


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If driving is imperfect  $\epsilon > 0$ , the magnetization dynamics shows an envelope. Imperfect periodicity.

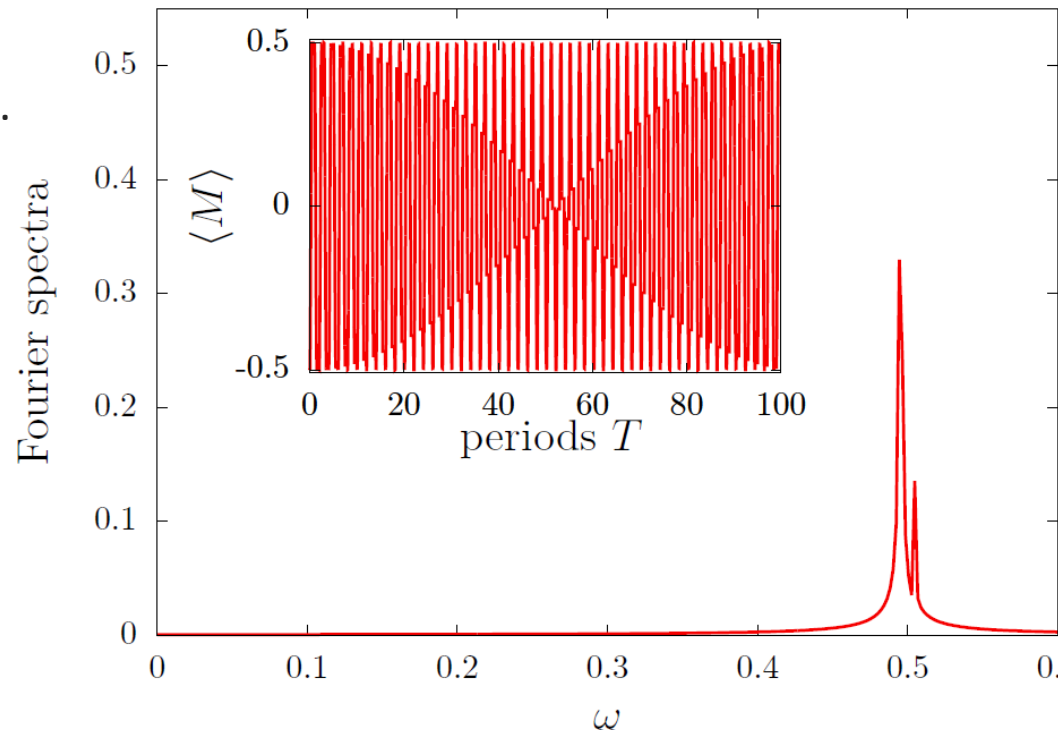
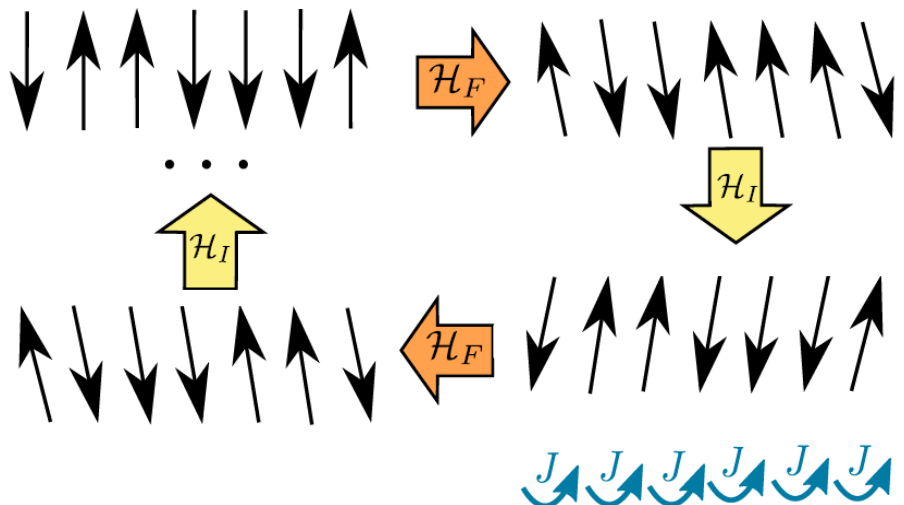


Illustration of a discrete time-crystal

$$\mathcal{H}_F = (\Omega - \epsilon) \sum_{i=1}^N \sigma_i^x$$

$$\mathcal{H}_I = \sum_{i=1}^{N-1} J \sigma_i^z \sigma_{i+1}^z$$

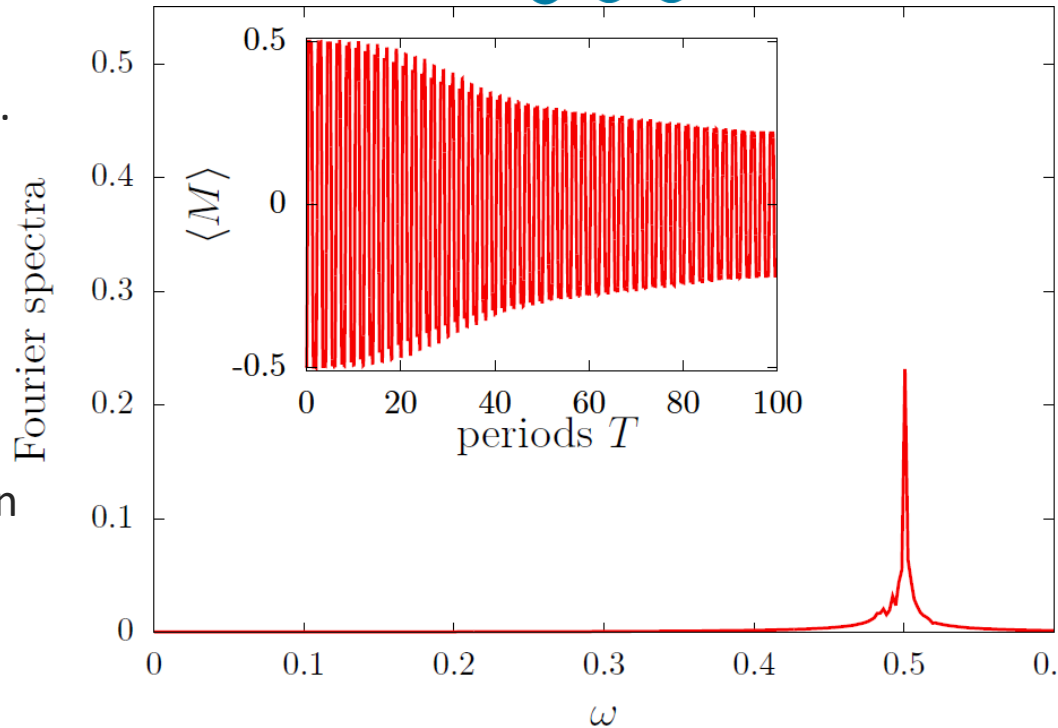


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If driving is imperfect  $\epsilon > 0$ , and interaction switched on, single peak appears but is damped due to thermalization within chain. Vanishing periodicity for large N.

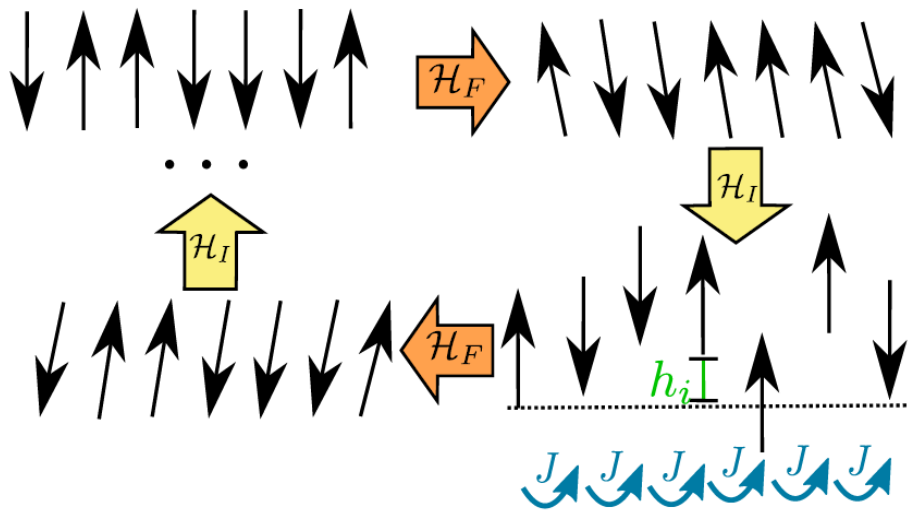


# Examples: (ii) Stabilized time-crystal

Illustration of a discrete time-crystal

$$\mathcal{H}_F = (\Omega - \epsilon) \sum_{i=1}^N \sigma_i^x$$

$$\mathcal{H}_I = \sum_{i=1}^{N-1} J \sigma_i^z \sigma_{i+1}^z + \sum_{i=1}^N h_i \sigma_i^z$$

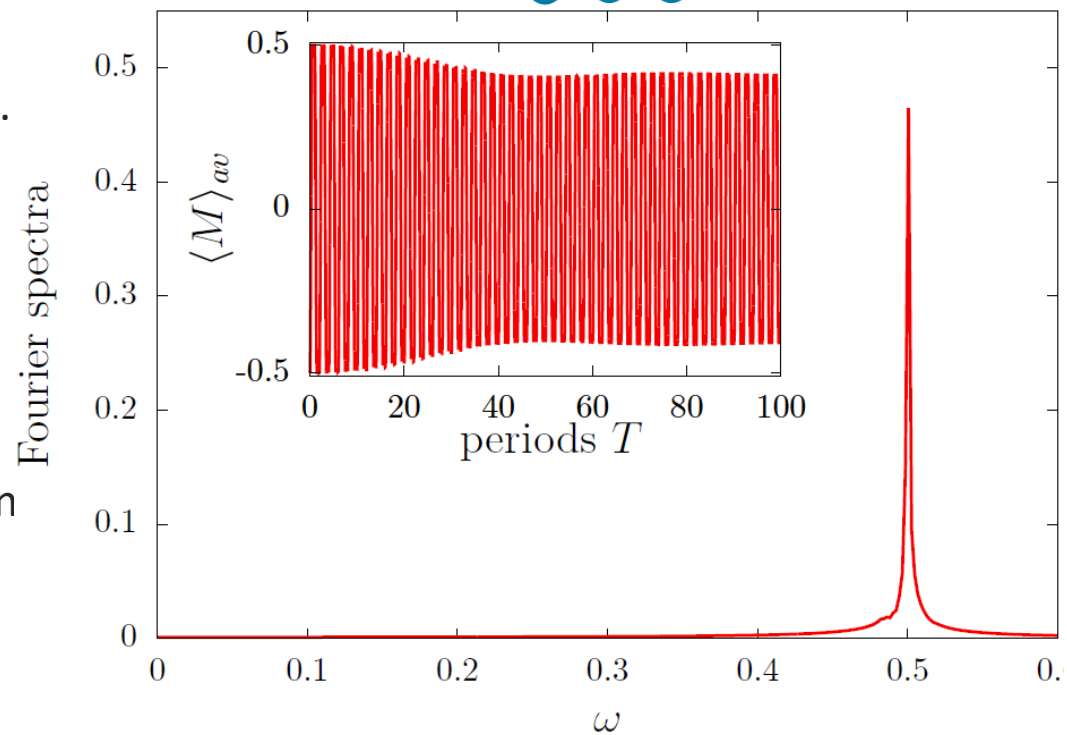


The spin chain of N spins returns despite imperfect rotation back to its initial state. Figure of merit and observable (staggered magnetization):

Yao et al, Phys. Rev. Lett. 118, 030401 (2017)

$$\langle M \rangle = \frac{1}{N} \sum_{i=1}^N (-1)^i \frac{\langle \sigma_i^z \rangle}{2}$$

If driving is imperfect  $\epsilon > 0$ , and interaction switched and disorder is present, thermalization is prevented. Periodicity even for large N.

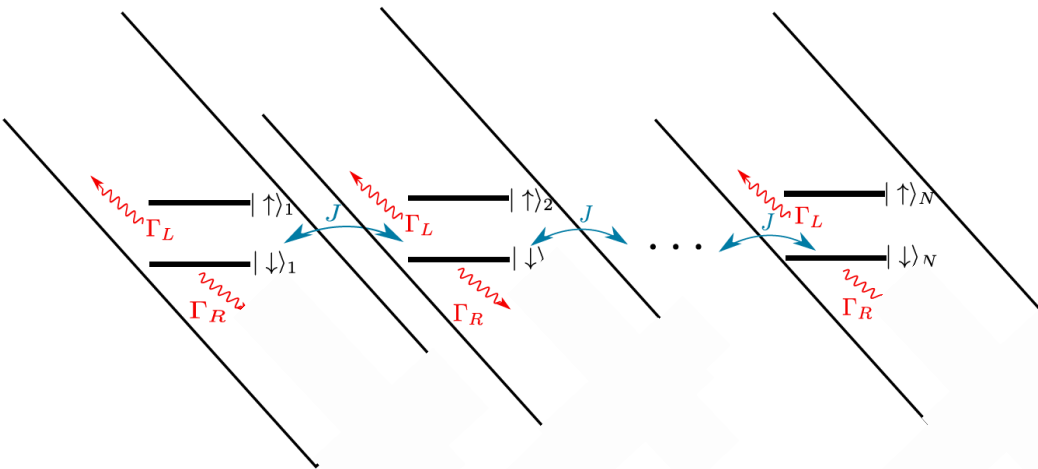




## Time-crystal in the presence of losses

Lazarides and Moessner, Phys. Rev. B 95, 195135 (2017)

Periodicity is lost when Markovian reservoir (bath) is coupled to the chain. Thermalization within chain is prevented due to many-body localization but thermalization with bath is inevitable

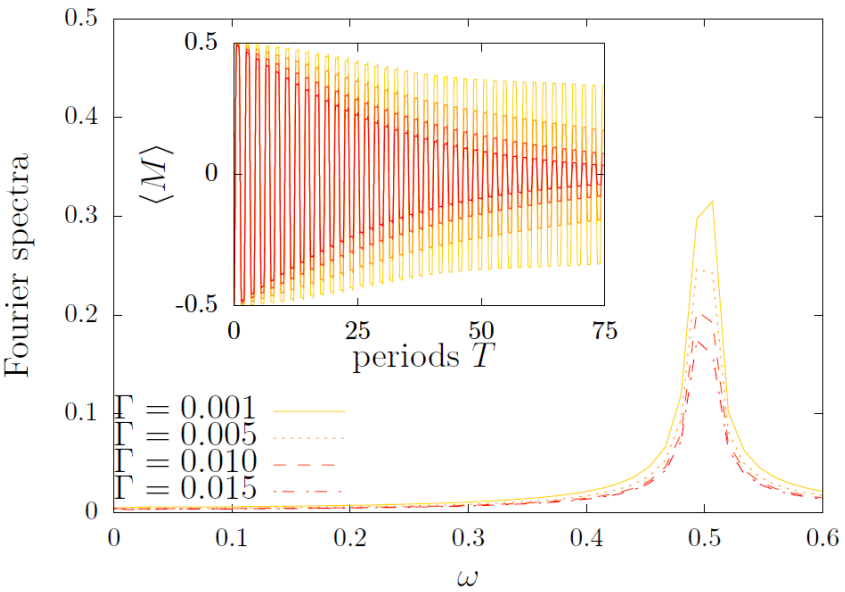
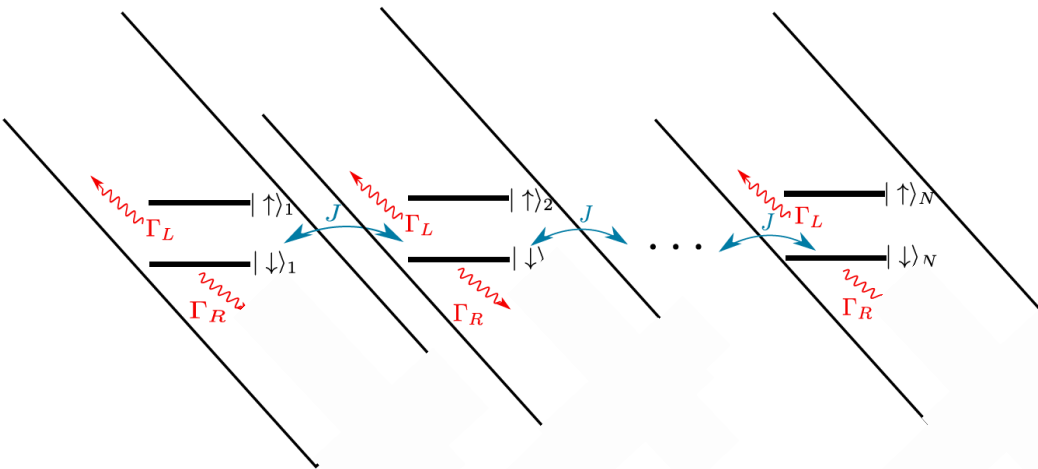




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# Examples: (ii) Stabilized time-crystal

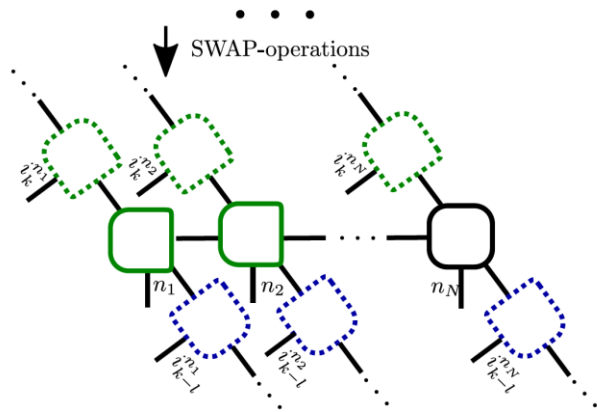
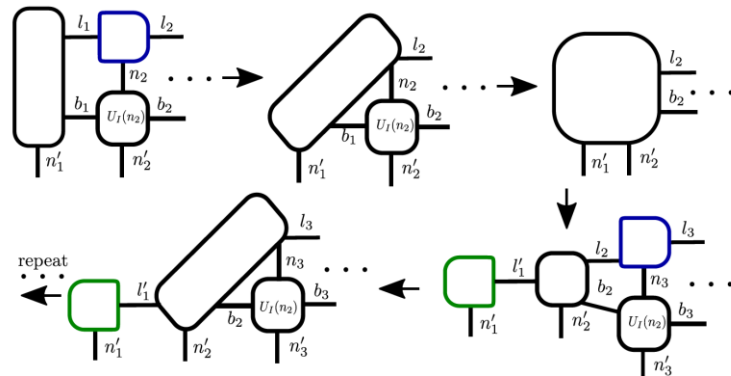
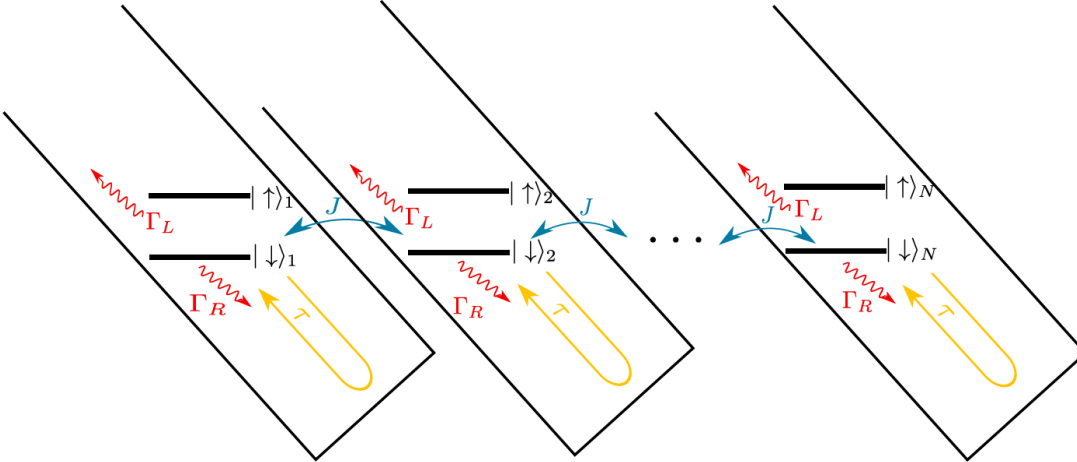


## Time-crystal in the presence of losses

Lazarides and Moessner, Phys. Rev. B 95, 195135 (2017)

But non-Markovian dissipation, such as quantum feedback interaction allows self-stabilizing system-reservoir dynamics and prevents again thermalization.

Droenner, AC in preparation (2018)





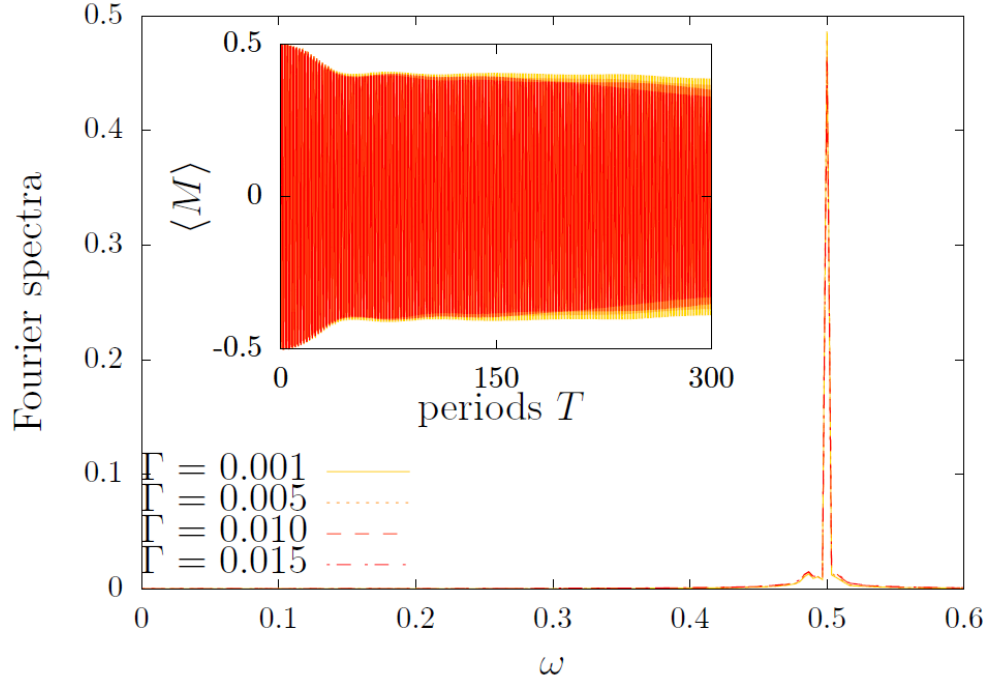
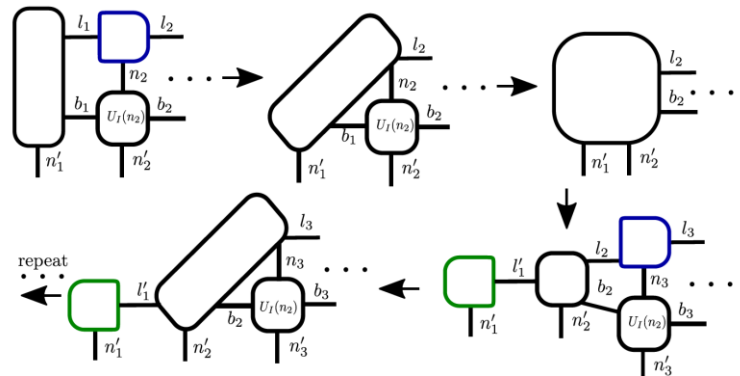
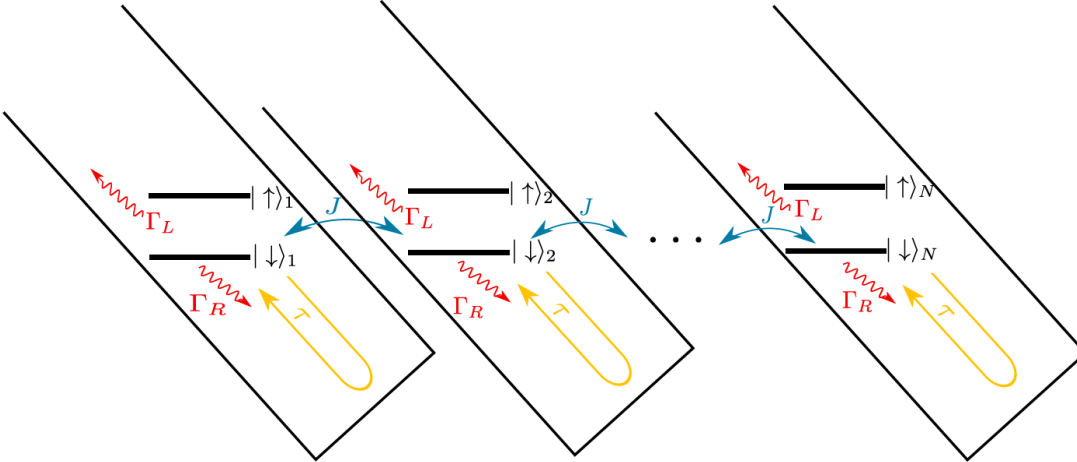
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**Thank you for the attention!**

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