Theory for strongly coupled quantum dot cavity quantum electrodynamics

-- Photon statistics and phonon signatures in quantum light emission --

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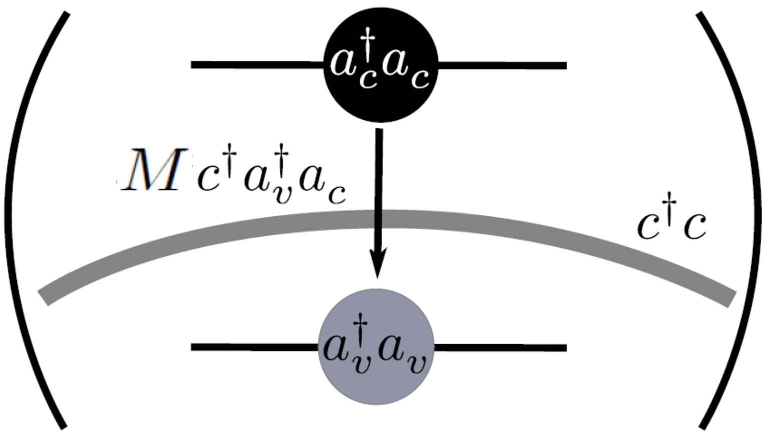
OUTLINE

Jaynes-Cummings model

Atom cavity-QED,
solved by the Jaynes-Cummings model [1]:

- Isolated two-level system (no losses)
- One-electron assumption
- One interaction: electron-light



„The simplest fully quantized model of interest“ (J.H. Eberly)

$$H_0 = \hbar\omega_0 c^\dagger c + \hbar\omega_v a_v^\dagger a_v + \hbar\omega_c a_c^\dagger a_c$$

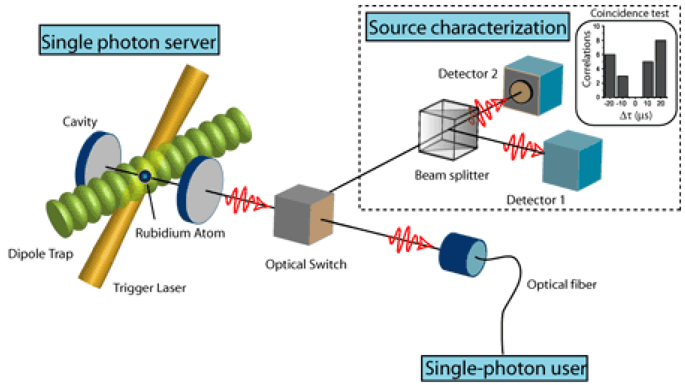
$$H_{el-pt} = -\hbar M (a_v^\dagger a_c c^\dagger + a_c^\dagger a_v c)$$

analytically solvable, e.g.

$$\langle a_c^\dagger a_c \rangle (t) = \cos^2(M\sqrt{N+1}\; t)$$

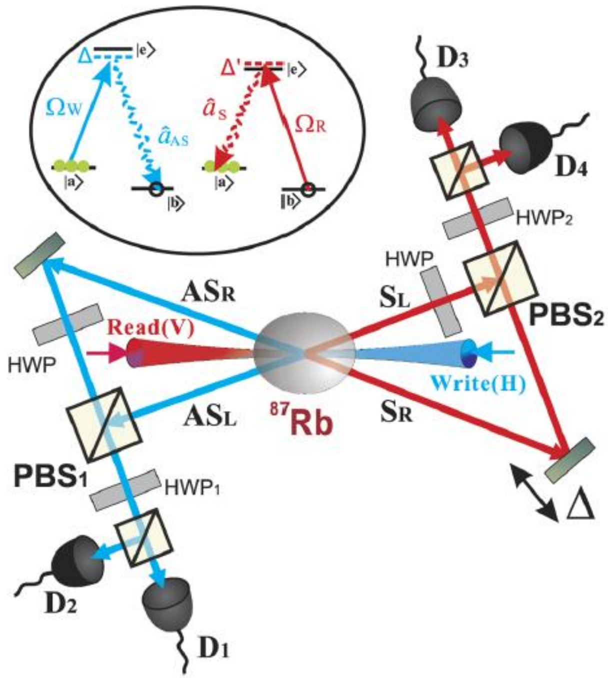
[1] E. Jaynes and F. Cummings, Proc.IEEE 51, 89 (1963)

Atom-photon interfaces: experimental realizations



Single-photon server with just one atom,
Hijlkema et al., Nat. Phys. 3, 253 (2007)

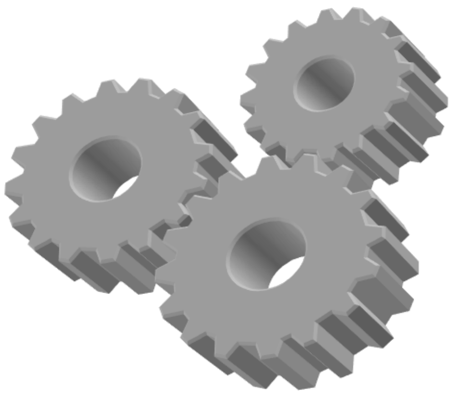
Typical realization based on:
(i) Trapped atoms
(ii) Atom beam



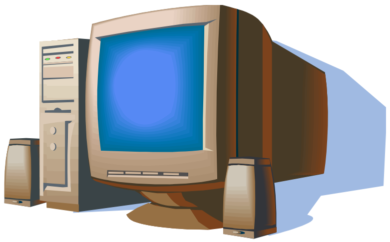
Entanglement source for quantum repeaters,
Chen et al., Phys.Rev.Lett. 99, 18505 (2007)

Semiconductor nanostructures for device fabrication

Advantageous semiconductor QD properties for future technological applications in microcavity systems:



- ❑ Fixed position, tailorable coupling strengths and frequencies,
- ❑ Cavity-system ultra-small,
- ❑ Electrical pumping,

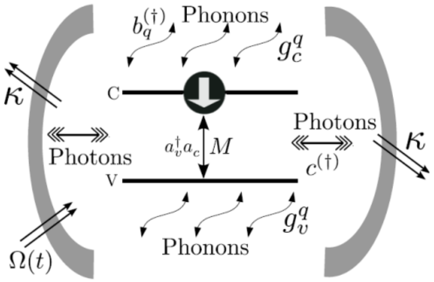


Semiconductor QD cavity-QED:
Theoretical simulations for device optimization are desirable

BUT:
more interactions
and additional loss mechanisms need to be considered

Hierarchy problem and cluster expansion

1.) more interactions (electron-phonon, electron-electron)
2.) number of carriers may not be fixed



$$i\hbar\partial_t\langle O\rangle = \langle[H,O]\rangle$$

$$\langle a_v^\dagger a_c c^\dagger\rangle$$

$$\langle a_c^\dagger a_c c^\dagger c\rangle$$

$$\langle a_v^\dagger a_c c^\dagger c^\dagger c\rangle$$

For non-Markovian description, equation of motion approach

Hierarchy problem

Typical approach: Factorization / truncation scheme via cluster expansion

$$\langle a^\dagger a c^\dagger c\rangle = \langle a^\dagger a\rangle\langle c^\dagger c\rangle + \langle a^\dagger\rangle\langle a\rangle\langle c^\dagger\rangle\langle c\rangle \ldots$$

Example and breakdown of cluster expansion

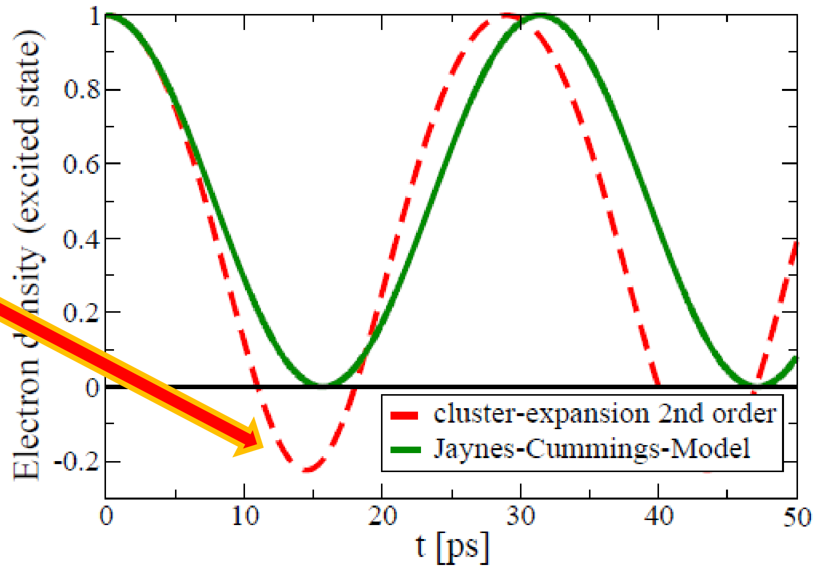
$$\partial_t \langle a_c^\dagger a_c \rangle = 2\ \mathrm{Im}\left[M \langle a_v^\dagger a_c c^\dagger \rangle \right]$$

$$\partial_t \langle a_v^\dagger a_c c^\dagger \rangle = -iM \langle a_c^\dagger a_c \rangle - iM \left(\langle a_c^\dagger a_c c^\dagger c \rangle - \langle a_v^\dagger a_v c^\dagger c \rangle \right)$$

Assuming Fock photons, no coherent contributions:

$$\partial_t \langle a_v^\dagger a_c c^\dagger \rangle = -iM \langle a_c^\dagger a_c \rangle - iM \langle c^\dagger c \rangle \left(\langle a_c^\dagger a_c \rangle - \langle a_v^\dagger a_v \rangle \right)$$

Vacuum Rabi oscillation (N=0) are not approximated good enough !!



Cluster expansion in weak correlated dynamics

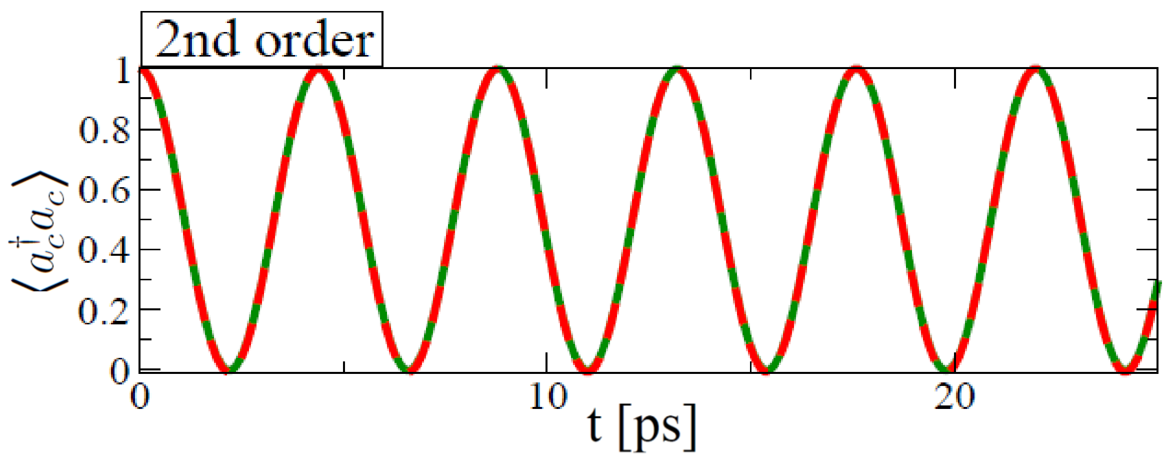
$$\partial_t \langle a_c^\dagger a_c \rangle = 2\ \mathrm{Im}\left[M \langle a_v^\dagger a_c c^\dagger \rangle \right]$$

$$\partial_t \langle a_v^\dagger a_c c^\dagger \rangle = -iM \langle a_c^\dagger a_c \rangle - iM \left(\langle a_c^\dagger a_c c^\dagger c \rangle - \langle a_v^\dagger a_v c^\dagger c \rangle \right)$$

Assuming Fock photons, no coherent contributions:

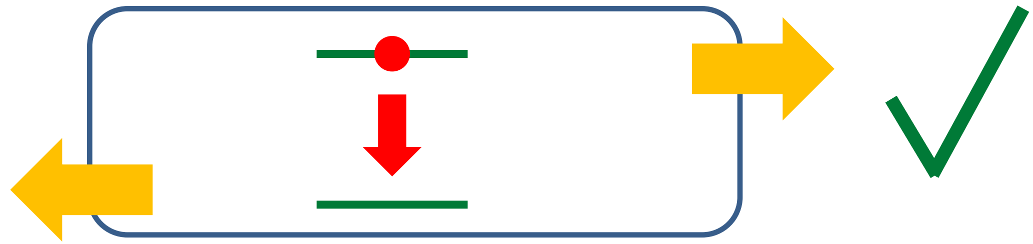
$$\partial_t \langle a_v^\dagger a_c c^\dagger \rangle = -iM \cancel{\langle a_c^\dagger a_c \rangle} - iM \langle c^\dagger c \rangle \left(\langle a_c^\dagger a_c \rangle - \langle a_v^\dagger a_v \rangle \right)$$

But:
Rabi oscillations with N=50 are well approximated !!



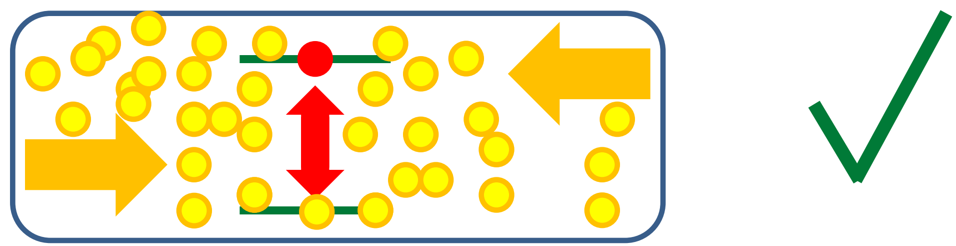
Limit of cluster expansion approach

Weak coupling regime:
(e.g. resonance fluorescence, biexciton cascade)



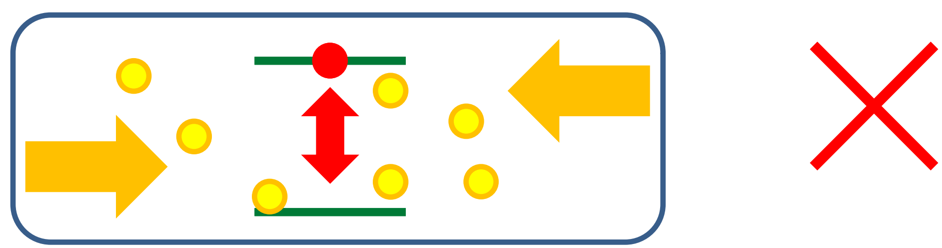
Strong coupling regime (dynamics weakly correlated):

(e.g. laser, superradiance effects)

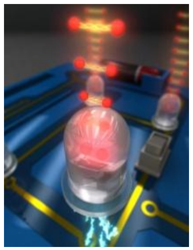


Strong coupling regime (dynamics strongly correlated):

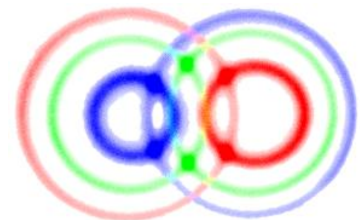
(e.g. single photon emitter)



Technological application of single photons



Since single-photon of interest for technological application,
(i) improved measurement
(ii) quantum cryptography
(iii) quantum information processing (entanglement)



New approaches for device simulations become necessary

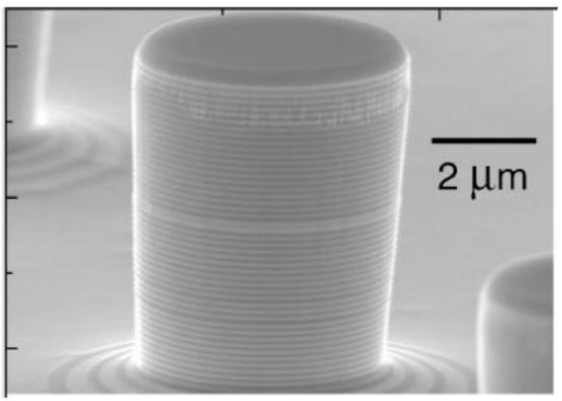
(i): mathematical induction method (atom-like QDs)

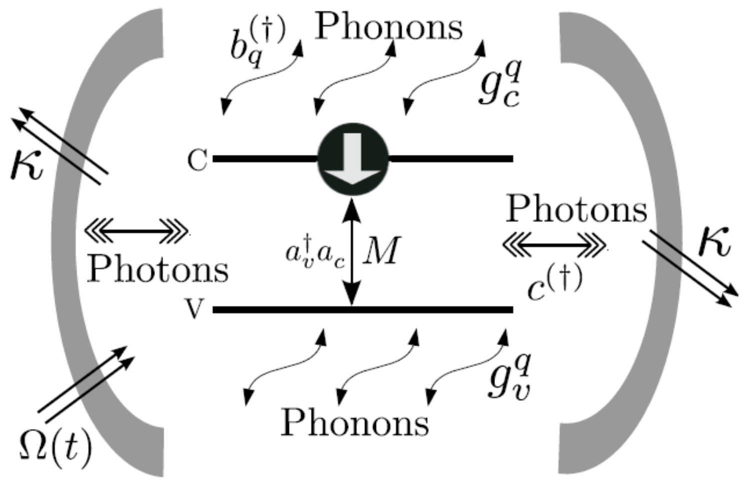
(ii): photon-probability cluster expansion (electrical pumping)

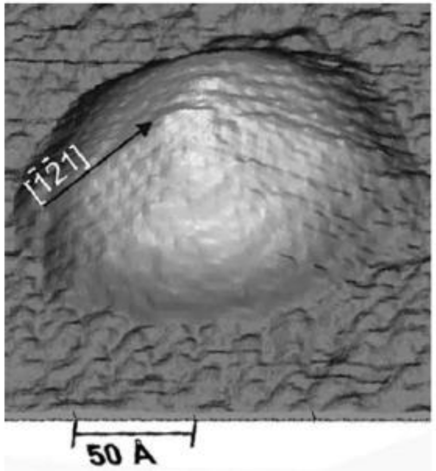
(i) mathematical induction method

Semiconductor QD cavity-QED Hamiltonian

Sanvitto et al., Appl.Phys.Lett. 86 (2005)







Marquez et al., Appl.Phys.Lett. 78 (2001)

$$
\begin{aligned}
H \;=\;& \hbar\omega_v a_v^\dagger a_v + \hbar\omega_c a_c^\dagger a_c - \hbar\Omega(t)(a_v^\dagger a_c + a_c^\dagger a_v) \\
&+\hbar\omega_0 c^\dagger c - \hbar M(a_v^\dagger a_c c^\dagger + a_c^\dagger a_v c) \\
&+\hbar\sum_q \omega_{LO} b_q^\dagger b_q + a_c^\dagger a_c\left(g_q^c b_q + g_q^{c*} b_q^\dagger\right) + a_v^\dagger a_v\left(g_q^v b_q + g_q^{v*} b_q^\dagger\right)
\end{aligned}
$$

QD, assumed as a 2-level system with one electron,

interacts with the cavity photons, bulk LO-phonons, a classical pump field

Solving LO-phonon QD cavity-QED without factorization

Using product rule for operators: $\partial_t \left(a_c^\dagger a_c \boxed{c^\dagger c} \boxed{b_q^\dagger b_q} \right) = \left(\partial_t \boxed{a_c^\dagger a_c c^\dagger c} \right) \boxed{b_q^\dagger b_q} + \boxed{c^\dagger c} \left(\partial_t \boxed{a_c^\dagger a_c b_q^\dagger b_q} \right)$

and generalized commutation relations: $[A, F(B)] = [A, B] F'(B)$

for every possible combination of phonon, photon, and electron operators:

$$G^{p,s}_{m,n} := a_v^\dagger a_v c^{\dagger p} c^s \bar{b}^{\dagger m} \bar{b}^n$$
$$E^{p,s}_{m,n} := a_c^\dagger a_c c^{\dagger p} c^s \bar{b}^{\dagger m} \bar{b}^n$$
$$T^{p,s}_{m,n} := a_v^\dagger a_c c^{\dagger p} c^s \bar{b}^{\dagger m} \bar{b}^n$$

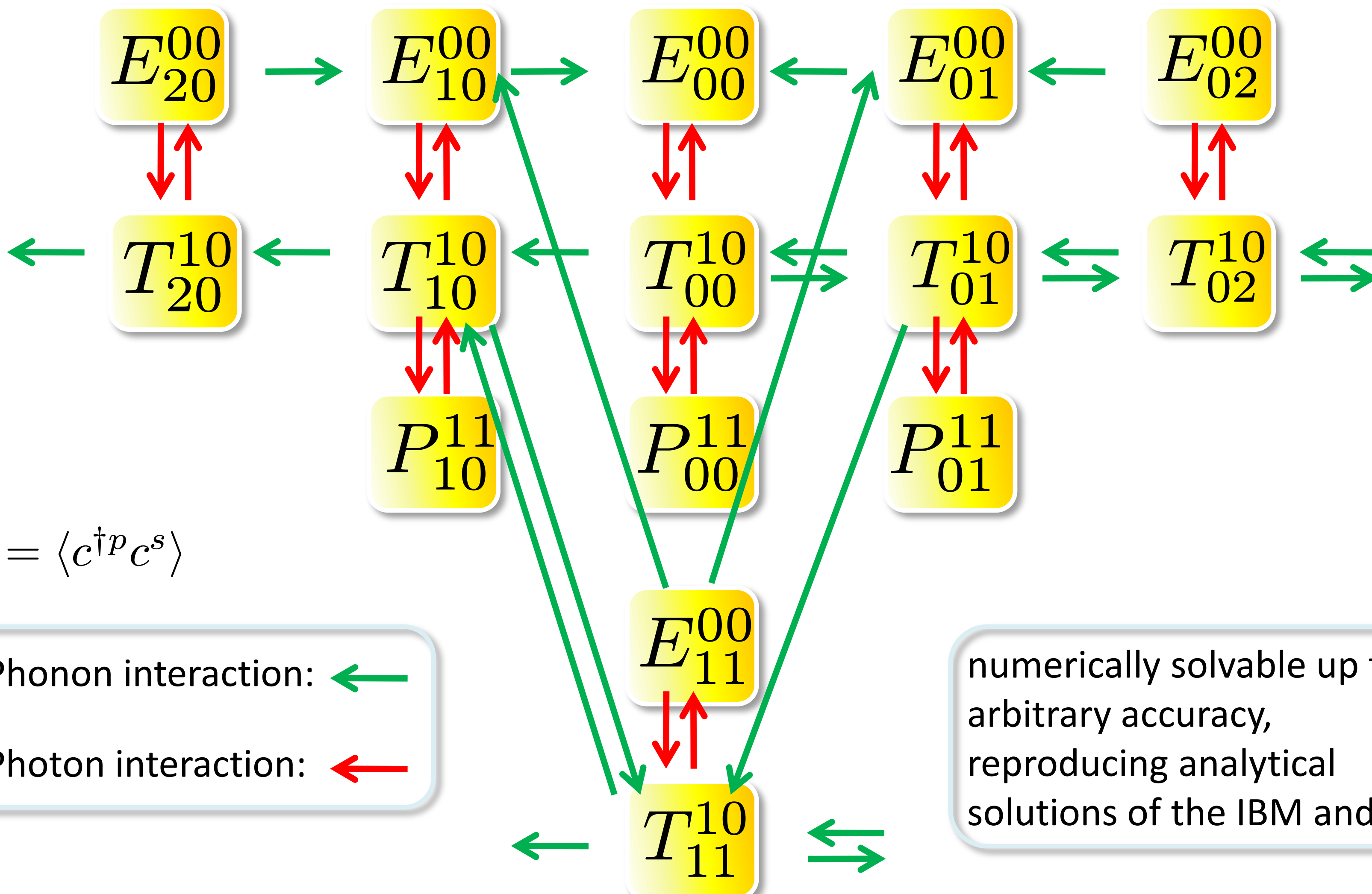
and

their dynamics, e.g.

$$\begin{aligned}
\partial_t \langle T^{p,s}_{m,n} \rangle = \\
= -i \left[\omega_{cv} - (p-s)\omega_0 - (m-n)\omega_{LO} - i(p+s)\kappa - i\gamma \right] \langle T^{p,s}_{m,n} \rangle \\
- ip\, M \langle E^{p-1,s}_{m,n} \rangle - iM(\langle E^{p,s+1}_{m,n} \rangle - \langle G^{p,s+1}_{m,n} \rangle) - i\Omega(t) \left(\langle E^{p,s}_{m,n} \rangle - \langle G^{p,s}_{m,n} \rangle \right) \\
- i \langle T^{p,s}_{m,n+1} \rangle - i \langle T^{p,s}_{m+1,n} \rangle + i\, m\, g_v \langle T^{p,s}_{m-1,n} \rangle - i\, n\, g_c \langle T^{p,s}_{m,n-1} \rangle,
\end{aligned}$$

General set of equations of motion

For example, in the case of LO-phonon assisted vacuum Rabi oscillations (E^{11}_{00} = 0) :



$$P^{ps}_{00} = \langle c^{\dagger p} c^{s} \rangle$$

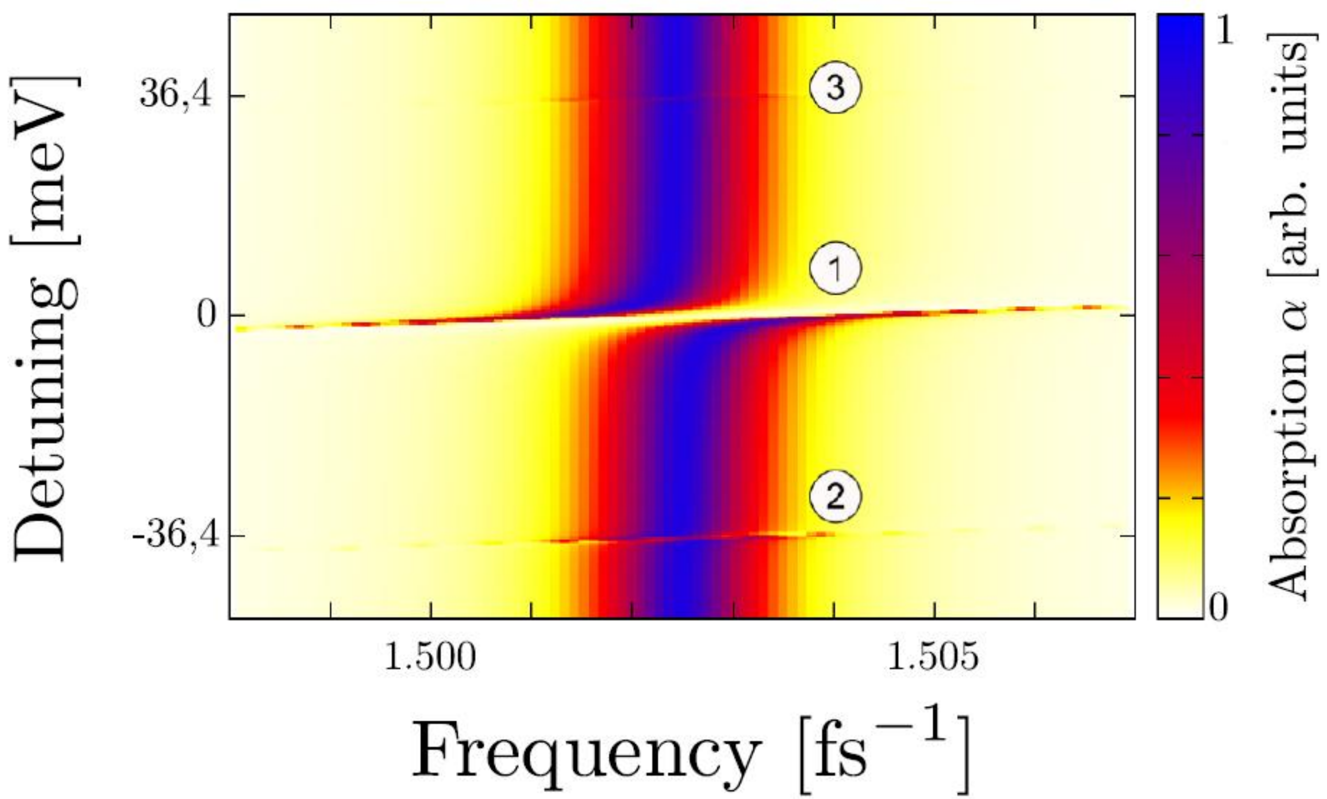
Phonon interaction: ←

Photon interaction: ←

numerically solvable up to arbitrary accuracy, reproducing analytical solutions of the IBM and JCM

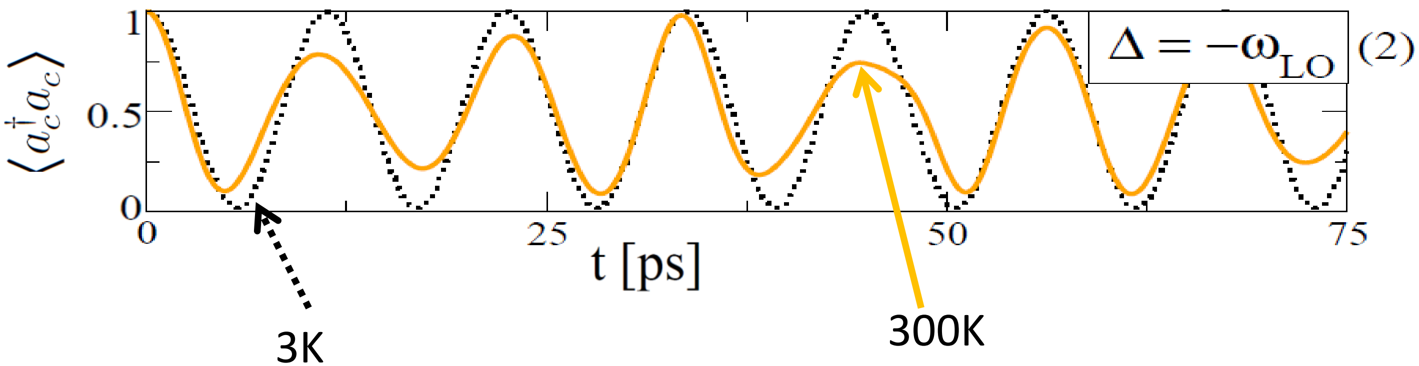
LO-phonon QD cavity-QED: additional anti-crossings

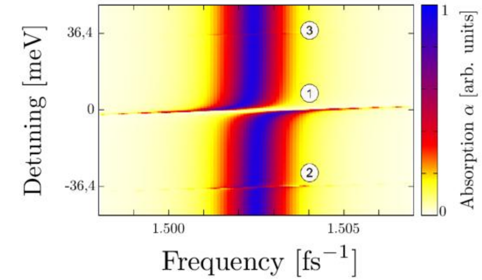
Probing with δ-pulse at 300K: $\alpha(\omega) \propto \mathrm{Re}\left[\frac{\langle T^{0,0}_{0,0}\rangle(\omega)}{\rho^T_0}\right]$



Modified Rabi frequency due to phonon cavity feeding

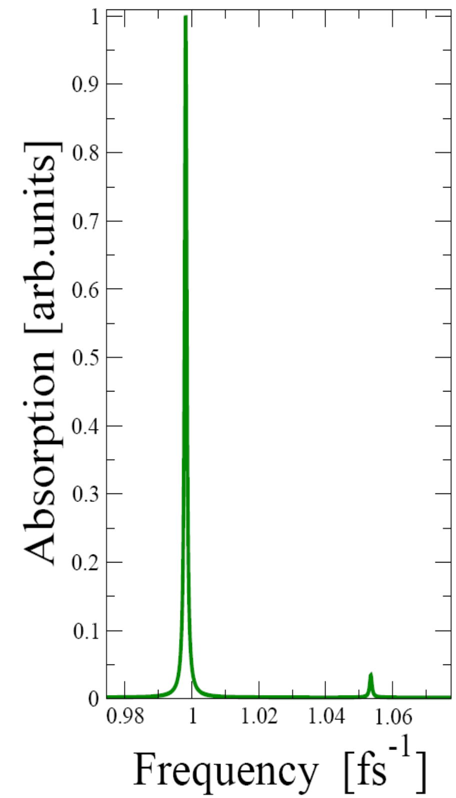
Modified Rabi frequency at the Stokes-position (2):
(temperature dependence weak)



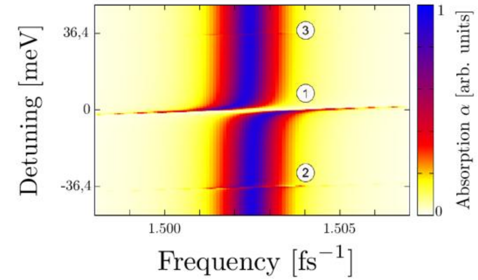


Effective oscillator strength (0 K) via Huang-Rhys factor:

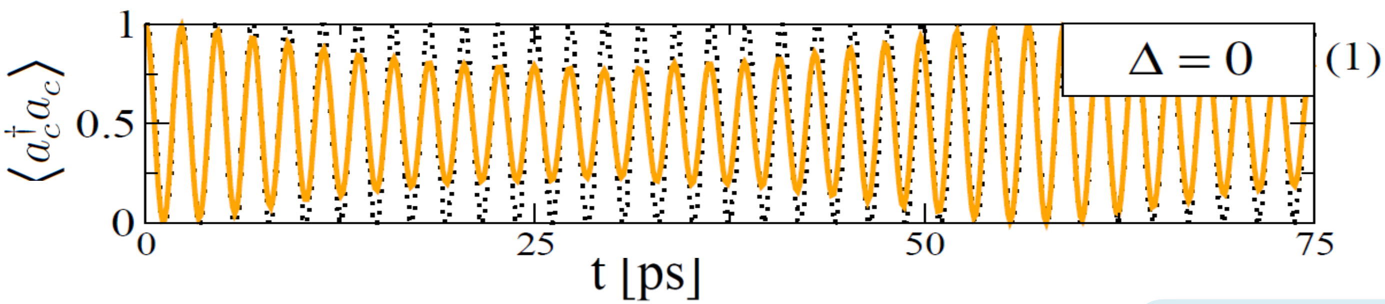
$$\frac{\Omega_2}{\Omega_1} \approx \xi_1 = \frac{g_{\mathrm{eff}}^2}{\omega_{LO}^2} e^{-\frac{\Delta_{LO}}{\omega_{LO}}} \longrightarrow \Omega_2 \approx \frac{M\ g_{\mathrm{eff}}^2}{\omega_{LO}^2}\ e^{-\frac{\Delta_{LO}}{\omega_{LO}}}$$



Quantum beating at resonance position



Quantum beating at resonance position (1):

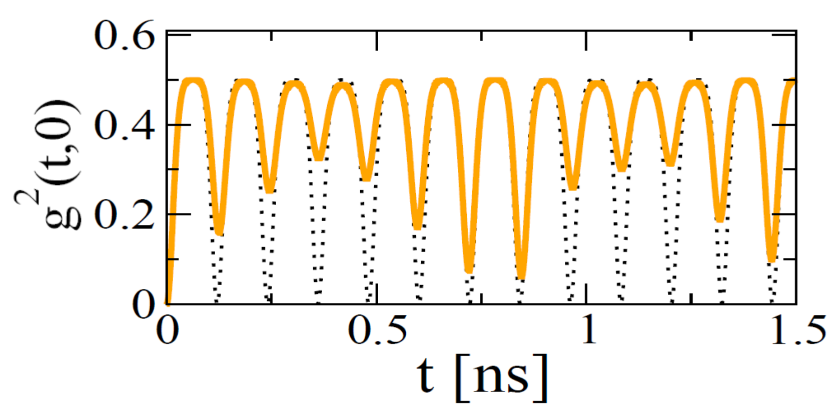


Impact on intensity-intensity correlation function:

$$g^{(2)}(t, \tau = 0) = \frac{\langle c^\dagger c^\dagger c c \rangle}{\langle c^\dagger c \rangle^2}$$

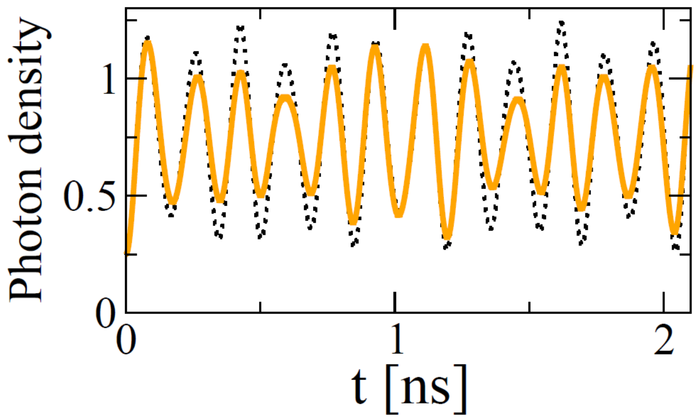
FOCK STATE:

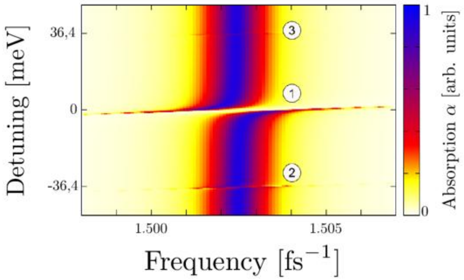
initially one fock photon in the cavity and an excited QD



Quantum beating at resonance position

Quantum beating at resonance position (1) :



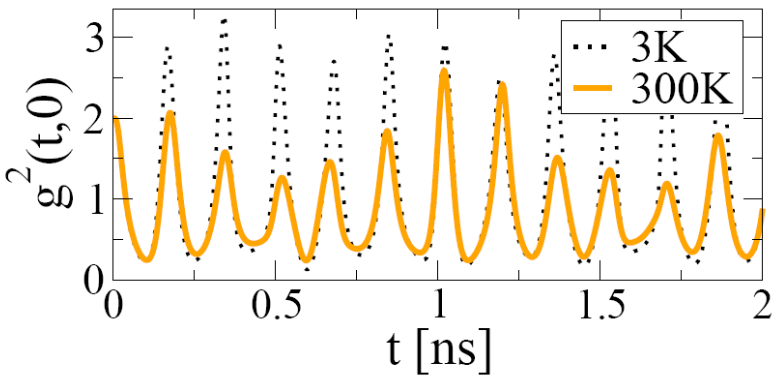


Impact on intensity-intensity correlation function:

$$g^{(2)}(t,\tau=0)=\frac{\langle c^\dagger c^\dagger c c\rangle}{\langle c^\dagger c\rangle^2}$$

THERMAL STATE:

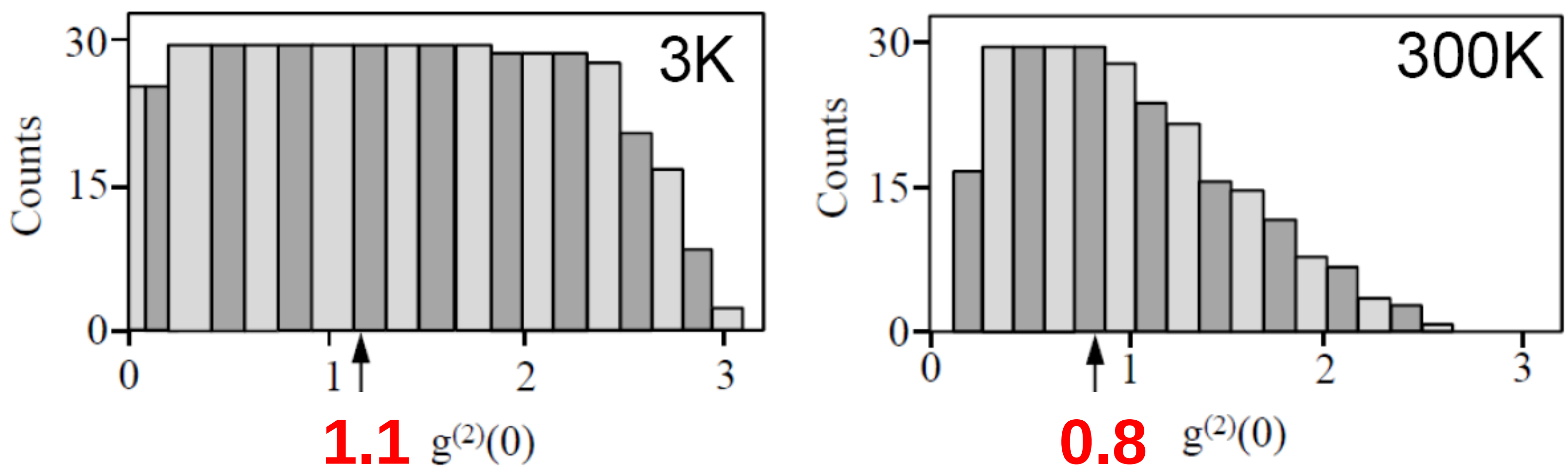
for a cavity field, prepared initially in the thermal state



LO-phonon induced thermal anti-bunching

Impact on intensity-intensity correlation function mean value:

$$\bar{g}^{(2)} := \frac{1}{T_a}\int_0^{T_a} \mathrm{d}t \; g^{(2)}(t,0)$$



Thermal cavity field is transformed into a non-classical field via LO-phonon interaction [1]

Semiconductor environment enforces non-classical light features

[1] PRL **104**, 156801 (2010)

Applications of the mathematical induction method

Systems, for which the induction method is successfully applied:

1. QD as a two-level system with one-electron (photon statistics) [1]
2. QD as a three-level system (quantum coherence) [2]
3. QD as a four-level system with two electrons (biexciton cascade) [3]

[1] PRL **104**, 156801 (2010) [2] PSSB, accepted (2010) [3] PRB **81**, 195319 (2010)

(ii) photon-probability cluster expansion

QD cavity-QED in the presence of a carrier reservoir

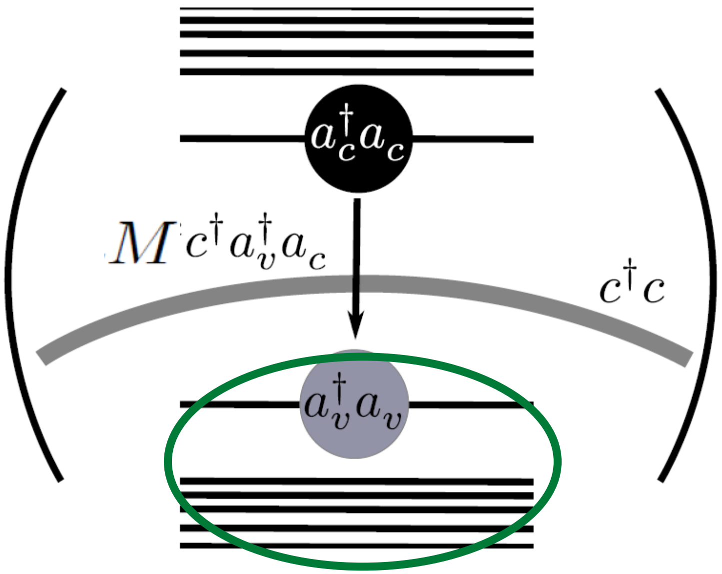
In the presence of a carrier reservoir, the number of carriers inside the QD is not fixed.

The one-electron assumption is not valid anymore:

$$\langle a_1^\dagger a_2^\dagger a_3 a_4 \rangle \neq 0$$

Typically, a Hartree – Fock factorization is applied:

$$\langle a_1^\dagger a_2^\dagger a_3 a_4 \rangle \approx \langle a_1^\dagger a_4 \rangle \langle a_2^\dagger a_3 \rangle - \langle a_1^\dagger a_3 \rangle \langle a_2^\dagger a_4 \rangle$$



But in case of strongly correlated electron-photon dynamics:

$$\langle a_1^\dagger a_2^\dagger a_3 a_4 c^\dagger c \rangle \neq 0$$

Photon-probability cluster expansion

A factorization approach for strongly correlated systems, the photon probability cluster expansion (PPCE) is introduced.[1]

Expansion of observables:

$$p_n = \langle |n\rangle\langle n| \rangle \qquad f_n^h = p_n - \langle |n\rangle\langle n| a_v^\dagger a_v \rangle$$

$$\langle c^\dagger c \rangle = \sum_{n=1}^{\infty} n\, p_n \qquad f_n^e = \langle |n\rangle\langle n| a_c^\dagger a_c \rangle$$

Introducing a modified Hartree – Fock approximation[1]:

$$\rho_{pn} = \frac{\langle n|\rho|n\rangle}{p_n} \approx \frac{1}{Z} e^{-\sum_{ij} \lambda_{ij}^n a_i^\dagger a_j}$$

$$\begin{aligned}
\langle |n\rangle\langle n| a_1^\dagger a_2^\dagger a_3 a_4 \rangle &\approx p_n \Big\{ \langle \frac{|n\rangle\langle n|}{p_n} a_1^\dagger a_4 \rangle \langle \frac{|n\rangle\langle n|}{p_n} a_2^\dagger a_3 \rangle - \langle \frac{|n\rangle\langle n|}{p_n} a_1^\dagger a_3 \rangle \langle \frac{|n\rangle\langle n|}{p_n} a_2^\dagger a_4 \rangle \Big\} \\
&\approx \frac{1}{p_n} \Big\{ \langle |n\rangle\langle n| a_1^\dagger a_4 \rangle \langle |n\rangle\langle n| a_2^\dagger a_3 \rangle - \langle |n\rangle\langle n| a_1^\dagger a_3 \rangle \langle |n\rangle\langle n| a_2^\dagger a_4 \rangle \Big\}
\end{aligned}$$

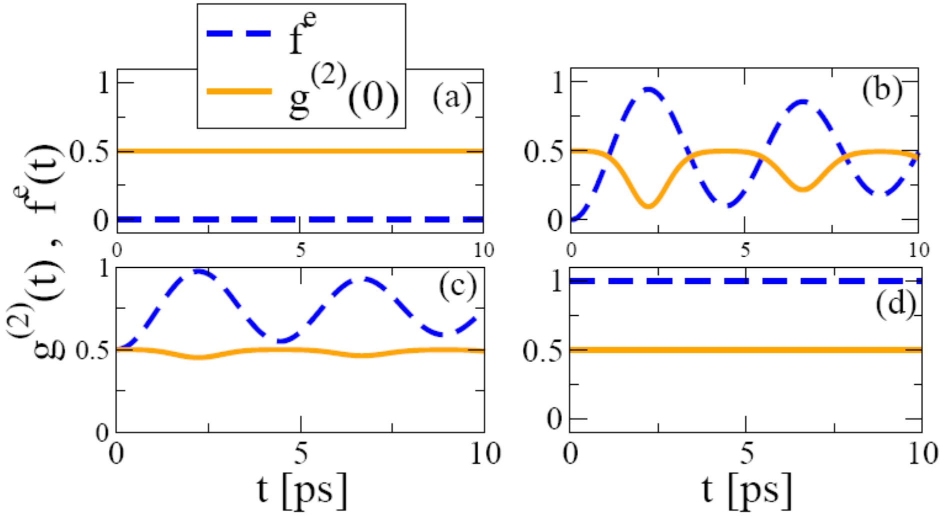
[1]PRL **103**, 087407 (2009)

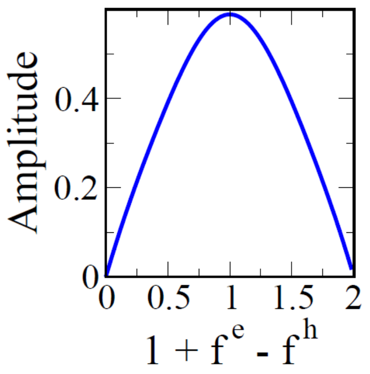
Modified Rabi oscillation amplitude

Equation of motion changed due to many-particle contribution[1] :

$$\partial_t \langle a_v^\dagger a_c |n+1\rangle\langle n| \rangle = +i\ M\ \sqrt{n+1}\ (p_{n+1} - f_{n+1}^h) - i\ M\ \sqrt{n+1}\ f_n^e$$

$$\partial_t \langle a_v^\dagger a_c |n+1\rangle\langle n| \rangle = -i\ M \sqrt{n+1}\ \left[\frac{f_n^h f_n^e}{p_n} - \frac{(p_{n+1} - f_{n+1}^h)(p_{n+1} - f_{n+1}^e)}{p_{n+1}} \right]$$





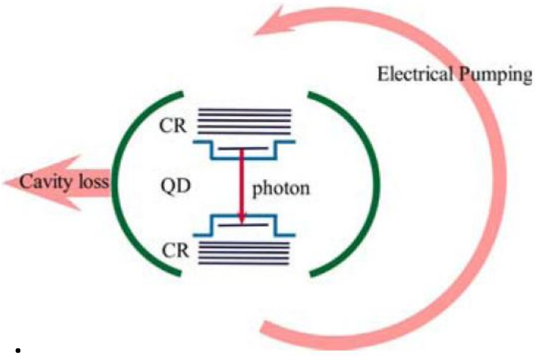
The amplitude of the Rabi flops depends on the number of electrons and holes in the QD.

[1]PSSB **247**, 809 (2010)

Simulation of an electrically driven single photon source

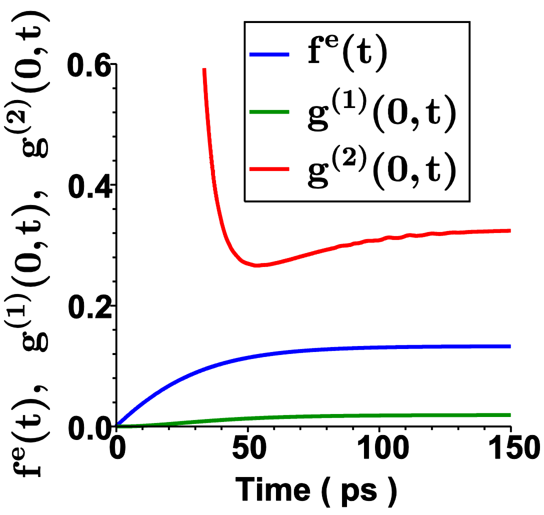
Inclusion of phenomenological pumping terms:

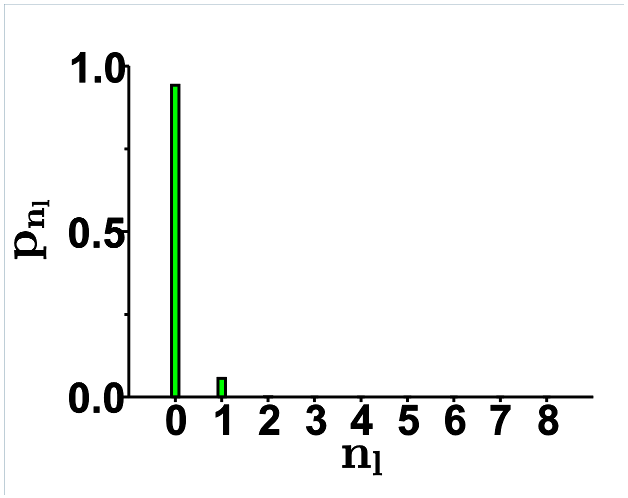
$$\partial_t f^e_n|_{pump} = S^{in}_e(p_n - f^e_n) - S^{out}_e f^e_n$$
$$\partial_t f^h_n|_{pump} = S^{in}_h(p_n - f^h_n) - S^{out}_h f^h_n$$



microscopical derived with corresponding interaction Hamiltonian

Pump mechanism and losses balance into a stationary state[1]





[1]RRL **4**, 289 (2010)

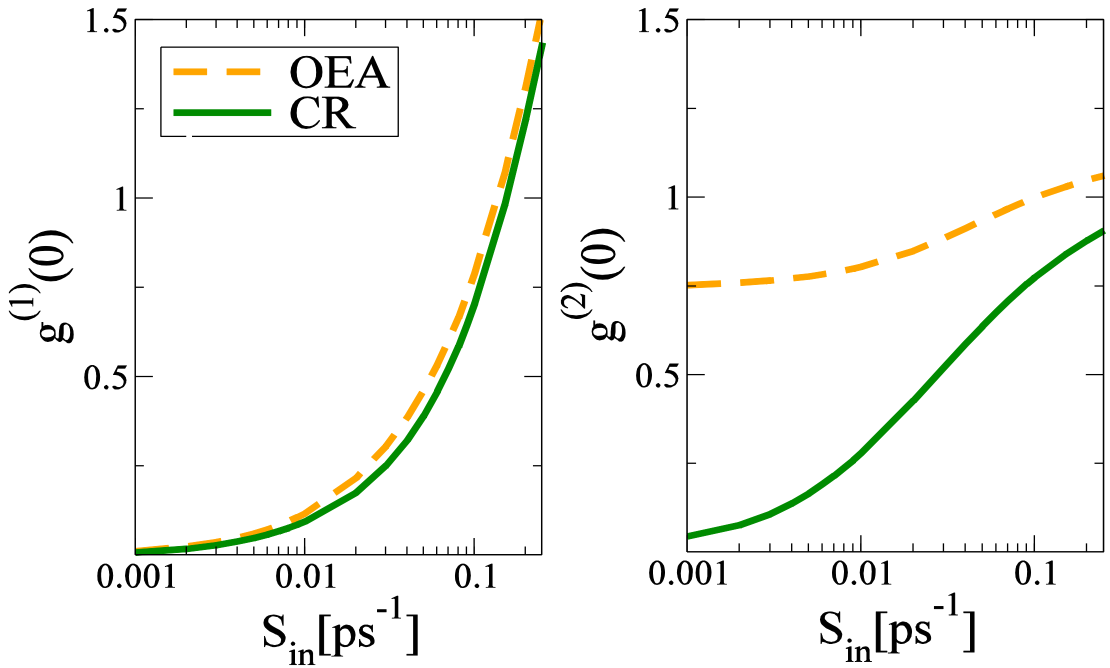
Parameter studies of single QD laser

Microscopic model reveals: Why are QD very promising single-photon sources? [1]

Comparison with one-electron approximation (OEA):

OEA: $$f^e_{sp} = f^e$$

CR: $$f^e_{sp} = \sum_n \frac{f^e_n f^h_n}{p_n}$$



Due to the WL, enhanced Pauli-blocking in the QD states occurs

Advantageous for single-photon emission in a wide pump interval

[1] Semic.Sci.Technology, accepted (2010)

Conclusion

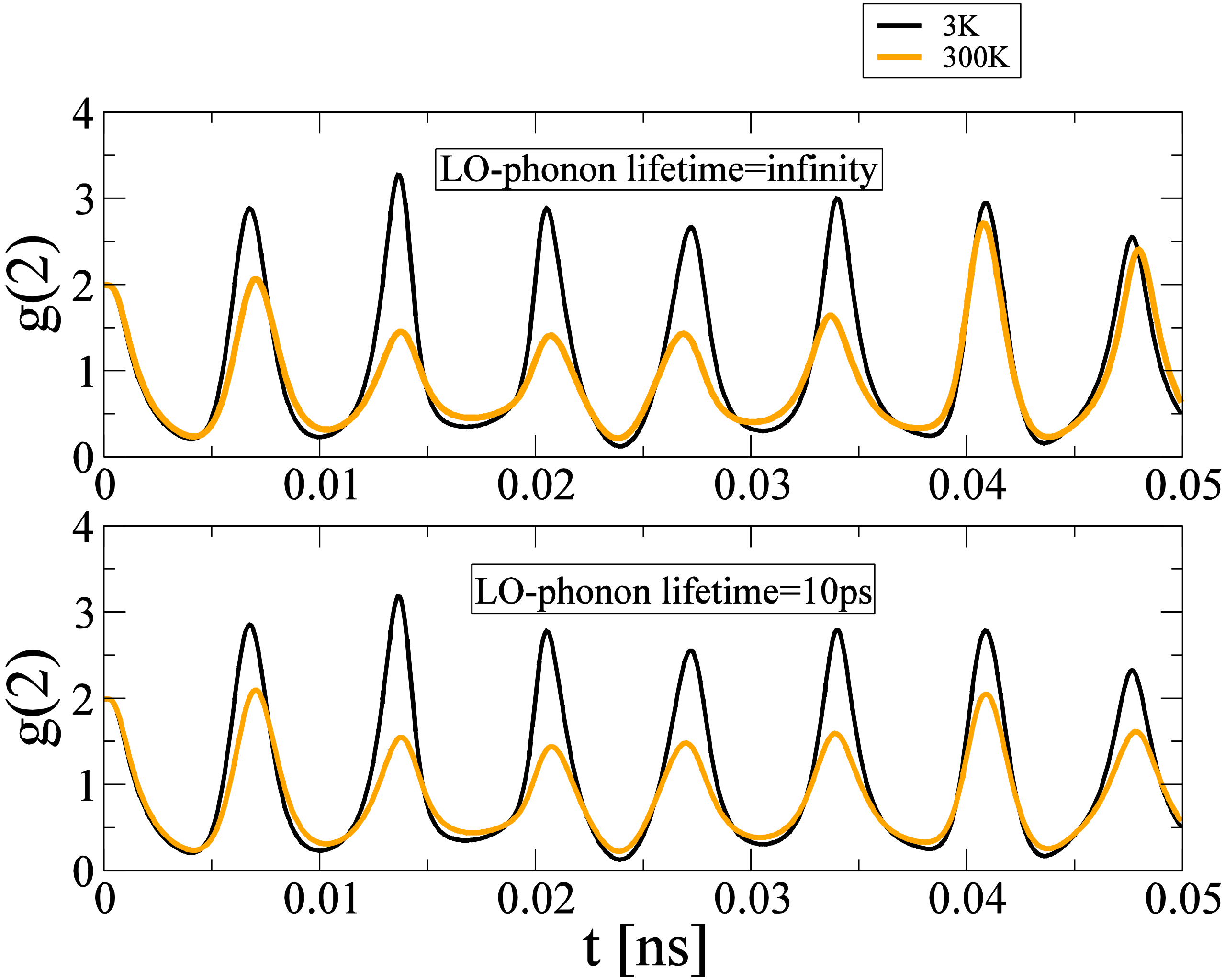
Microscopic theory helps to reveal advantageous quantum optical properties of semiconductor QD, for example:

- LO-phonon cavity feeding (phonon induced strong coupling)
- LO-phonon induced anti-bunching of a thermal cavity field
- Many-particle induced enhanced Pauli-blocking enforces single-photon emission

Theory for strongly coupled quantum dot cavity quantum electrodynamics

-- Photon statistics and phonon signatures in quantum light emission --

Thermal antibunching with LO-phonon life time



PPCE: general canonical statistical operator

$$
\begin{aligned}
\Big\langle |n\rangle\langle n| a_i^\dagger a_j^\dagger a_k a_l \Big\rangle &= \sum_{m,\{n_i\}} \langle n_1 ... n_N, m| \; |n\rangle\langle n| a_i^\dagger a_j^\dagger a_k a_l \, \rho | n_1 ... n_N, m\rangle \\
&\approx \sum_{\{n_i\}} \langle n_1 ... n_N | a_i^\dagger a_j^\dagger a_k a_l \, \langle n|\rho|n\rangle | n_1 ... n_N\rangle .
\end{aligned}
$$

$$
\begin{aligned}
\rho_{pn} &= \frac{1}{p_n}\langle n|\rho|n\rangle = \frac{1}{p_n}\langle n|\rho_{pt}\otimes\rho_{el}|n\rangle = \frac{1}{p_n}\langle n|\sum_m p_m |m\rangle\langle m| \otimes \rho_{el}|n\rangle \\
&= \frac{1}{p_n}\sum_m p_m \langle n|m\rangle\langle m|\rho_{el}|n\rangle = \frac{p_n}{p_n}\langle n|n\rangle \rho_{el} = \rho_{el} .
\end{aligned}
$$