Theory of quantum dot cavity-QED

-- LO-phonon induced cavity feeding and antibunching of thermal radiation --

Alexander Carmele, Julia Kabuss, Marten Richter, Andreas Knorr, and Weng W. Chow

OUTLINE

I: Introduction and Motivation

II: Mathematical induction method

III: Application: LO-phonon induced cavity feeding

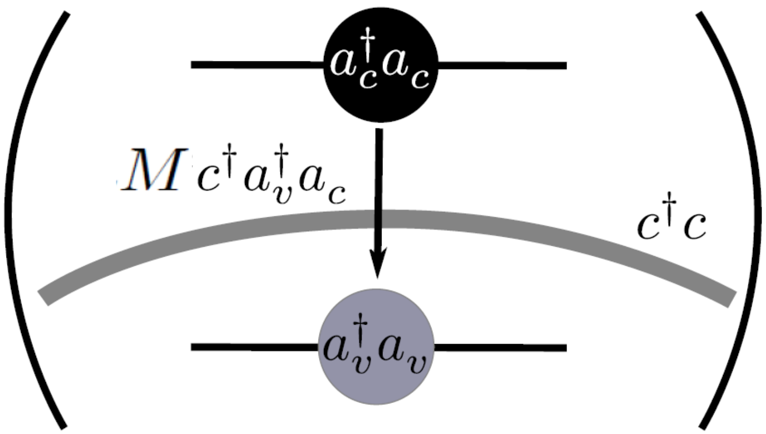
IV: Application: LO-phonon induced antibunching

V: Conclusion

Jaynes-Cummings model

Cavity-QED with an atom,
solved by the Jaynes-Cummings model [1]:

- ❑ Isolated two-level system (no losses)
- ❑ One-electron assumption
- ❑ One interaction: electron-light



„The simplest fully quantized model of interest" (J.H. Eberly)

[1] E. Jaynes and F. Cummings, Proc.IEEE 51, 89 (1963)

Jaynes-Cummings model

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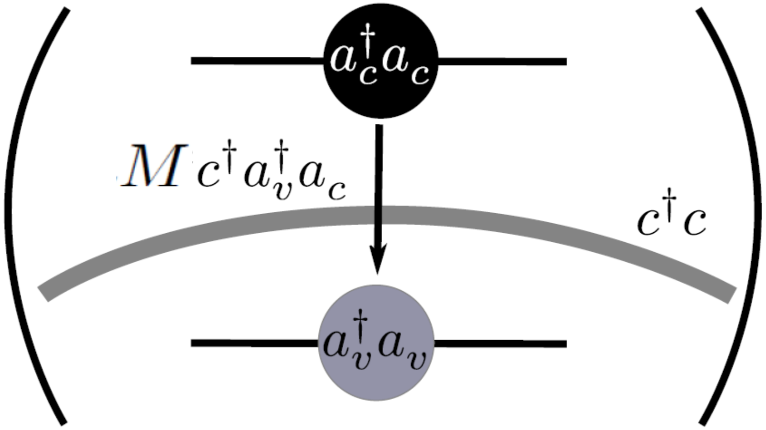

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„The simplest fully quantized model of interest“ (J.H. Eberly)

$$H_0 = \hbar\omega_0 c^\dagger c + \hbar\omega_v a_v^\dagger a_v + \hbar\omega_c a_c^\dagger a_c$$

$$H_{el-pt} = -\hbar M(a_v^\dagger a_c c^\dagger + a_c^\dagger a_v c)$$

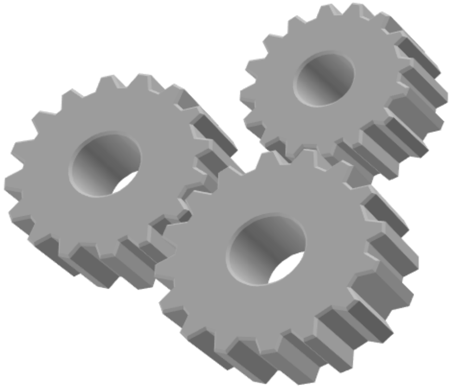
→ analytically solvable, e.g.

$$\langle a_c^\dagger a_c\rangle(t) = \cos^2(M\sqrt{N+1}\; t)$$

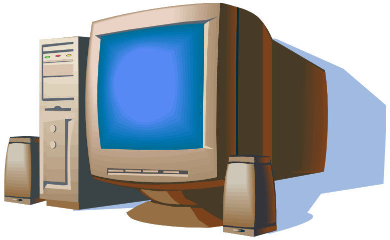
[1] E. Jaynes and F. Cummings, Proc.IEEE 51, 89 (1963)

Semiconductor nanostructures for device fabrication

Advantageous semiconductor QD properties for future technological applications in microcavity systems:



- ❑ Fixed emitter position,
- ❑ Cavity-emitter system ultra-small,
- ❑ tailorable coupling strength and optical properties

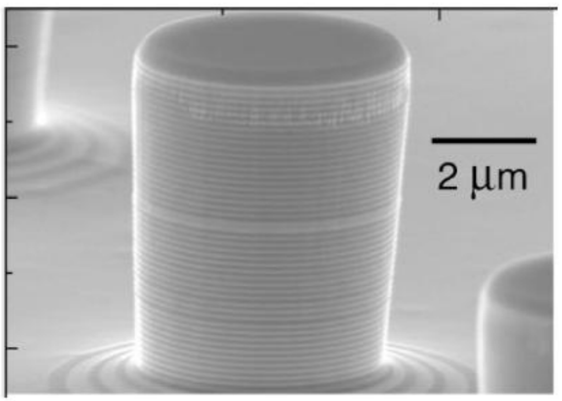


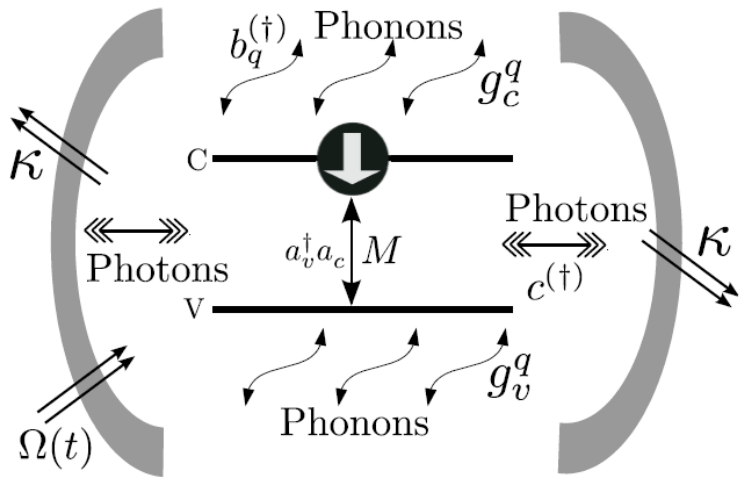
Semiconductor QD cavity-QED:
Theoretical simulations for device optimization are desirable

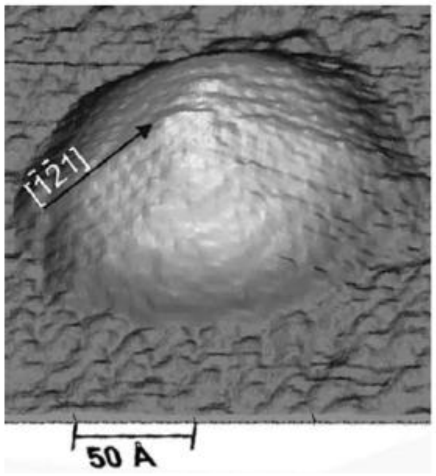
BUT:
more interactions need to be considered

Semiconductor QD cavity-QED Hamiltonian

Sanvitto et al., Appl.Phys.Lett. 86 (2005)







Marquez et al., Appl.Phys.Lett. 78 (2001)

$$
\begin{aligned}
H \quad = \quad & \hbar\omega_v a_v^\dagger a_v + \hbar\omega_c a_c^\dagger a_c - \hbar\Omega(t)(a_v^\dagger a_c + a_c^\dagger a_v) \\
& + \hbar\omega_0 c^\dagger c \; - \hbar M \left(a_v^\dagger a_c c^\dagger + a_c^\dagger a_v c\;\right) \\
& + \hbar \sum_q \omega_{LO} b_q^\dagger b_q + a_c^\dagger a_c \left(g_q^c b_q + g_q^{c*} b_q^\dagger\right) + a_v^\dagger a_v \left(g_q^v b_q + g_q^{v*} b_q^\dagger\right)
\end{aligned}
$$

QD, assumed as a 2-level system with one electron,

interacts with the cavity photons, bulk LO-phonons , a classical pump field

Solving LO-phonon QD cavity-QED without factorization

for every possible combination of phonon, photon, and electron operators:

$$G^{p,s}_{m,n} := a^\dagger_v a_v c^{\dagger p} c^s \bar{b}^{\dagger m} \bar{b}^n$$

$$E^{p,s}_{m,n} := a^\dagger_c a_c c^{\dagger p} c^s \bar{b}^{\dagger m} \bar{b}^n$$

$$T^{p,s}_{m,n} := a^\dagger_v a_c c^{\dagger p} c^s \bar{b}^{\dagger m} \bar{b}^n$$

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Using product rule for operators:

$$\partial_t \left(a^\dagger_c a_c c^\dagger c b^\dagger_q b_q \right) = \left(\partial_t\, a^\dagger_c a_c c^\dagger c \right) b^\dagger_q b_q + c^\dagger c \left(\partial_t\, a^\dagger_c a_c b^\dagger_q b_q \right)$$

and generalized commutation relations:

$$[A, F(B)] = [A, B] F'(B)$$

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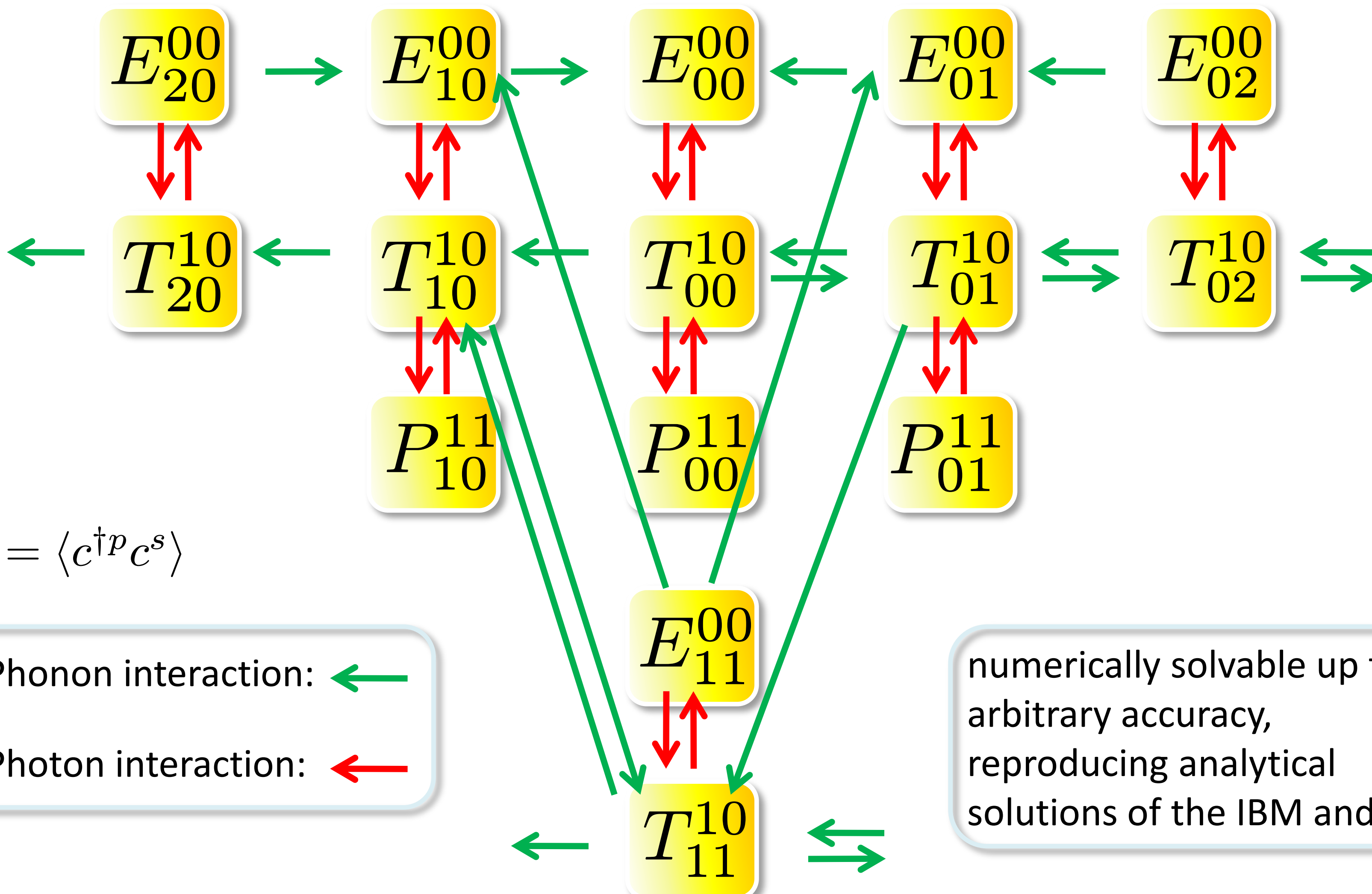
their dynamics can be calculated:

$$\begin{aligned} &\partial_t \langle T^{p,s}_{m,n} \rangle = \\ &= -i \left[\omega_{cv} - (p-s)\omega_0 - (m-n)\omega_{LO} - i(p+s)\kappa - i\gamma \right] \langle T^{p,s}_{m,n} \rangle \\ &\quad - ip\, M \langle E^{p-1,s}_{m,n} \rangle - iM (\langle E^{p,s+1}_{m,n} \rangle - \langle G^{p,s+1}_{m,n} \rangle) - i\Omega(t) \left(\langle E^{p,s}_{m,n} \rangle - \langle G^{p,s}_{m,n} \rangle \right) \\ &\quad - i \, \langle T^{p,s}_{m,n+1} \rangle - i \, \langle T^{p,s}_{m+1,n} \rangle + i \; m \; g_v \, \langle T^{p,s}_{m-1,n} \rangle - i \; n \; g_c \, \langle T^{p,s}_{m,n-1} \rangle, \end{aligned}$$

General set of equations of motion

For example, in the case of LO-phonon assisted vacuum Rabi oscillations ($E^{11}_{00} = 0$) :



$$P^{ps}_{00} = \langle c^{\dagger p} c^s \rangle$$

Phonon interaction: ←

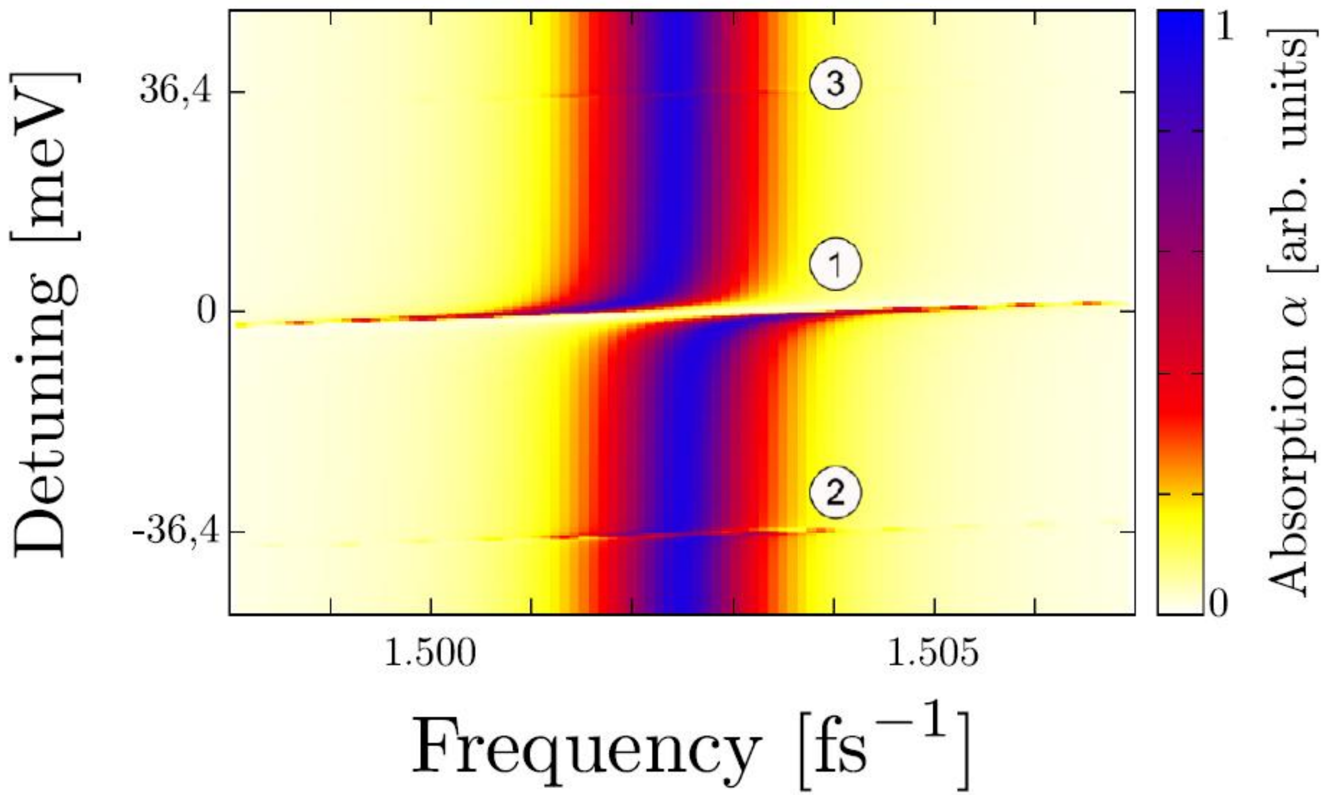
Photon interaction: ←

numerically solvable up to arbitrary accuracy, reproducing analytical solutions of the IBM and JCM

LO-phonon induced cavity feeding

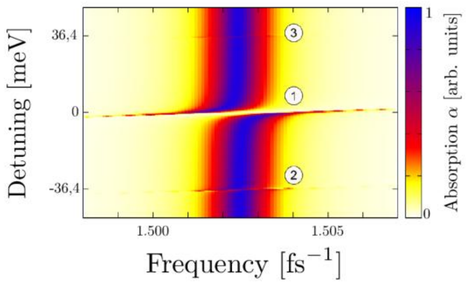
LO-phonon QD cavity-QED: additional anti-crossings

Probing with δ-pulse at 300K: $\alpha(\omega) \propto \mathrm{Re}\left[\frac{\langle T_{0,0}^{0,0}\rangle(\omega)}{\rho_0^T}\right]$

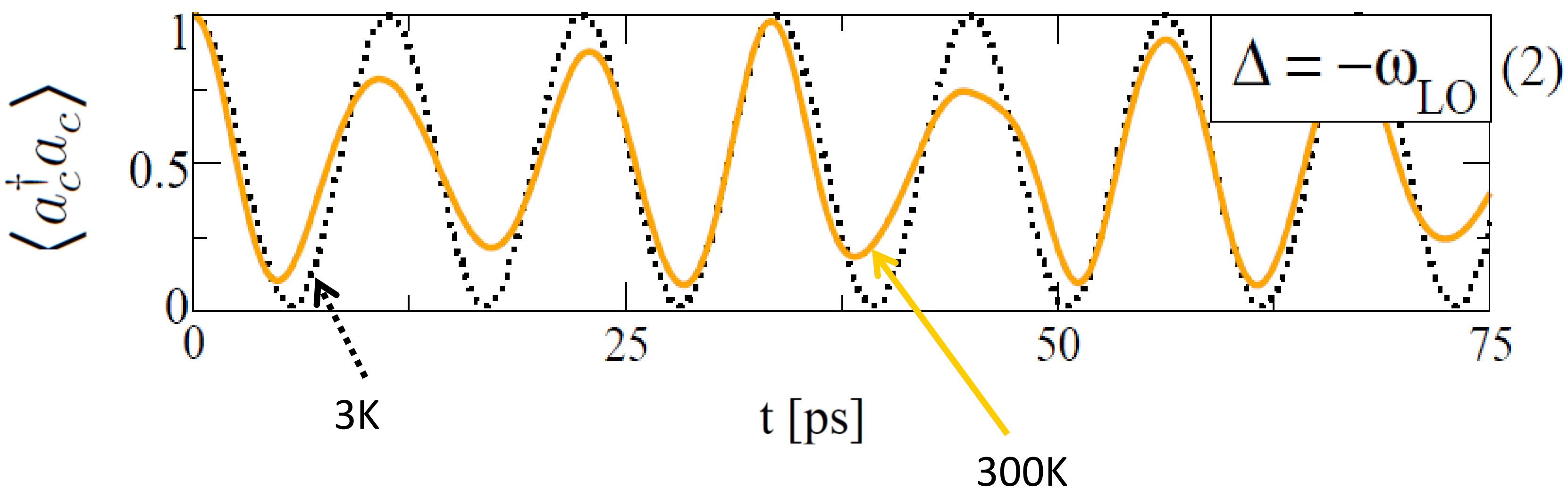


[1] in preparation (2011)

Modified Rabi frequency due to phonon cavity feeding



Modified Rabi frequency[1] at the Stokes-position (2), assuming an excited QD and no cavity photon (vacuum Rabi oscillations):



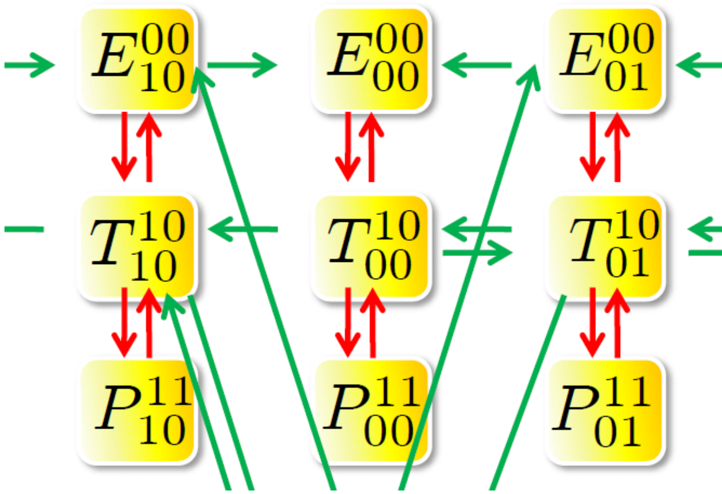
(i) Temperature dependence of frequency weak, but affects amplitude strongly
(ii) Rabi frequency now determined by the LO-phonon coupling strength, also

[1] in preparation (2011)

Modified Rabi frequency analytical expression

Modified Rabi frequency at the Stokes-position (2), assuming an excited QD and no cavity photon (vacuum Rabi oscillations):

$$\langle E^{00}_{00}\rangle(t) - \langle E^{00}_{00}\rangle(0) = \int_0^t \mathrm{d}t_1 2\mathrm{Im}\left[M\langle T^{10}_{00}\rangle(t_1)\right]$$



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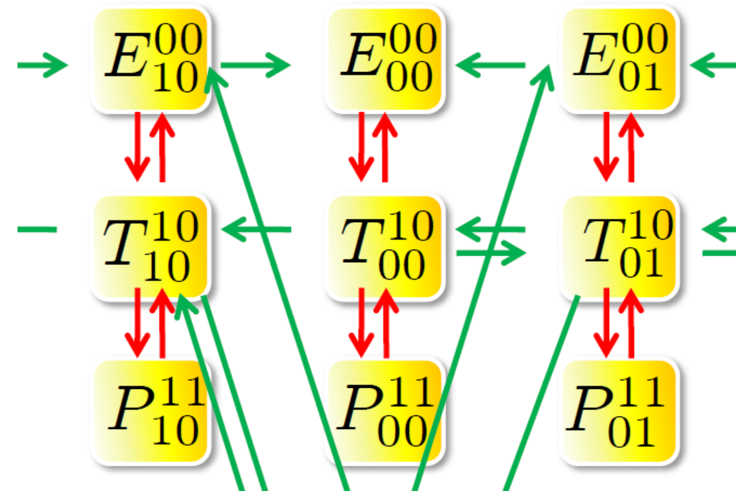

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$$\langle T^{10}_{00}\rangle(t_1) - \langle T^{10}_{00}\rangle(0) = e^{i\Delta t_1}\int_0^{t_1} \mathrm{d}t_2 e^{-i\Delta t_2}\left[-iM\langle E^{00}_{00}\rangle(t_2) - iM\langle E^{11}_{00}\rangle(t_2) + iM\langle P^{11}_{00}\rangle(t_2) - i\langle T^{10}_{10}\rangle(t_2) - i\langle T^{10}_{01}\rangle(t_2)\right]$$

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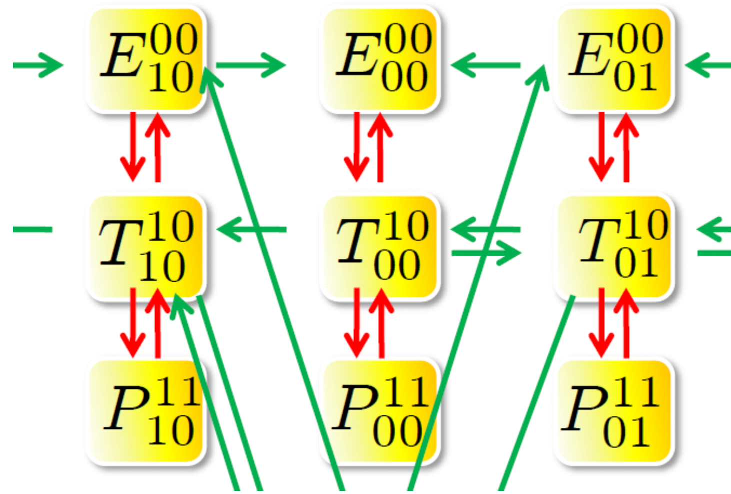

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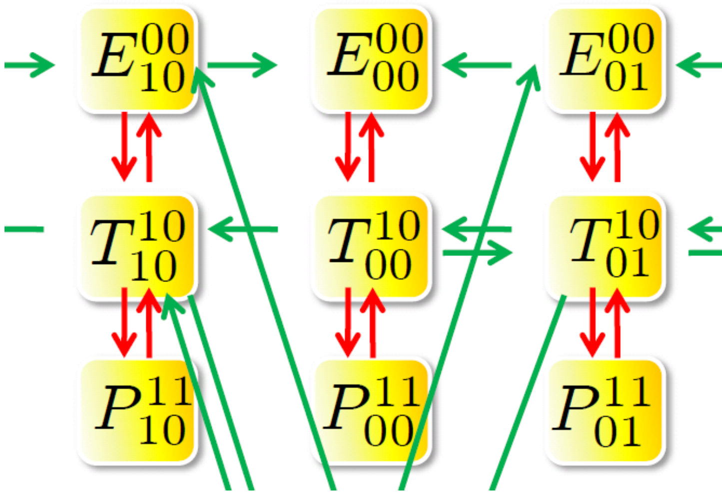

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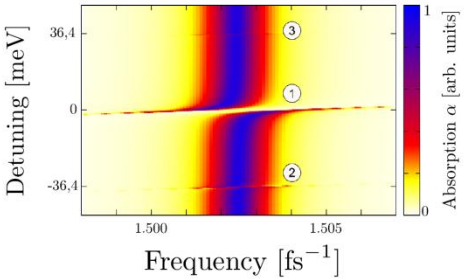
$$\langle E^{00}_{10}\rangle(t_3) - \langle E^{00}_{10}\rangle(0) = e^{i\omega_{LO}t_3}\int_0^{t_3} \mathrm{d}t_4 e^{-i\omega_{LO}t_4}\left[-iM\langle T^{10}_{10}\rangle(t_4) + iM\langle T^{10}_{01}\rangle^*(t_4) - i(\sum_q |g^q_{vc}|^2)\langle E^{00}_{00}\rangle(t_4)\right]$$

$$\langle E^{00}_{00}\rangle(t) - 1 = \left[-M^2\frac{g^2_{\mathrm{eff}}}{\omega^2_{\mathrm{LO}}}\right] t^2 = -\Omega^2 t^2$$

$$\Omega \approx \frac{M\ g_{\mathrm{eff}}}{\omega_{LO}}$$

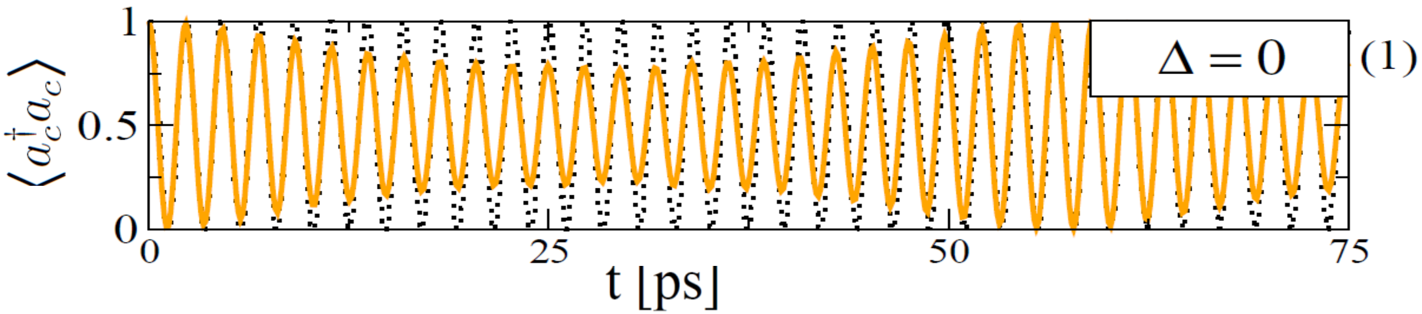
LO-phonon enhanced anti-bunching

Quantum beating at resonance position



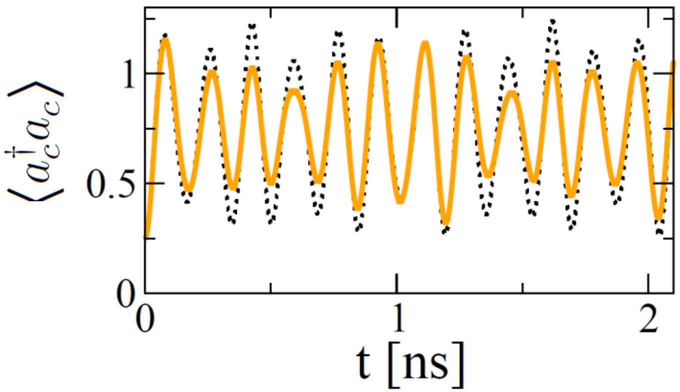
Quantum beating at resonance position (1):

Vacuum Rabi oscillations:



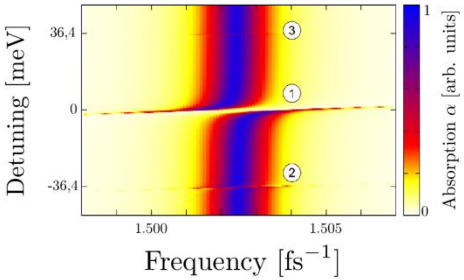
Thermal state:

for a cavity field, prepared initially in the thermal state



Impact small on Rabi oscillations

Quantum beating at resonance position



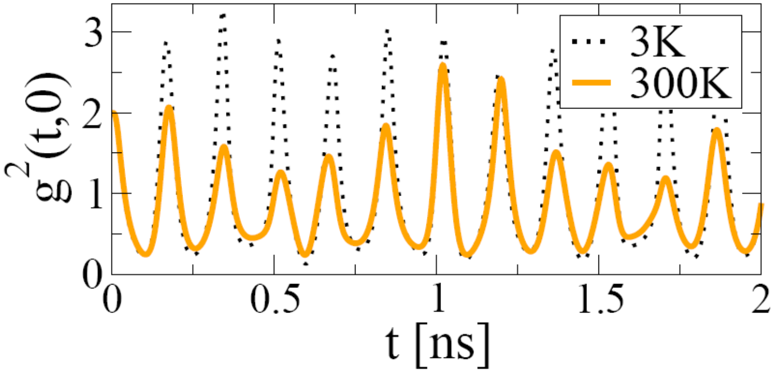
Quantum beating at resonance position (1):

Intensity-intensity correlation function:

$$g^{(2)}(t,\tau = 0) = \frac{\langle c^\dagger c^\dagger c c\rangle}{\langle c^\dagger c\rangle^2}$$

Thermal state:

for a cavity field, prepared
initially in the thermal state

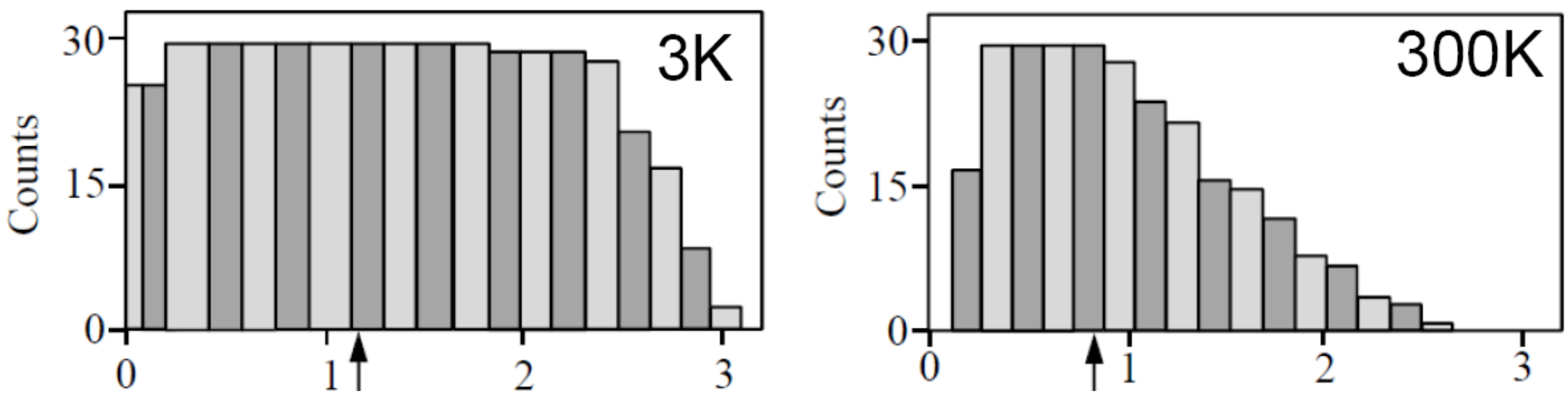


Impact strong on correlation function:

LO-phonon enhances quantum optical features

Impact on intensity-intensity correlation function mean value:

$$\bar{g}^{(2)} := \frac{1}{T_a}\int_0^{T_a} \mathrm{d}t \; g^{(2)}(t,0)$$



Thermal state: **1.1** $g^{(2)}(0)$ **0.8** $g^{(2)}(0)$

Cavity field is transformed into a non-classical field via LO-phonon interaction [1]

→ Semiconductor environment enforces non-classical light features

[1] PRL **104**, 156801 (2010)

Applications of the mathematical induction method

Systems, for which the induction method is successfully applied:

1. QD as a two-level system with one-electron (photon statistics) [1]
2. QD as a three-level system (quantum coherence) [2]
3. QD as a four-level system with two electrons (biexciton cascade) [3]

[1] PRL **104**, 156801 (2010) [2] PSSB, accepted (2010) [3] PRB **81**, 195319 (2010)

Conclusions

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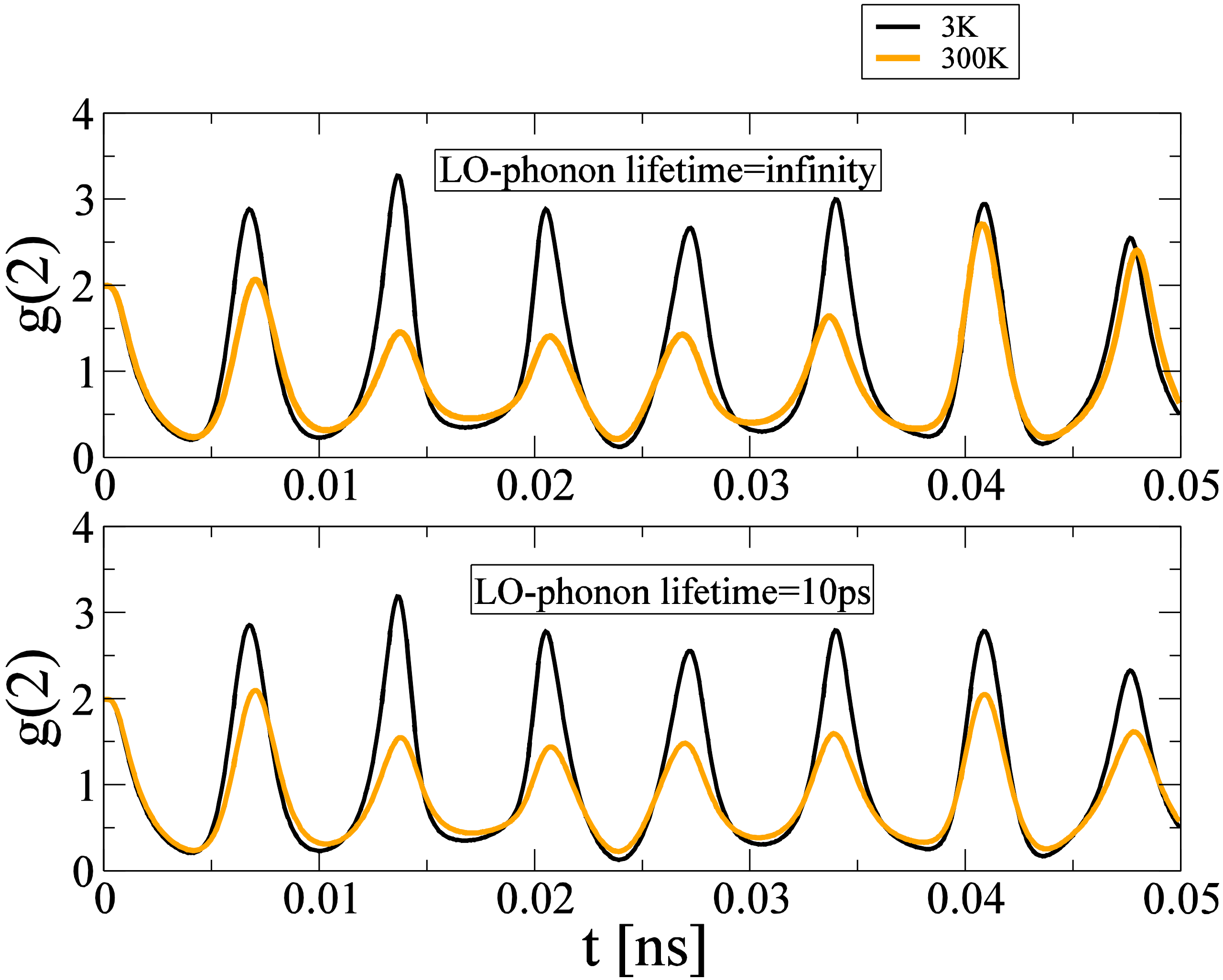
Thank you for your attention!!

[1] PRL **104**, 156801 (2010) [2] PSSB, accepted (2010) [3] PRB **81**, 195319 (2010)

Theory for strongly coupled quantum dot cavity quantum electrodynamics

-- Photon statistics and phonon signatures in quantum light emission --


3K
300K
LO-phonon lifetime=infinity
LO-phonon lifetime=10ps
g(2)
0
1
2
3
4
0
0.01
0.02
0.03
0.04
0.05
t [ns]

PPCE: general canonical statistical operator