Theory of quantum dot cavity-QED:

LO-phonon induced strong coupling and non-equilibrium phonon distributions

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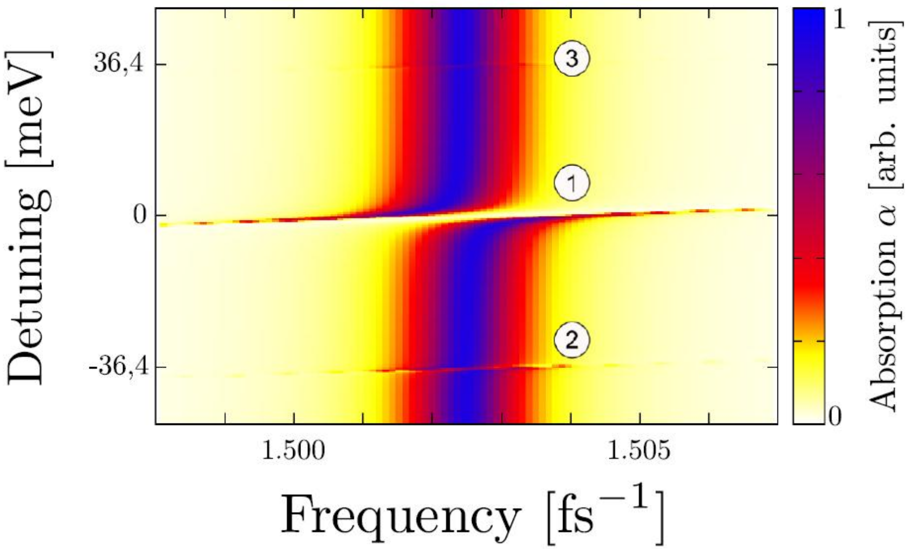
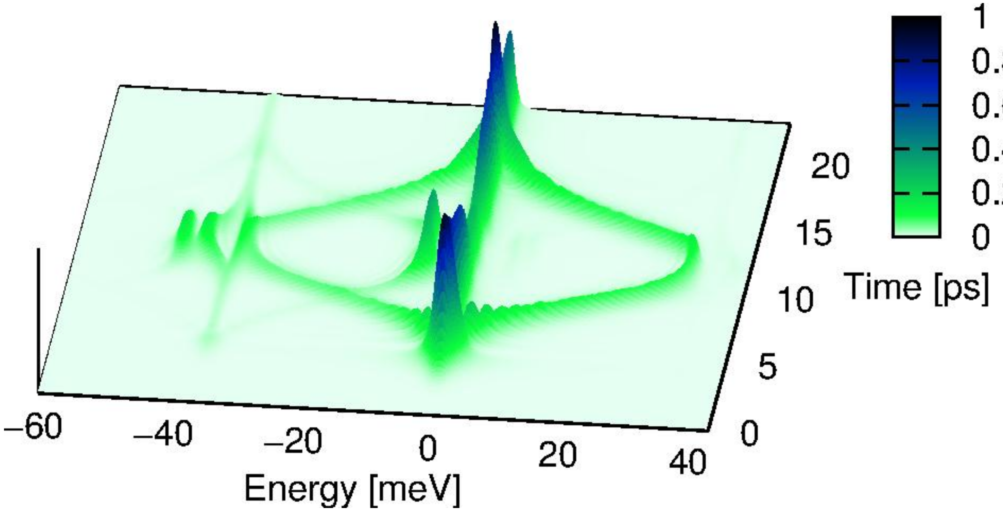
Strongy coupled photon-phonon-excitons:

Microcavity system:



Continuum case:



Outline

- ❑ Inductive equation of motion: Beyond typical perturbation approaches
- ❑ LO-phonon cavity-QED: Phonon-induced strong coupling
- ❑ QD-Luminescence: Phonon-assisted Mollow triplet

Inductive equation of motion method

Using product rule for operators: $\partial_t \left(a_c^\dagger a_c c^\dagger c\, b_q^\dagger b_q \right) = \left(\partial_t\, a_c^\dagger a_c c^\dagger c \right) b_q^\dagger b_q + c^\dagger c \left(\partial_t\, a_c^\dagger a_c b_q^\dagger b_q \right)$

and generalized commutation relations: $[A, F(B)] = [A, B]F'(B)$

for every possible combination of phonon, photon, and electron operators:

$$G^{p,s}_{m,n} := a_v^\dagger a_v c^{\dagger p} c^s \bar{b}^{\dagger m} \bar{b}^n$$
$$E^{p,s}_{m,n} := a_c^\dagger a_c c^{\dagger p} c^s \bar{b}^{\dagger m} \bar{b}^n$$
$$T^{p,s}_{m,n} := a_v^\dagger a_c c^{\dagger p} c^s \bar{b}^{\dagger m} \bar{b}^n$$

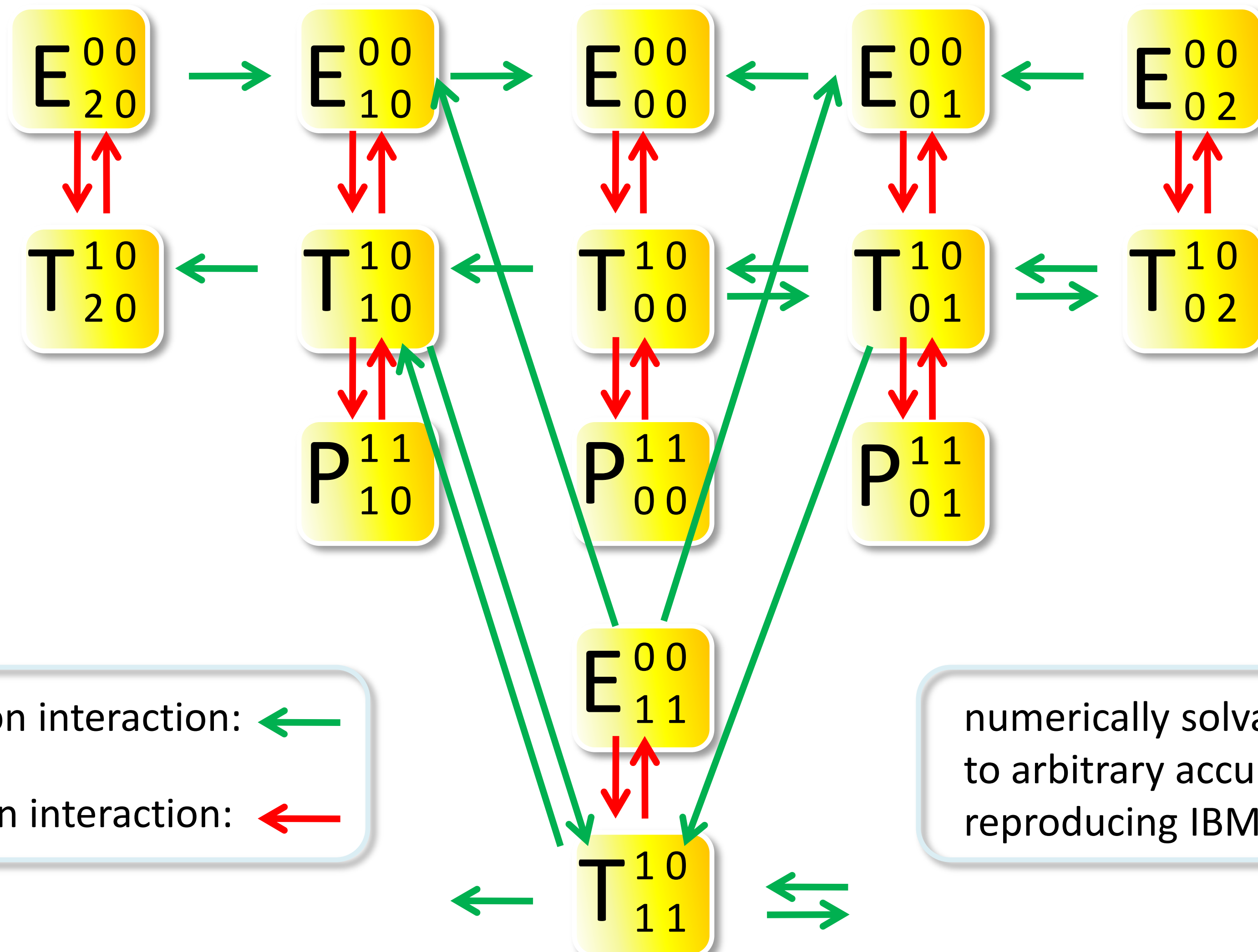
and

their dynamics, e.g.

$$\begin{aligned}
&\partial_t \langle T^{p,s}_{m,n} \rangle = \\
&= -i \left[\omega_{cv} - (p-s)\omega_0 - (m-n)\omega_{LO} - i(p+s)\kappa - i\gamma \right] \langle T^{p,s}_{m,n} \rangle \\
&\quad - ip\, M \langle E^{p-1,s}_{m,n} \rangle - iM (\langle E^{p,s+1}_{m,n} \rangle - \langle G^{p,s+1}_{m,n} \rangle) - i\Omega(t) \left(\langle E^{p,s}_{m,n} \rangle - \langle G^{p,s}_{m,n} \rangle \right) \\
&\quad - i \, \langle T^{p,s}_{m,n+1} \rangle - i \, \langle T^{p,s}_{m+1,n} \rangle + i\ m\ g_v \, \langle T^{p,s}_{m-1,n} \rangle - i\ n\ g_c \, \langle T^{p,s}_{m,n-1} \rangle,
\end{aligned}$$

Inductive equation of motion method

For example, in the case of LO-phonon assisted vacuum Rabi oscillations ($E^{1\,1}_{0\,0} = 0$) :



Phonon interaction: ←

Photon interaction: ←

numerically solvable up to arbitrary accuracy, reproducing IBM and JCM

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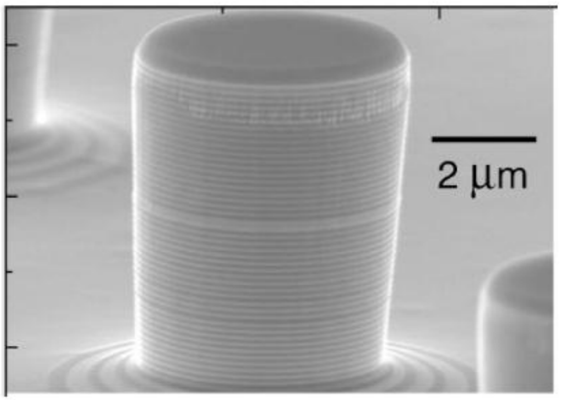
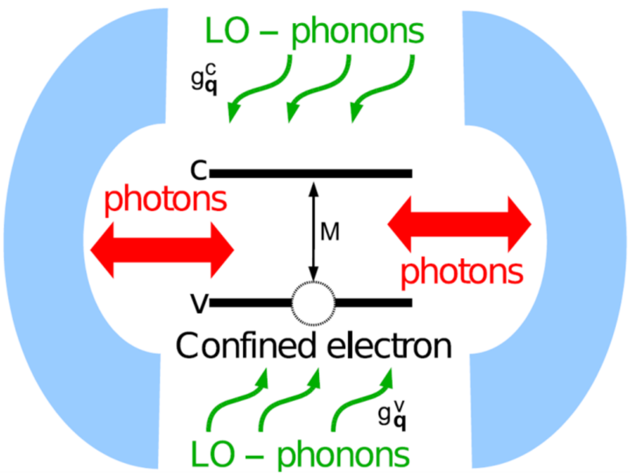
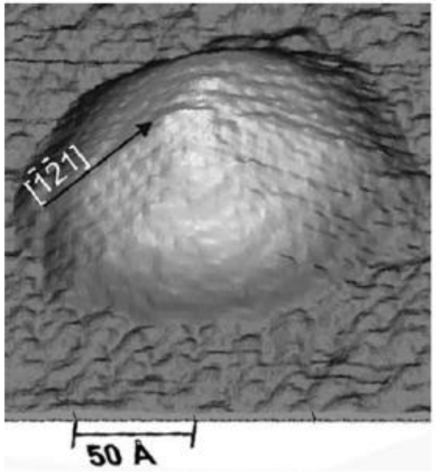
QD cavity-QED with bulk phonons (LO)

Sanvitto et al., Appl.Phys.Lett. 86 (2005)

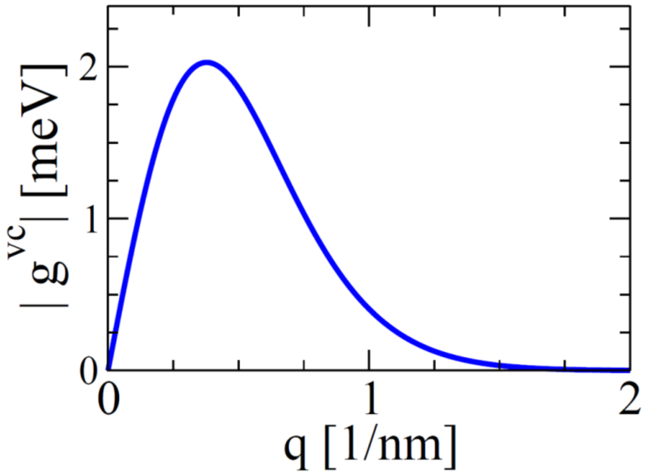






Marquez et al., Appl.Phys.Lett. 78 (2001)

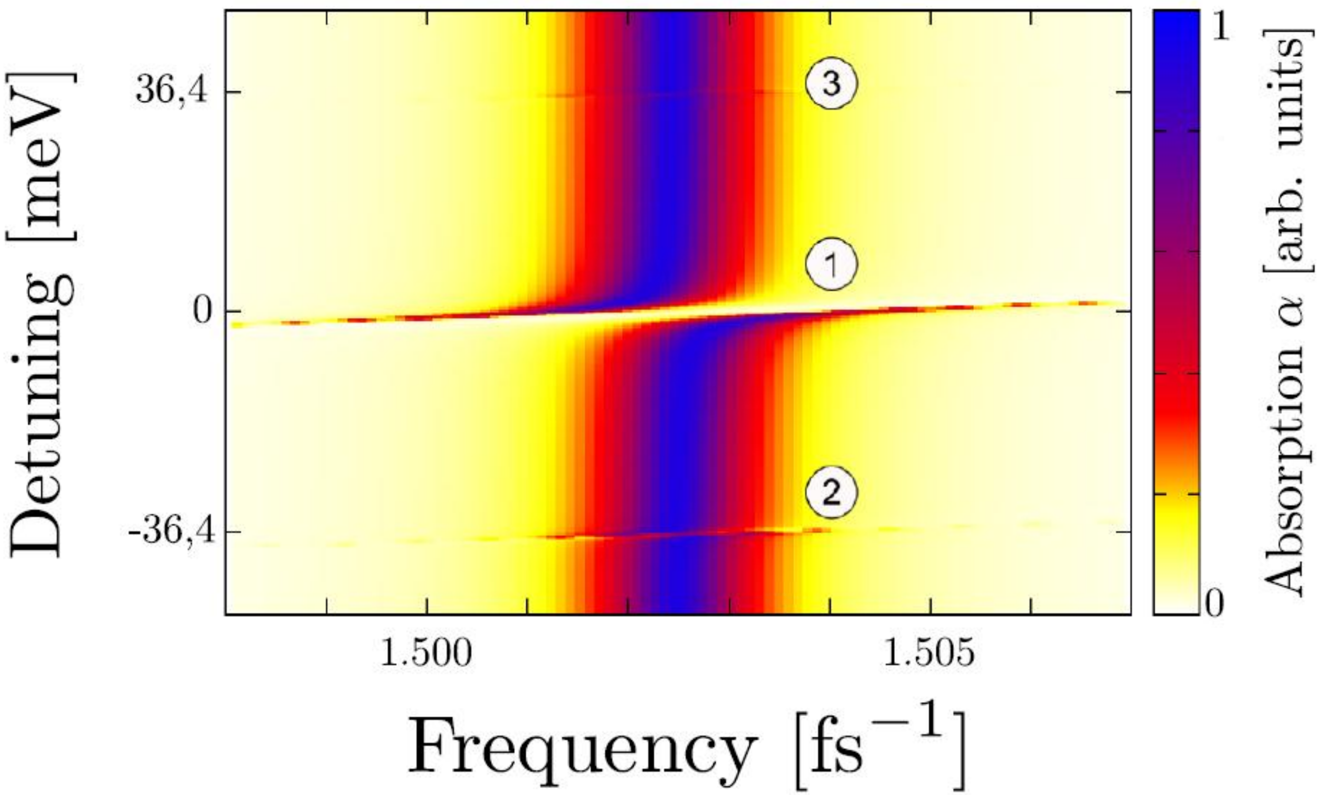
$$
\begin{aligned}
H \quad = \quad & \hbar\omega_v a_v^\dagger a_v + \hbar\omega_c a_c^\dagger a_c - \hbar\Omega(t)(a_v^\dagger a_c + a_c^\dagger a_v) \\
& + \hbar\omega_0 c^\dagger c \;\; - \hbar M (a_v^\dagger a_c c^\dagger + a_c^\dagger a_v c \;) \\
& + \hbar \sum_q \omega_{LO} b_q^\dagger b_q + a_c^\dagger a_c \left(g_q^c b_q + g_q^{c*} b_q^\dagger \right)
\end{aligned}
$$



Absorption signal

Probing with δ-pulse at 300K:

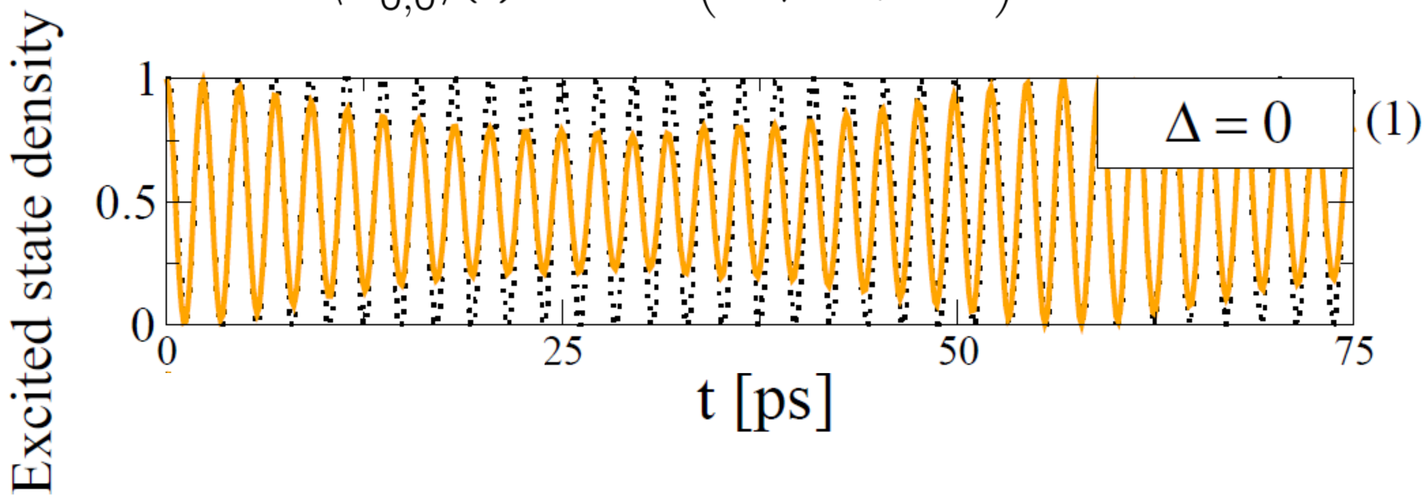
$$\alpha(\omega) \propto \mathrm{Re}\left[\frac{\langle T^{0,0}_{0,0}\rangle(\omega)}{\rho^T_0}\right]$$

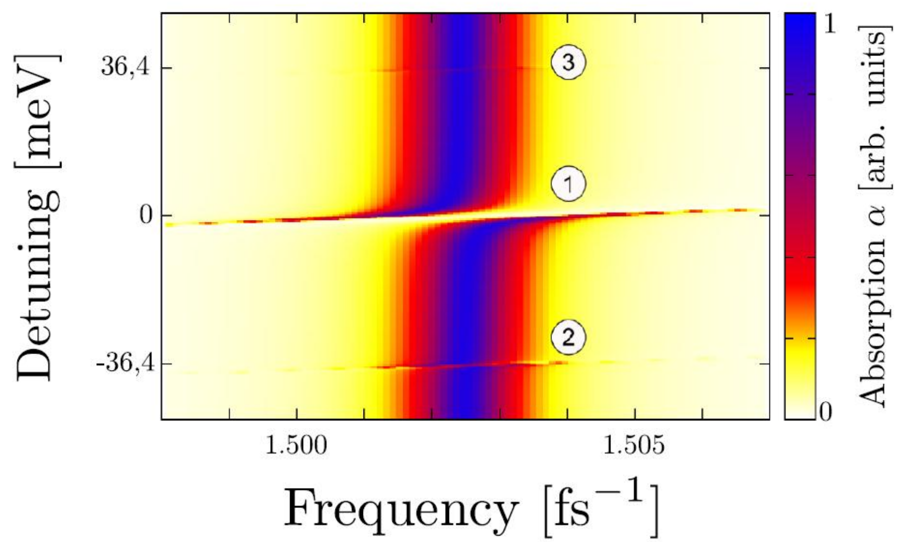


[1] in preparation (2012)

Rabi oscillation: different detunings

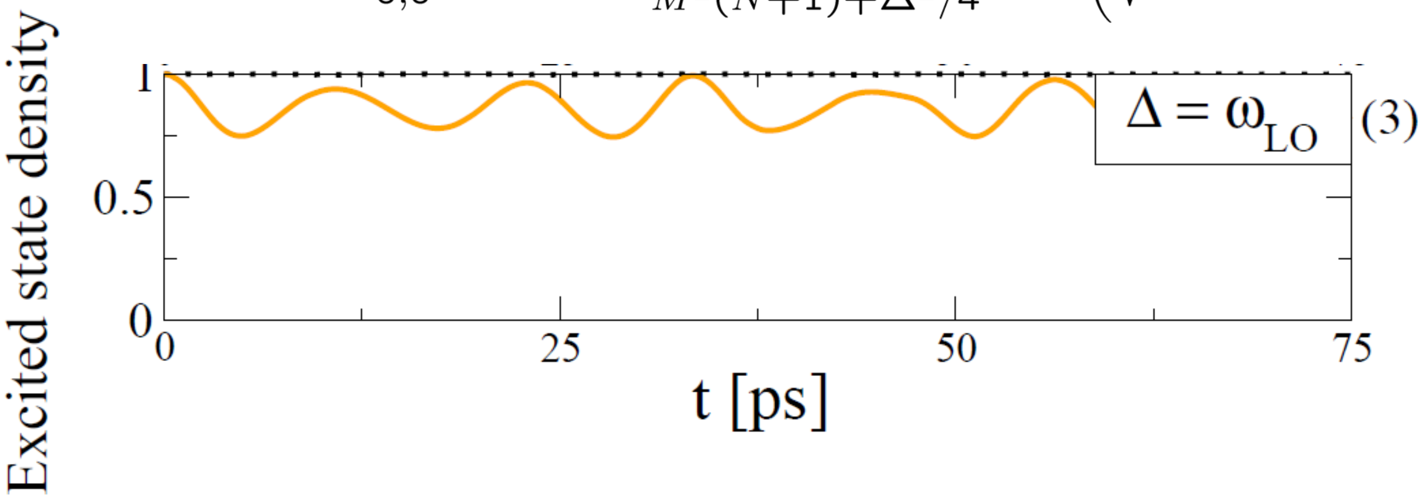
$$\langle E^{0,0}_{0,0}\rangle(t) = \cos^2\left(M\sqrt{N+1}\;\; t\right)$$

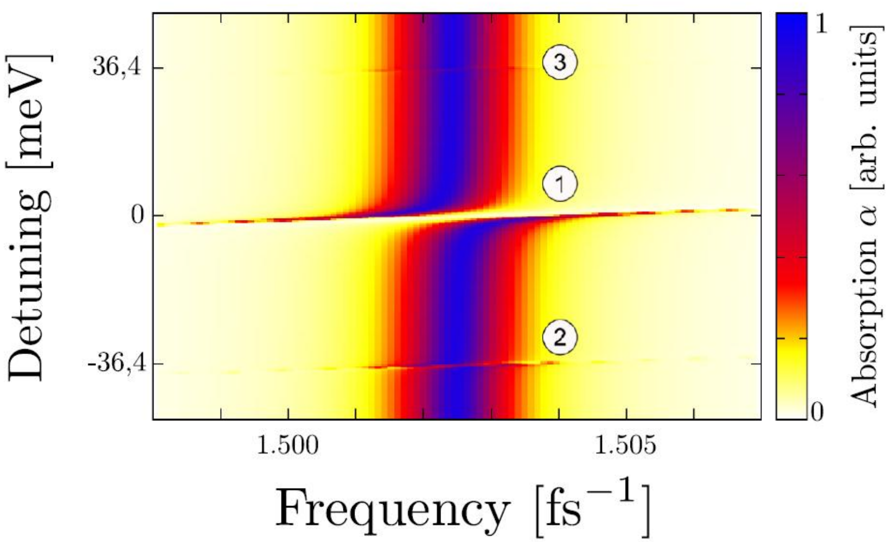




Rabi oscillation: different detunings

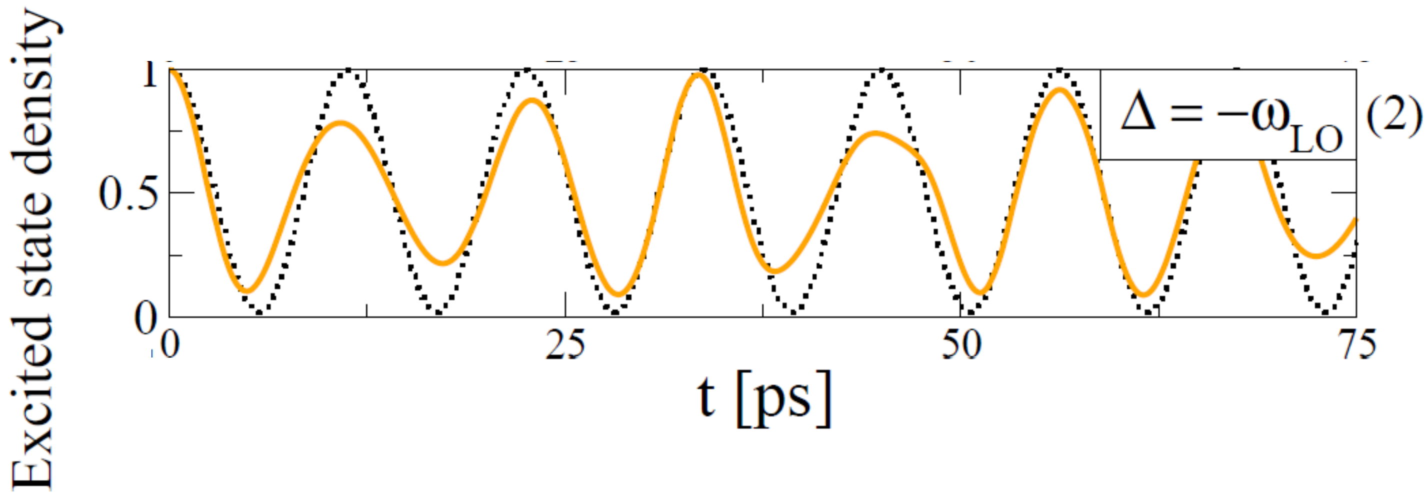
$$\langle E_{0,0}^{0,0}\rangle(t) = 1 - \frac{M^2(N+1)}{M^2(N+1)+\Delta^2/4}\sin^2\left(\sqrt{M^2(N+1)+\Delta^2/4}\;\; t\right)$$

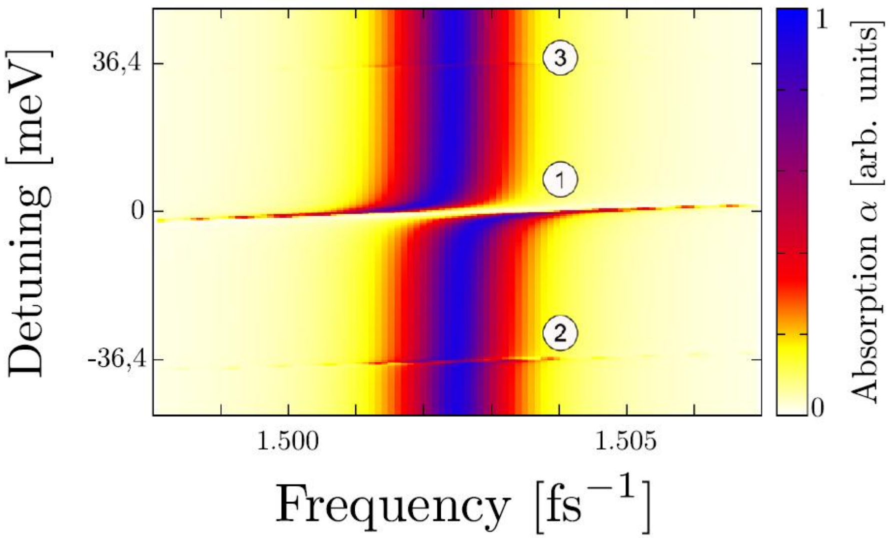




Rabi oscillation: different detunings

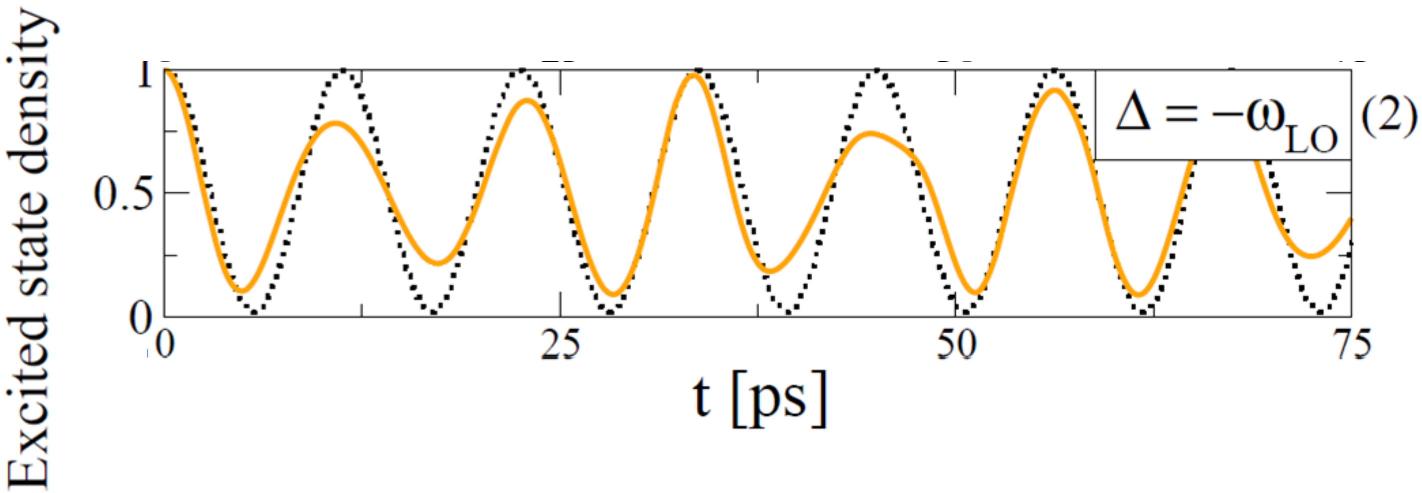
$$\langle E^{0,0}_{0,0}\rangle(t) = ????$$





Rabi oscillation: different detunings

$$\langle E^{0,0}_{0,0}\rangle(t) = ????$$

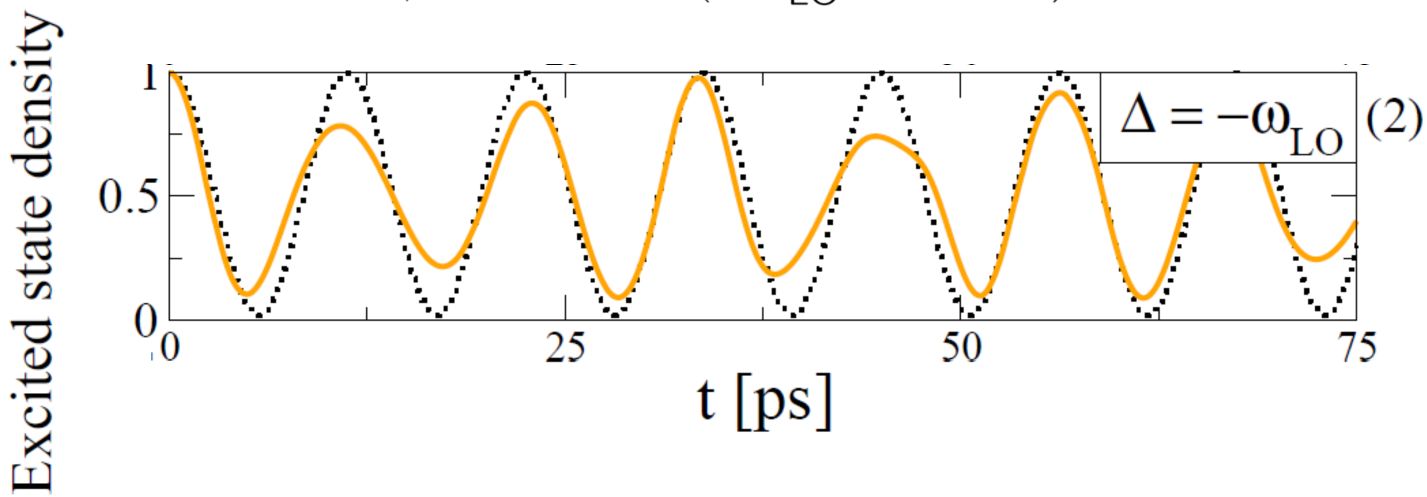


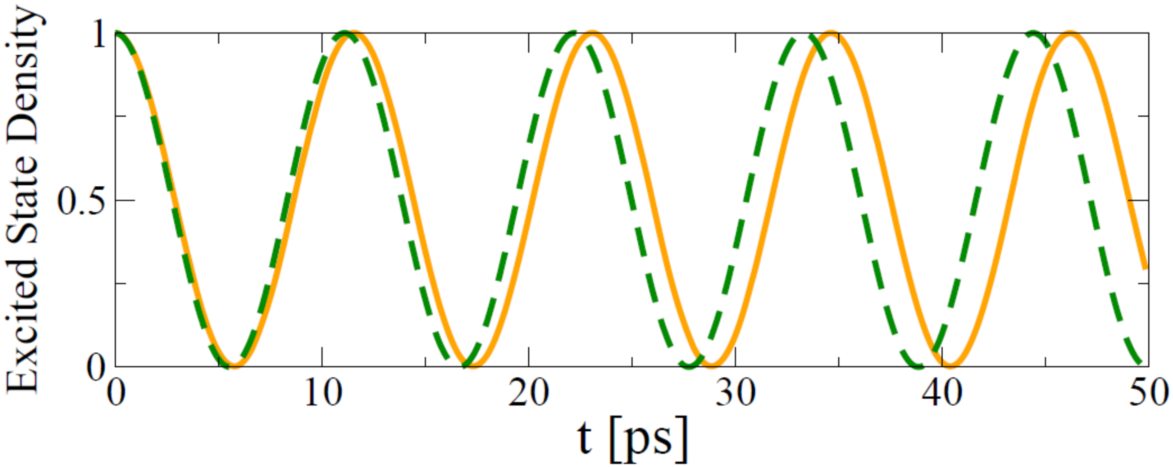
$$\begin{aligned}
\langle E^{0,0}_{0,0}\rangle(t) - \langle E^{0,0}_{0,0}\rangle(t_0) &= 2M \int_{t_0}^{t} \mathrm{d}t_1 \,\mathrm{Im}\left(\langle T^{1,0}_{0,0}\rangle(t_1)\right) \\
&= -2M^2 \int_{t_0}^{t} \mathrm{d}t_1 \int_{t_0}^{t_1} \mathrm{d}t_2 \,\langle E^{0,0}_{0,0}\rangle(t_2) \\
&= \ldots
\end{aligned}$$

$$\langle E^{0,0}_{0,0}\rangle(t) \approx 1 - \frac{M^2\, g^2_{\mathrm{eff}}}{\omega^2_{\mathrm{LO}}}\, t^2$$

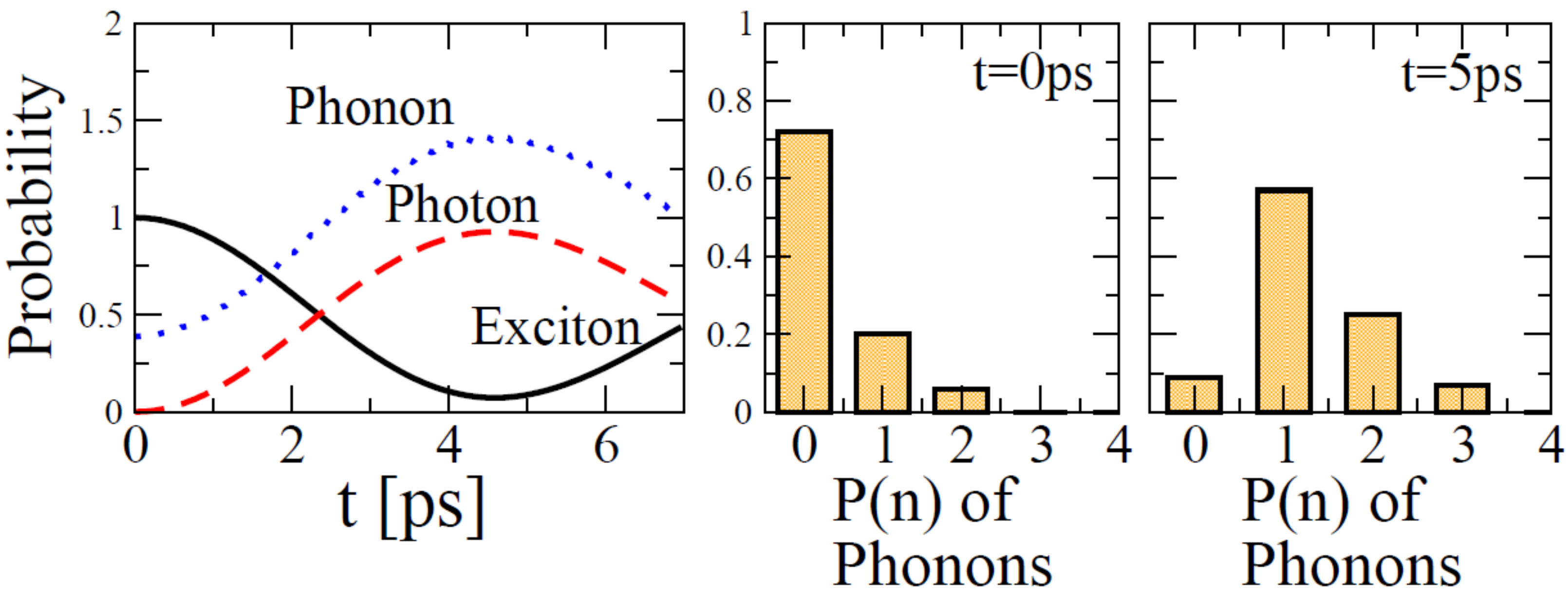
Rabi oscillation: different detunings

$$\langle E^{0,0}_{0,0}\rangle(t) \approx \cos^2\left(M\frac{g_{\text{eff}}}{\omega_{\text{LO}}}\sqrt{N+1}\ t\right)$$





LO-phonon-photon polariton



$$H_{epp} = \hbar M' \sum_k \left(a_v^\dagger a_c c_k^\dagger + h.a. \right)$$

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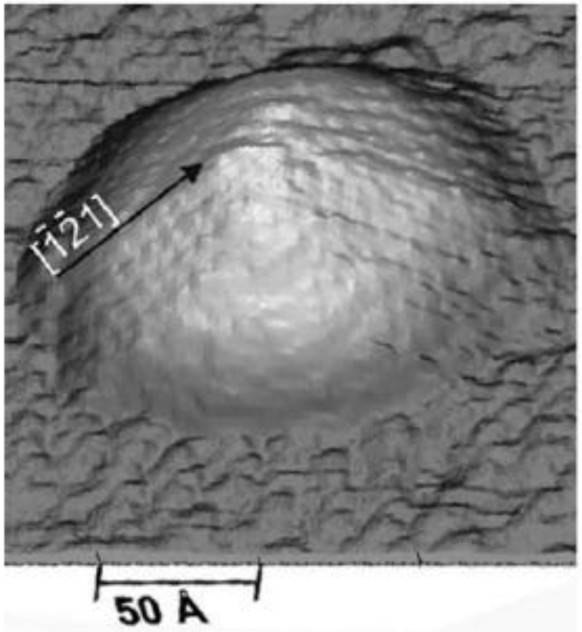
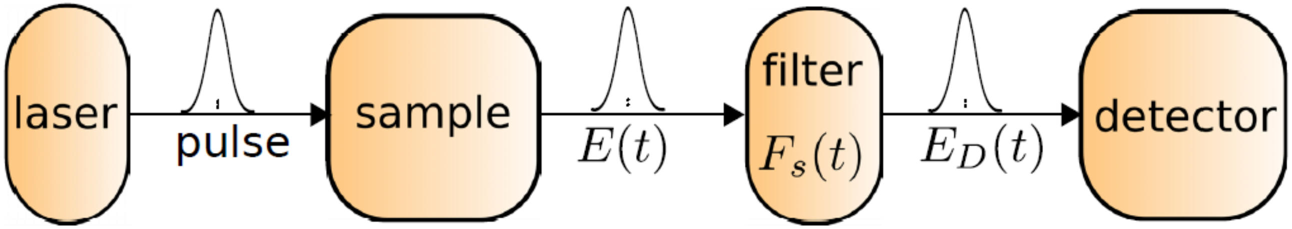
Semiconductor QD - Hamiltonian

Marquez et al., Appl.Phys.Lett. 78 (2001)

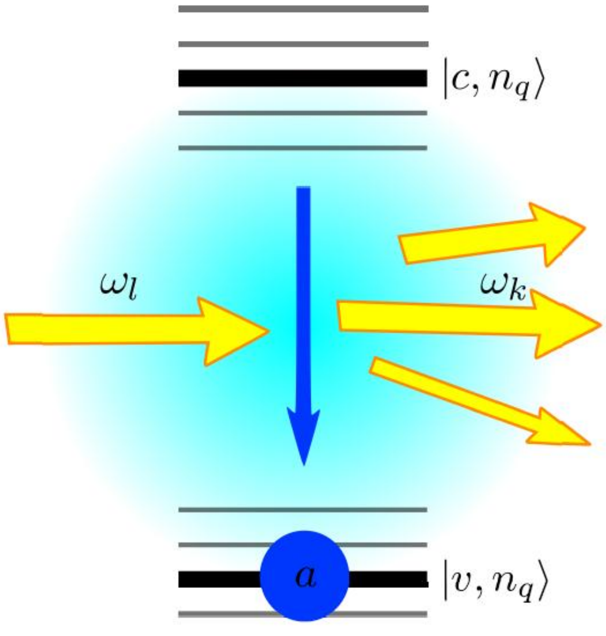


QD with one electron interacts with the bulk LO-phonons , continuum photons, and a classical pump field

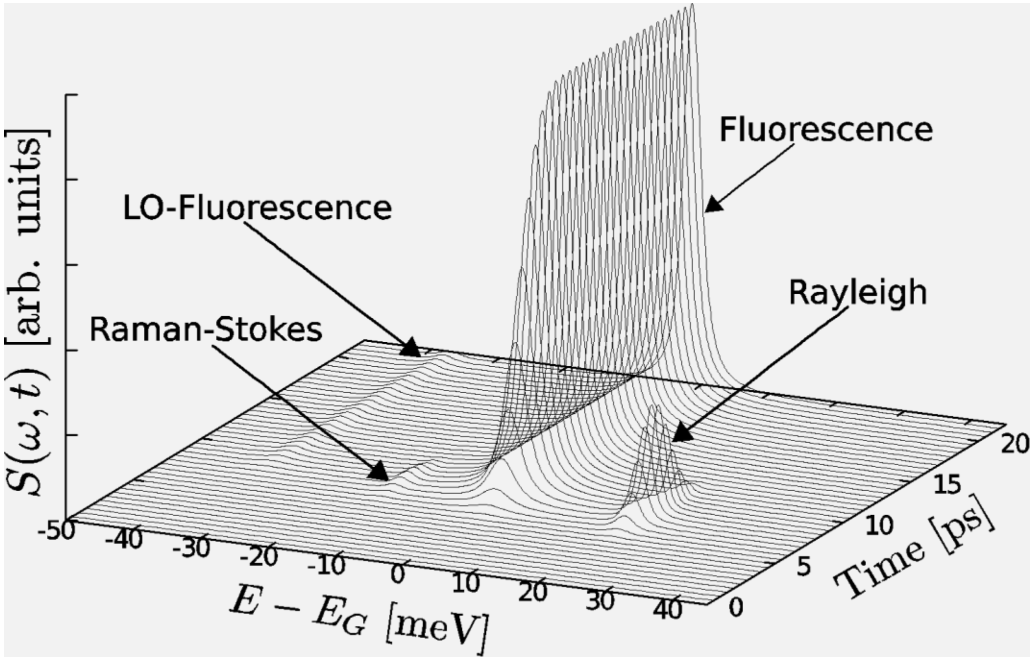


Externally excited QD:

Weak excitation:



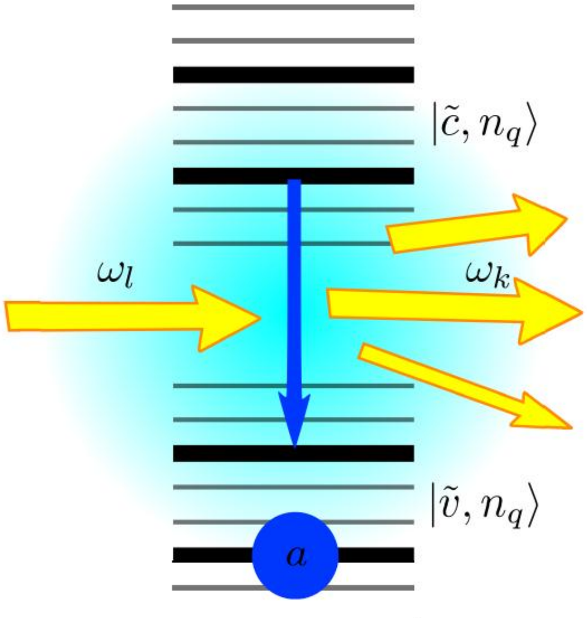
(a)weak excitation



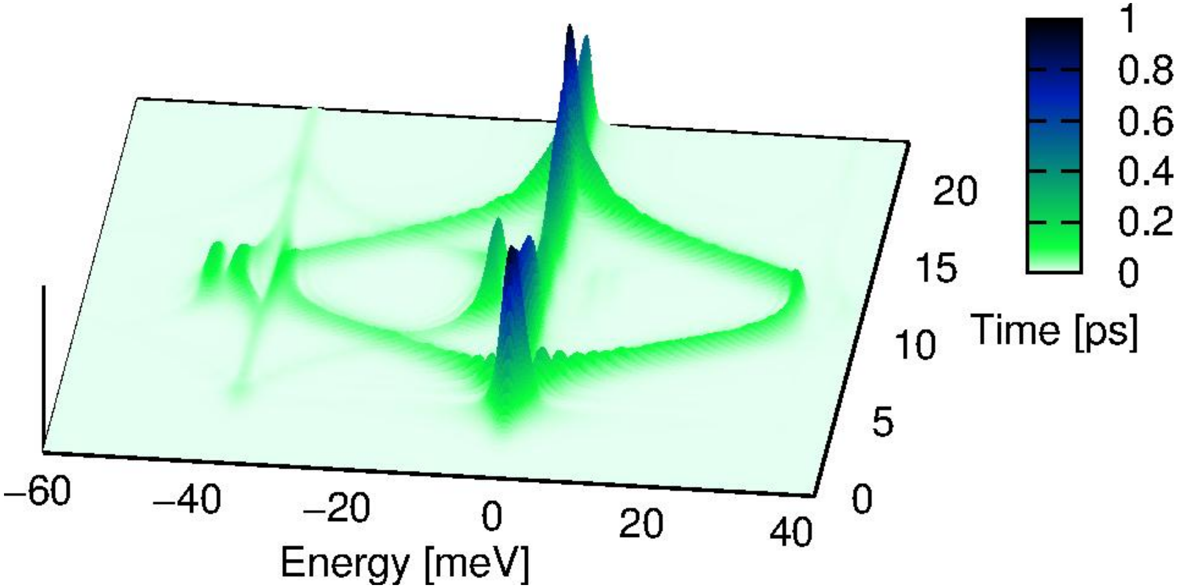
[1]PRB **81**, 075314 (2010)

Externally excited QD:

Strong excitation:

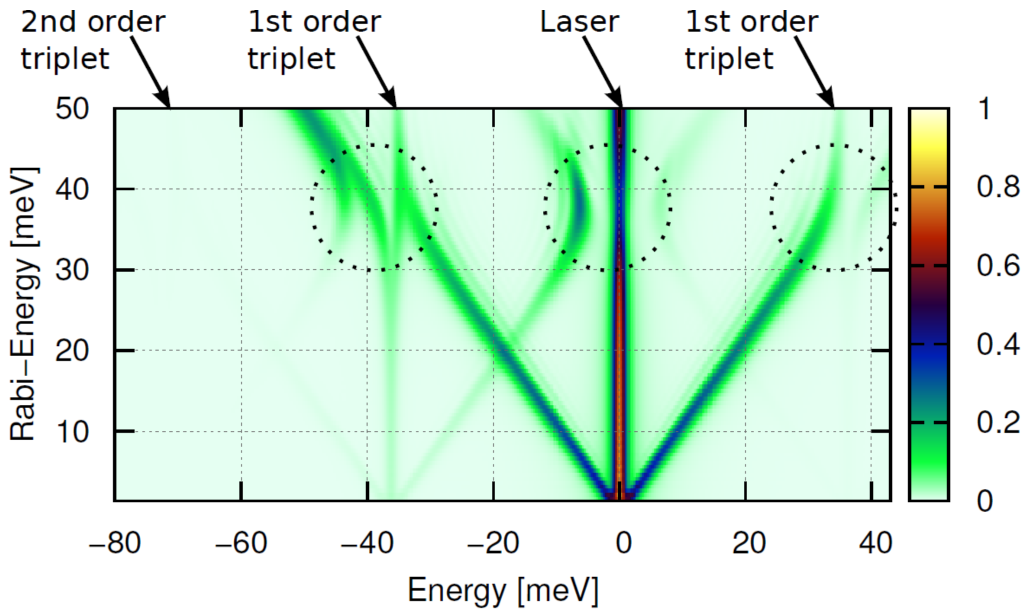


(b)strong excitation



[1]PRB **84**, 075314 (2011)

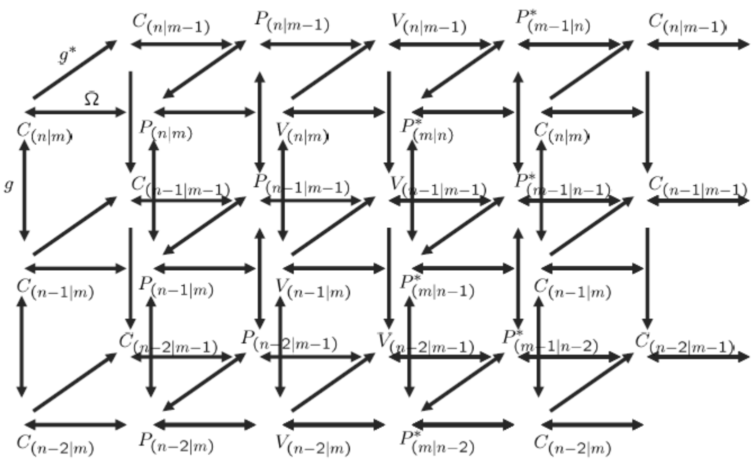
Emission under strong pulsed optical excitation

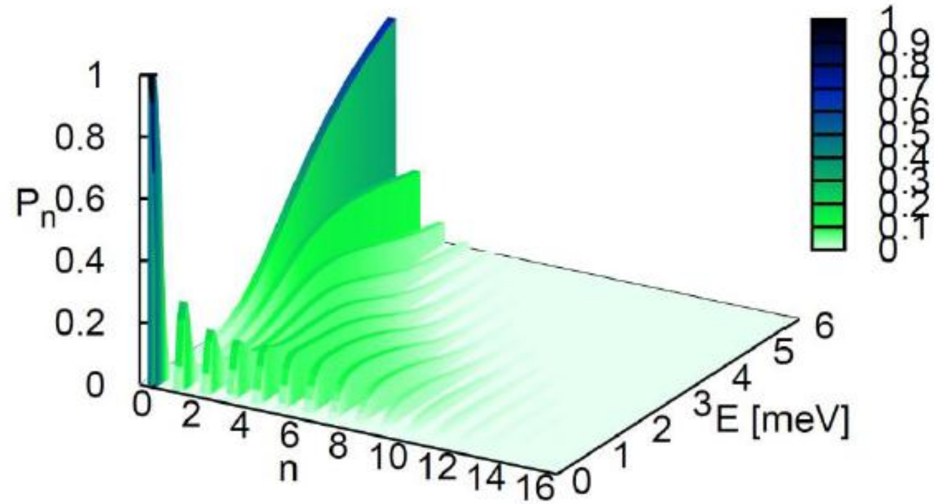


- Spectrum shows the usual Mollow triplet and phonon-assisted Mollow triplets
- Additional anticrossings, when the Rabi-energy matches the phonon energy (Here 36.4 meV for InGaAs/GaAs-QD)
- These anti-crossings scale with the electron-phonon coupling strength

[1]PRB **84**, 125324 (2011)

Laser induced non-equilibrium phonons





High orders of phonon operators become important [1,2]

[1]PRL **104**, 156801 (2010), [2]PSS(b) 248, **872** (2011)

Conclusion and outlook

- Microscopic calculation of electron-phonon interaction
- Arbitrary high order contributions via equation of motion method

- Phonon-assisted polariton: phonon-controlled strong coupling
- LO Phonon-assisted Mollow triplets: Huang Rhys Factor

Thank you for your attention !!