Theory of semiconductor quantum dot cavity-QED:

-- Addressing entanglement, non-equilibrium phonon, and photon distributions --

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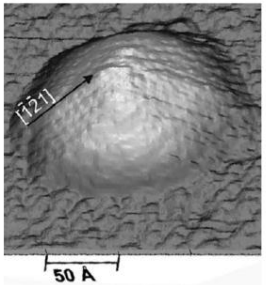
Nichtlineare Optik und Quantenelektronik
Technische Universität Berlin

OUTLINE



I: Semiconductor Quantum Dot

- Semiconductor physics and heterostructures
- Quantum Dot Hamiltonian and equation of motion approach

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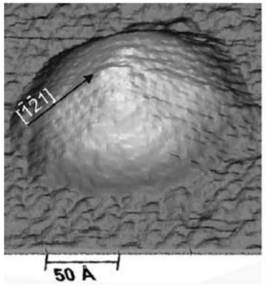

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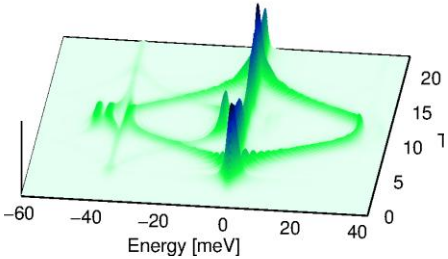

II: Quantum Dot as a Two-Level System

- ❑ Time-resolved phonon-assisted Mollow triplet
- ❑ Proposal for a phonon laser
- ❑ Photon-loss and induced quantum feedback

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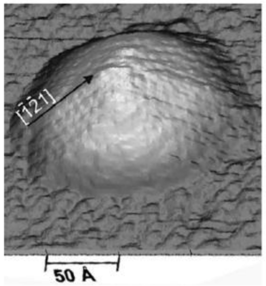

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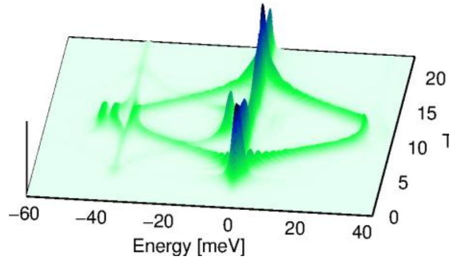

II: Quantum Dot as a Two-Level System

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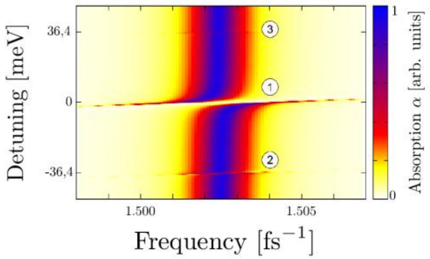
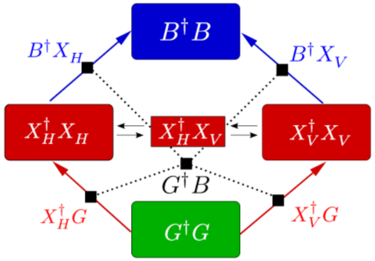
III: Quantum Dot as a Four-Level System

- Entangled photons: Pure dephasing does not matter
- Entangled photons: Multi-phonon scattering beyond 70K
- Strongly coupled cavity-QED: Crosscorrelation





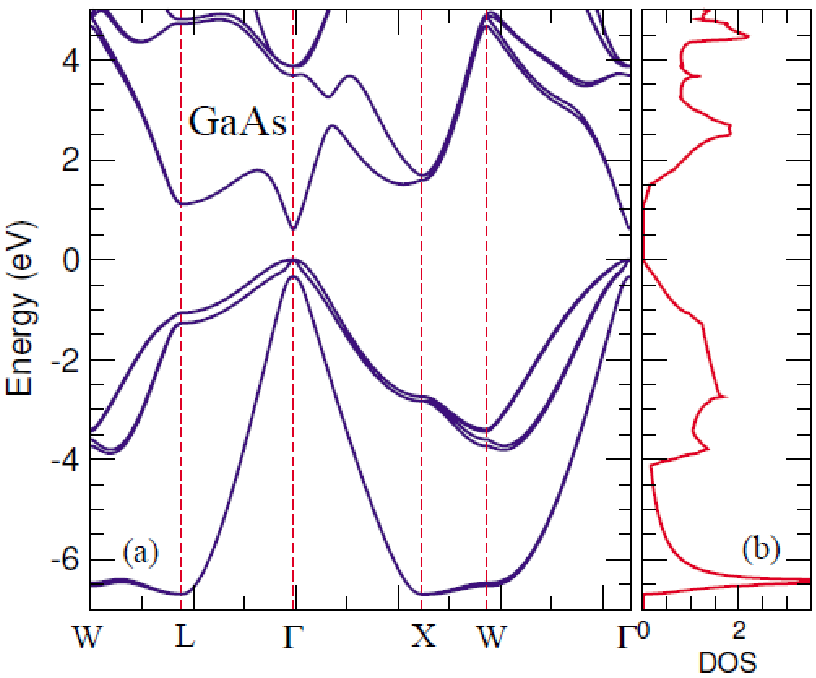
(i) semiconductor quantum dot

- Semiconductor physics and heterostructures
- Quantum Dot Hamiltonian and equation of motion approach

Semiconductor structures – effective mass approximation

$$\mathscr{H}_0 = -\frac{\hbar^2}{2m_0}\Delta + V_{\text{lat}}(\mathbf{r})$$

Bandstructure of GaAs:
Parabolic structure of lowest conduction and highest valence band



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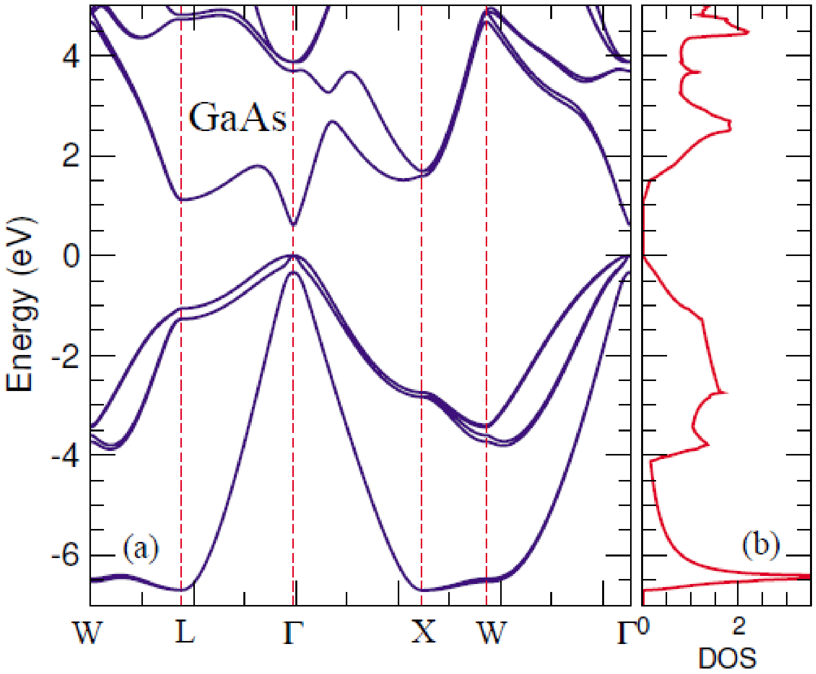


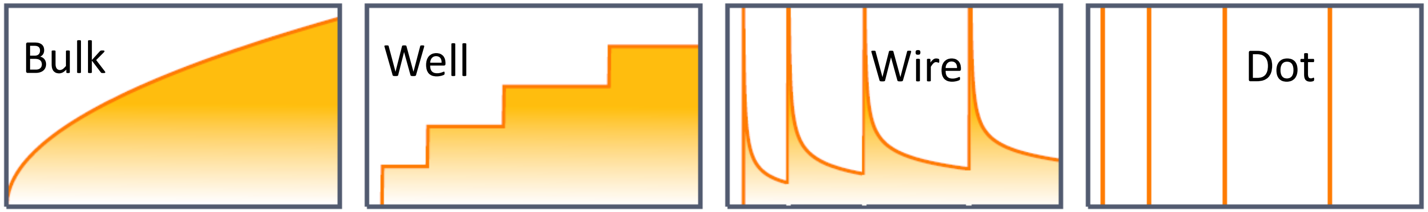
$$E_n(\mathbf{k}) \approx E_n(0) + \frac{\hbar^2 k^2}{2m_n^*} \quad \text{with} \quad \frac{1}{m_n^*} = \frac{1}{\hbar^2}\frac{\partial^2 E_n(\mathbf{k})}{\partial k^2}\bigg|_{\mathbf{k}_\Gamma=0}$$

Mixing semiconductors with different band gaps: nanostructures

Semiconductor Nanostructures and second quantization

Confinement potential:
Geometry, Size,
Material specifics

$$\left[-\frac{\hbar^2}{2m_n^*}\Delta + V_{\text{conf}}(\mathbf{r})\right]\xi_n(\mathbf{r}) = \varepsilon_n \xi_n(\mathbf{r}) \quad \text{with} \quad \varepsilon_n = E - E_n(0)$$



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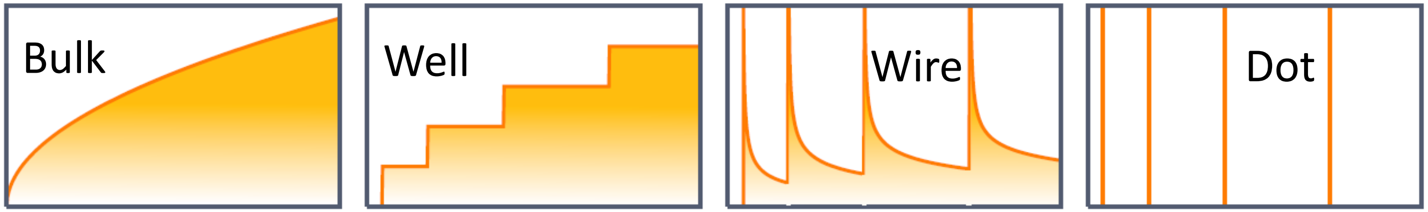

$$\varphi_n(\mathbf{r}) = u_{n,\mathbf{k}\approx 0}(\mathbf{r})\xi_n(\mathbf{r}) \qquad \hat{\Psi}(\mathbf{r}) = \sum_n \varphi_n(\mathbf{r})\hat{a}_n, \qquad \hat{\Psi}^\dagger(\mathbf{r}) = \sum_n \varphi_n^*(\mathbf{r})\hat{a}_n^\dagger$$

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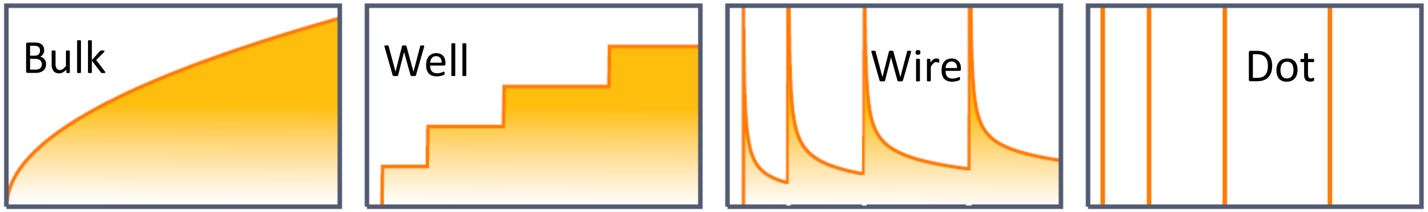

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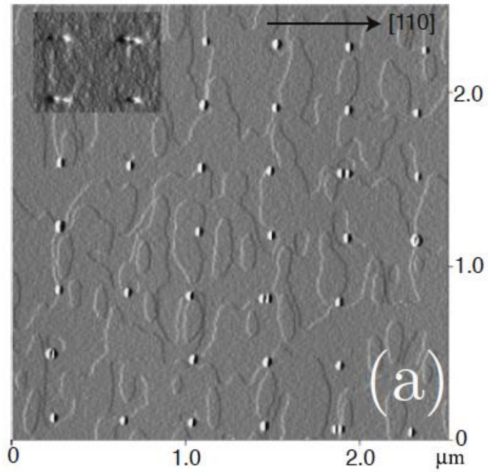
Microscopic calculated
Wave function for
2nd quantization

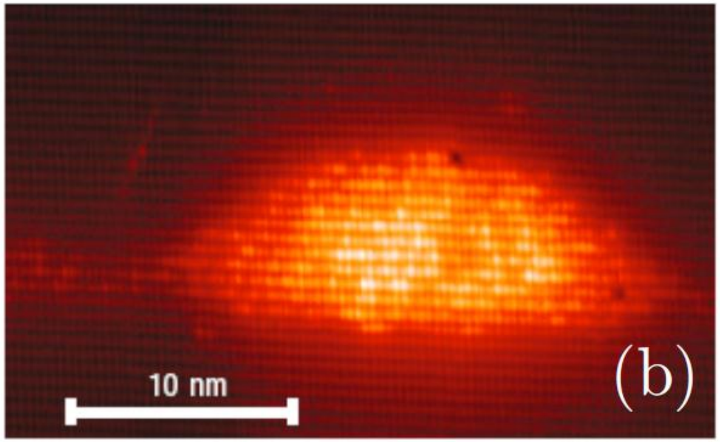
$$\hat{H}_0^c = \int d^3r\ \hat{\Psi}^\dagger(\mathbf{r})\left(-\frac{\hbar^2}{2m_0}\Delta + V_{\text{lat}}(\mathbf{r}) + V_{\text{conf}}(\mathbf{r})\right)\hat{\Psi}(\mathbf{r})$$
$$= \sum_n \varepsilon_n \hat{a}_n^\dagger \hat{a}_n.$$

Semiconductor quantum dots

Quantum dots:
An artifical atom



Atkinson et al.,Jpn. J. Appl. Phys. 45 (2006)



Shields, Nat. Photonics, 221 (2007)

Discrete energy levels
→ Optical properties by design
→ Electrical pumping possible

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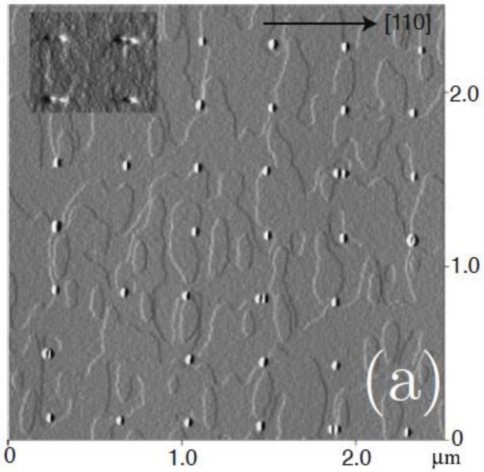

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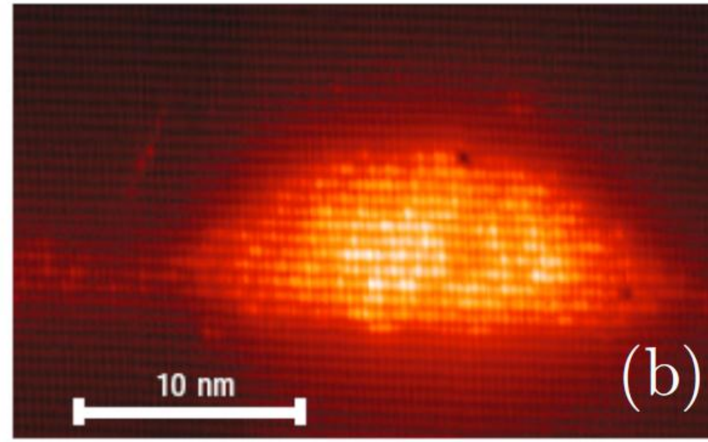

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Discrete energy levels
→ Optical properties by design
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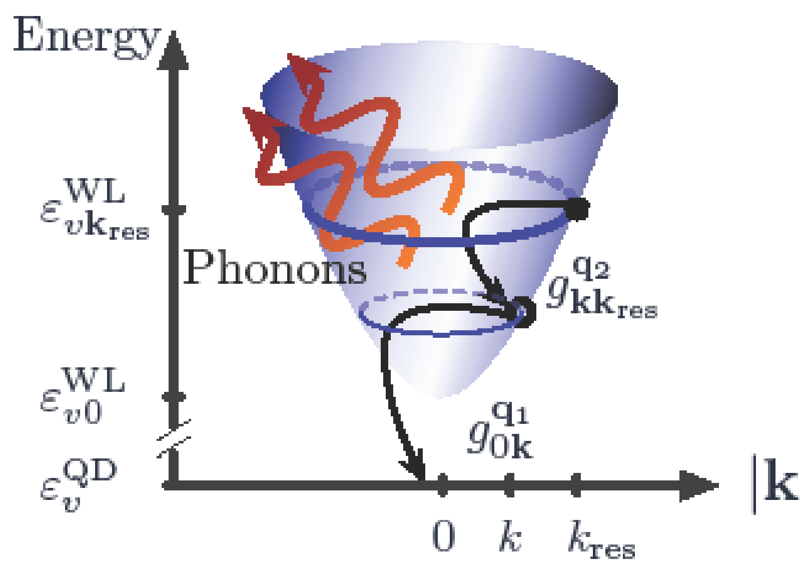
But:
Semiconductor environment (wetting layer, phonons) leads to dephasing!

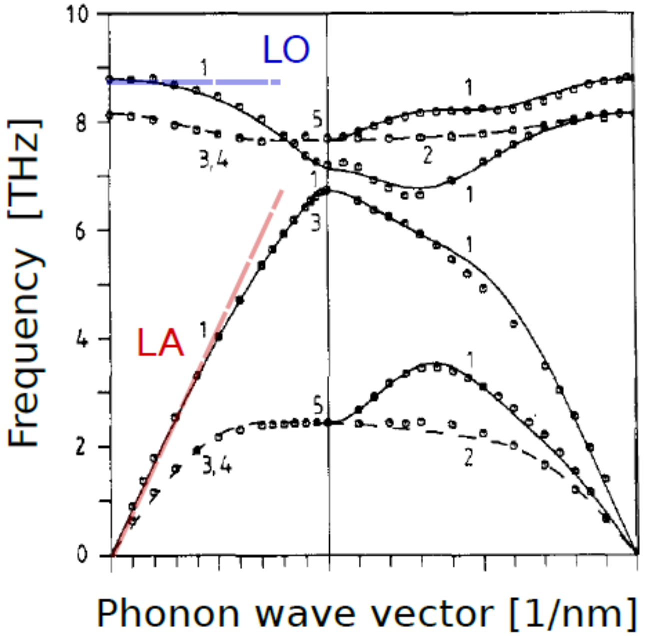


Dephasing mechanism in a semiconductor QD

Pure dephasing:

→ Deformation (LA) potential[1]

$$g_{\mathrm{LA},\mathbf{q}}^{\lambda\mu,3\mathrm{D}} = \delta_{\lambda,\mu}\sqrt{\frac{\hbar q}{2\rho c_s V}}D_\lambda$$



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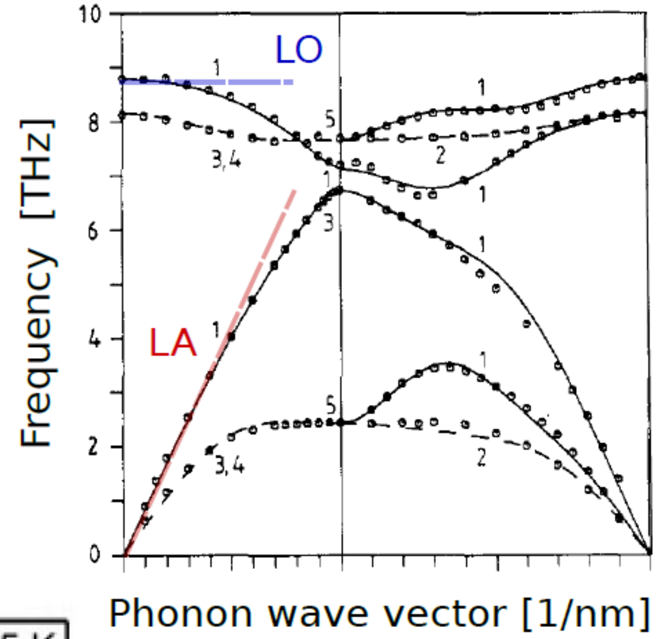
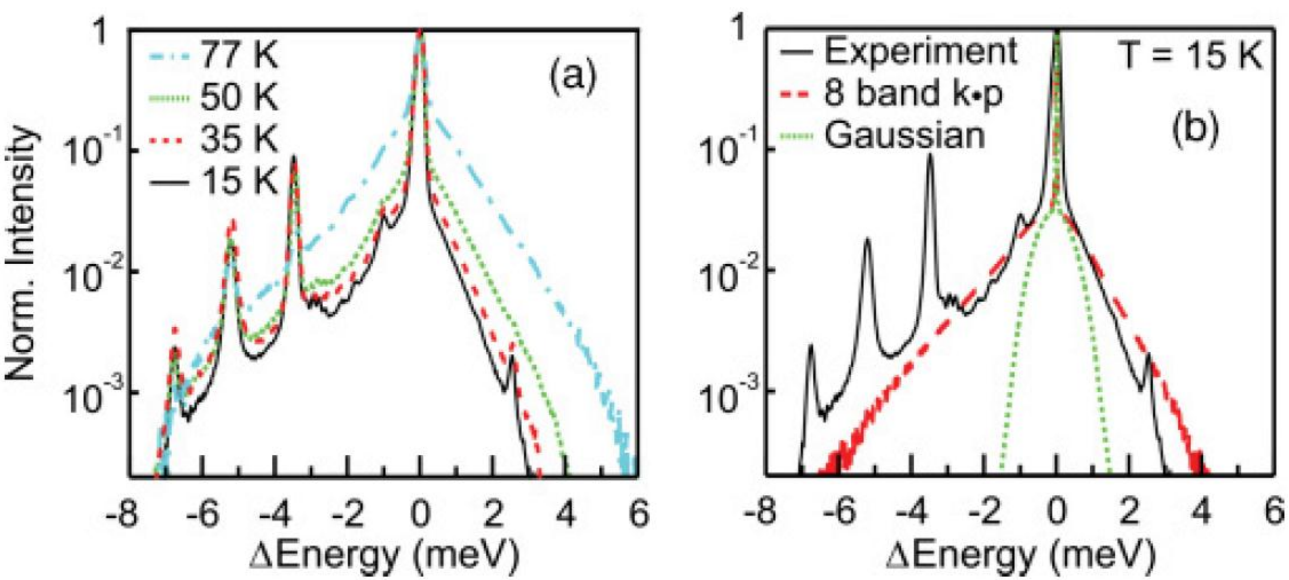




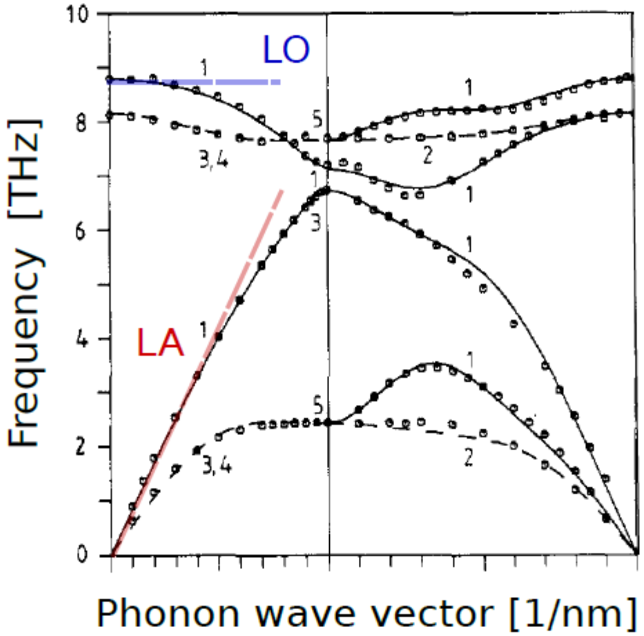
[1]PRB **83**, 041304(R) (2011)

Dephasing mechanism in a semiconductor QD

Pure dephasing:

→ Fröhlich (LO) potential[1]

$$g_{\mathrm{LO},q}^{\lambda\mu,\mathrm{3D}} = \frac{1}{q}\sqrt{\frac{e_0^2\hbar\omega_{\mathrm{LO}}}{2\varepsilon_0 V}\left(\frac{1}{\varepsilon_\infty}-\frac{1}{\varepsilon_{\mathrm{st}}}\right)}$$



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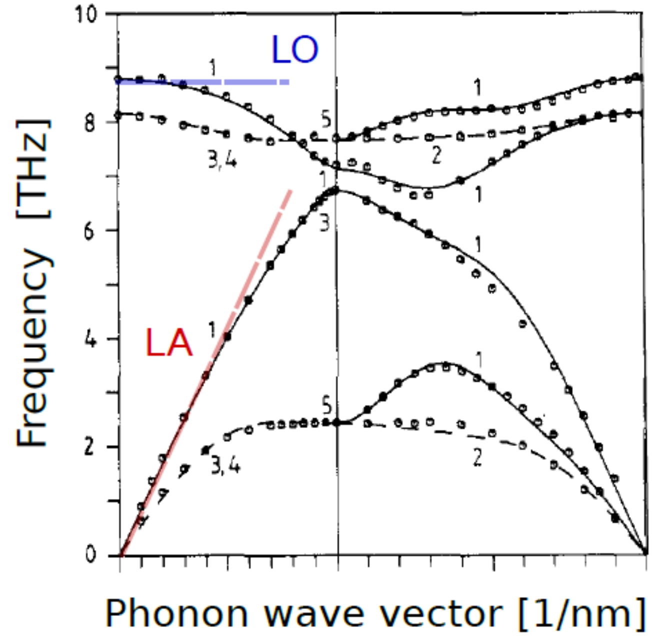
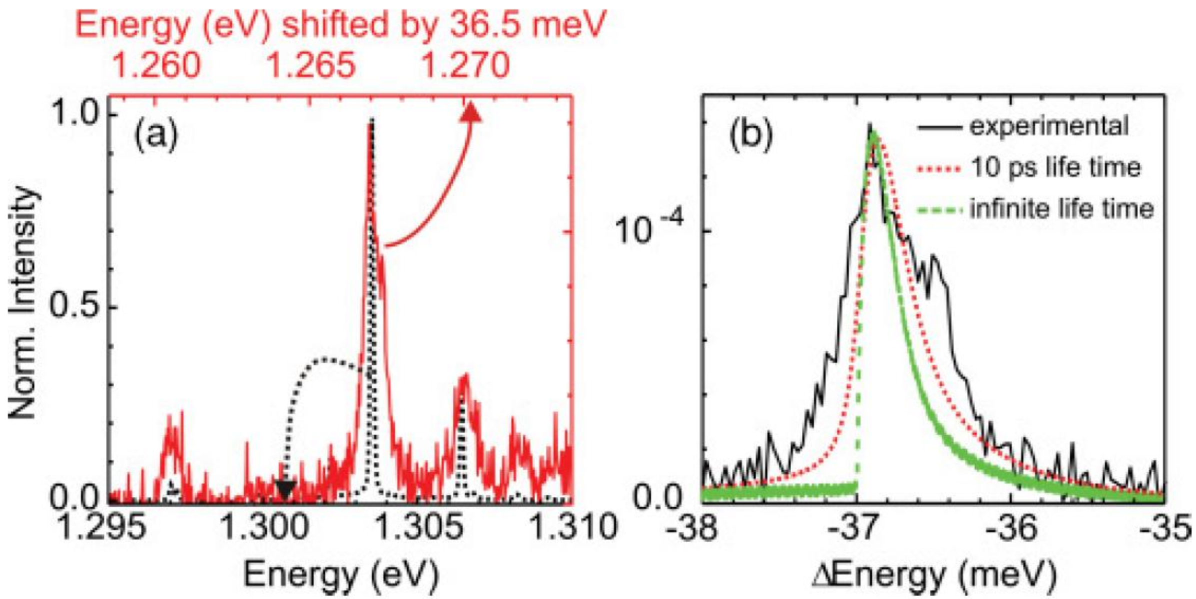




[1]PRB **83**, 041304(R) (2011)

Goal

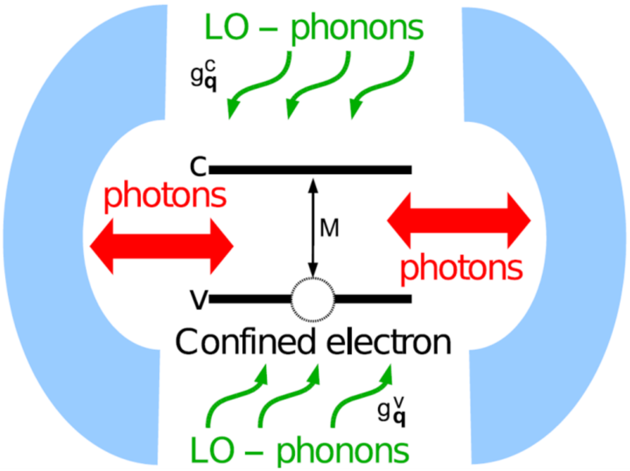
GOAL

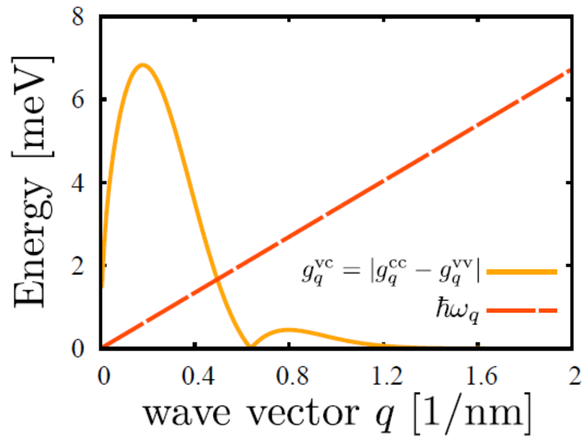
Developing a theoretical framework to find advantageous features of the semiconductor environment

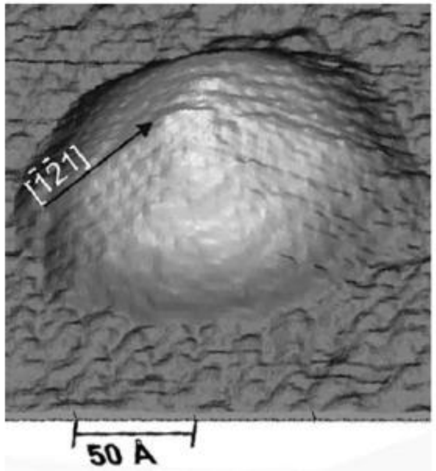
(i) semiconductor quantum dot

- ❑ Semiconductor physics and heterostructures
- ❑ Quantum Dot Hamiltonian and equation of motion approach

Semiconductor QD cavity-QED Hamiltonian







Marquez et al., Appl.Phys.Lett. 78 (2001)

$$H = H_{\text{el}} + H_{\text{phonon}} + H_{\text{photon}} + H_{\text{laser}}$$

$$H_{\text{el}} = \hbar \sum_i \omega_i a_i^\dagger a_i + \hbar \sum_{ijlm} V_{lm}^{ij} a_i^\dagger a_j^\dagger a_l a_m$$

$$H_{\text{phonon}} = \hbar \sum_{\mathbf{q}} \omega_q b_{\mathbf{q}}^\dagger b_{\mathbf{q}} + \hbar \sum_{i,j,\mathbf{q}} g_{\mathbf{q}}^{ij} \; a_i^\dagger a_j b_{\mathbf{q}}^\dagger + \text{H.c.}$$

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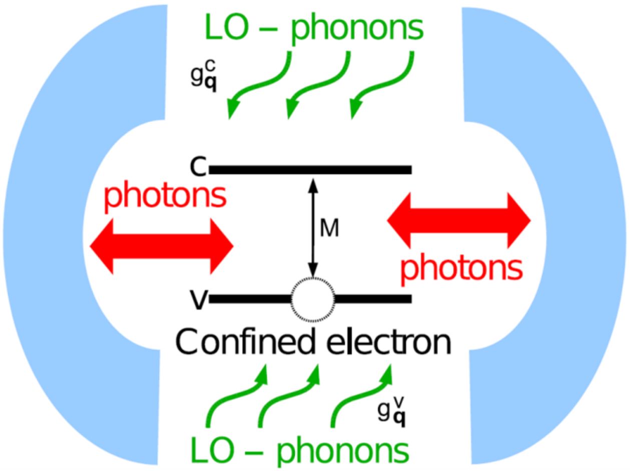

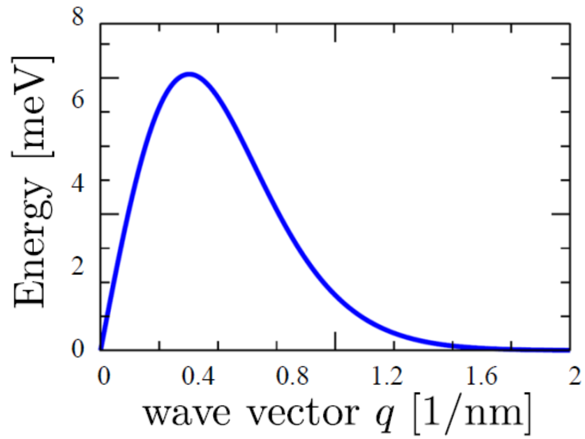

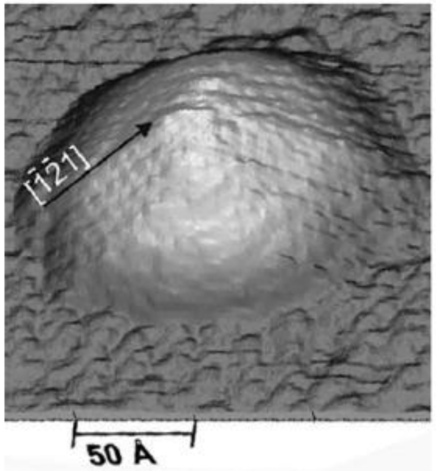

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Solving LO-phonon QD cavity-QED without factorization

for every possible combination of phonon, photon, and electron operators for example a two-level system:

$$G_{m,n}^{p,s} := a_v^\dagger a_v c^{\dagger p} c^s \bar{b}^{\dagger m} \bar{b}^n$$

$$E_{m,n}^{p,s} := a_c^\dagger a_c c^{\dagger p} c^s \bar{b}^{\dagger m} \bar{b}^n$$

$$T_{m,n}^{p,s} := a_v^\dagger a_c c^{\dagger p} c^s \bar{b}^{\dagger m} \bar{b}^n$$

Solving LO-phonon QD cavity-QED without factorization

Using product rule for operators: $\partial_t \left(a_c^\dagger a_c c^\dagger c b_q^\dagger b_q\right) = \left(\partial_t\ a_c^\dagger a_c c^\dagger c\right) b_q^\dagger b_q + c^\dagger c \left(\partial_t\ a_c^\dagger a_c b_q^\dagger b_q\right)$

and generalized commutation relations:

$$[A, F(B)] = [A, B]F'(B)$$

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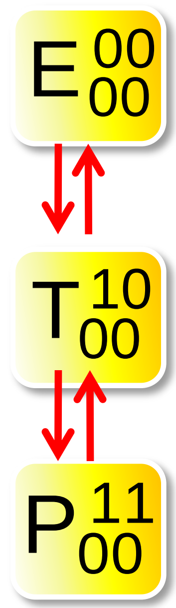
and

their dynamics, e.g.

$$\begin{aligned}
&\partial_t \langle T_{m,n}^{p,s} \rangle = \\
&= -i\left[\omega_{cv} - (p-s)\omega_0 - (m-n)\omega_{LO} - i(p+s)\kappa - i\gamma\right] \langle T_{m,n}^{p,s} \rangle \\
&\quad - ip\ M\langle E_{m,n}^{p-1,s} \rangle - iM(\langle E_{m,n}^{p,s+1} \rangle - \langle G_{m,n}^{p,s+1} \rangle) - i\Omega(t)\left(\langle E_{m,n}^{p,s} \rangle - \langle G_{m,n}^{p,s} \rangle\right) \\
&\quad - i\ \langle T_{m,n+1}^{p,s} \rangle - i\ \langle T_{m+1,n}^{p,s} \rangle + i\ m\ g_v\ \langle T_{m-1,n}^{p,s} \rangle - i\ n\ g_c\ \langle T_{m,n-1}^{p,s} \rangle,
\end{aligned}$$

General set of equations of motion

For example, in the case of LO-phonon assisted vacuum Rabi oscillations ($E^{11}_{00} = 0$) :



Photon interaction: ←

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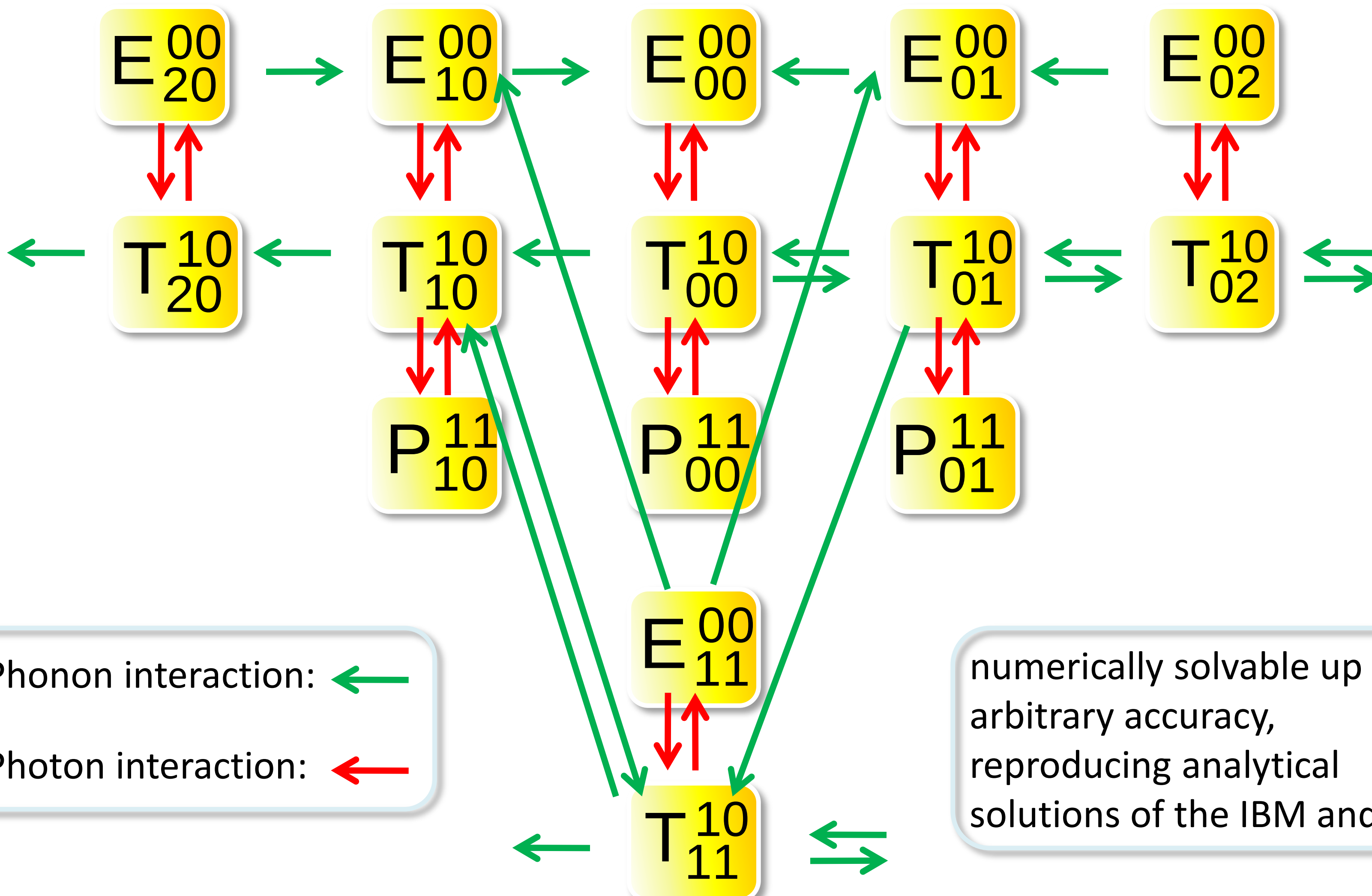

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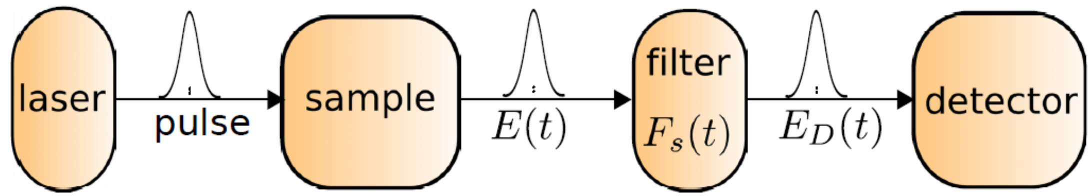
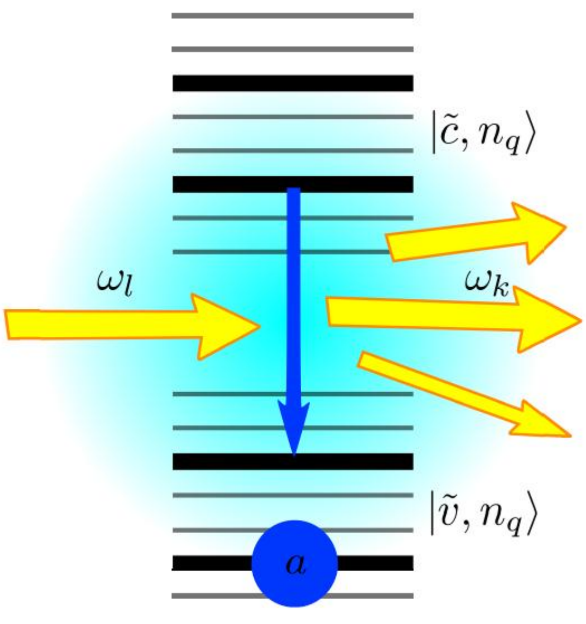
(ii) quantum dot as a two-level system

- ❑ Time-resolved phonon-assisted Mollow triplet
- ❑ Proposal for a phonon laser
- ❑ Photon-loss and induced quantum feedback

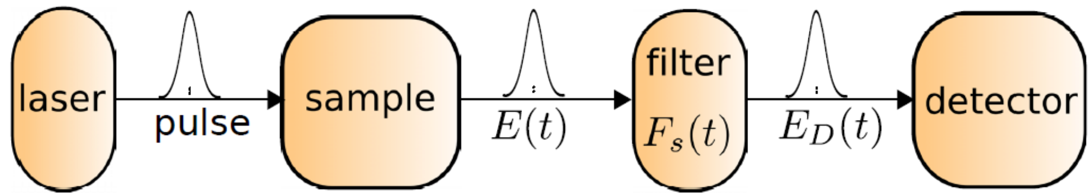
Strong excitation:

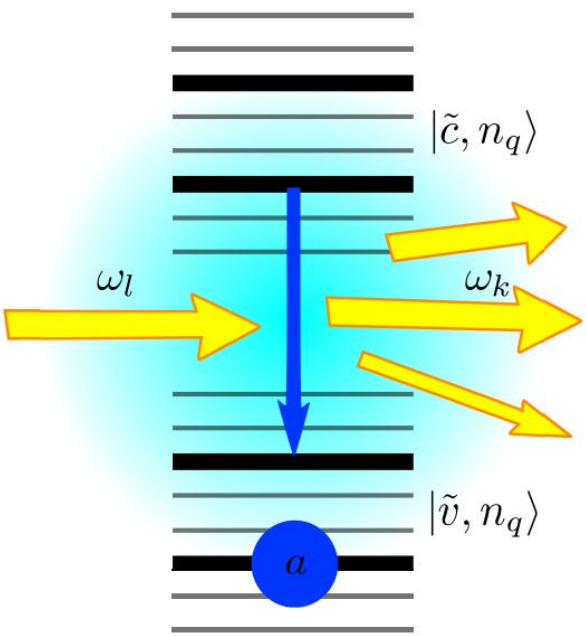




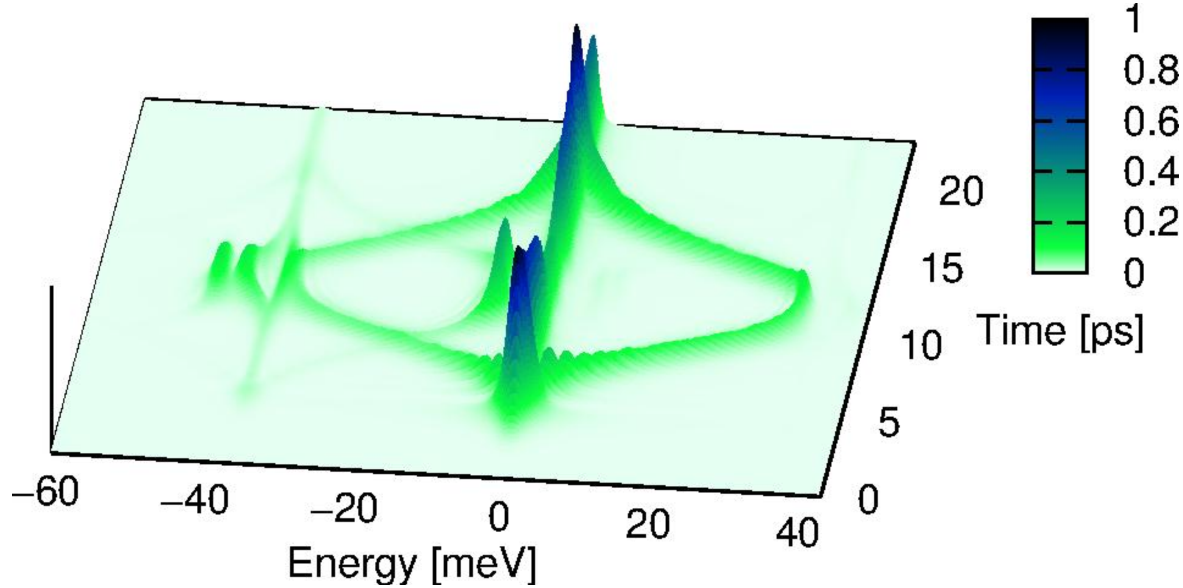
(b)strong excitation

Strong excitation:



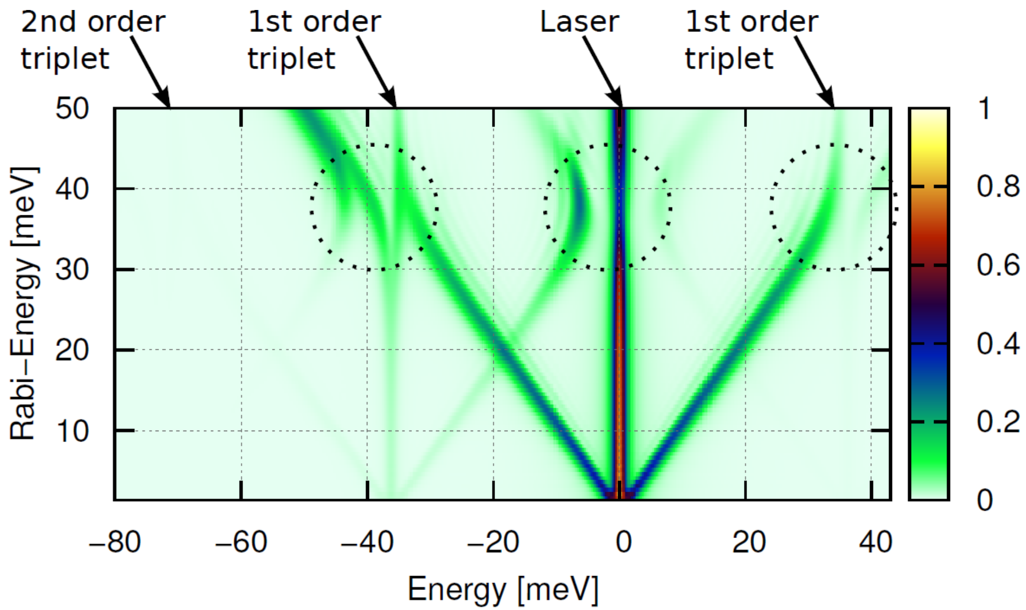


(b)strong excitation



[1]PRB **84**, 075314 (2011)

Phonon coupling strength via anti-crossing



- Spectrum shows the usual Mollow triplet and phonon-assisted Mollow triplets
- Additional anticrossings, when the Rabi-energy matches the phonon energy (Here 36.4 meV for InGaAs/GaAs-QD)
- These anti-crossings scale with the electron-phonon coupling strength

[1]PRB **84**, 125324 (2011)

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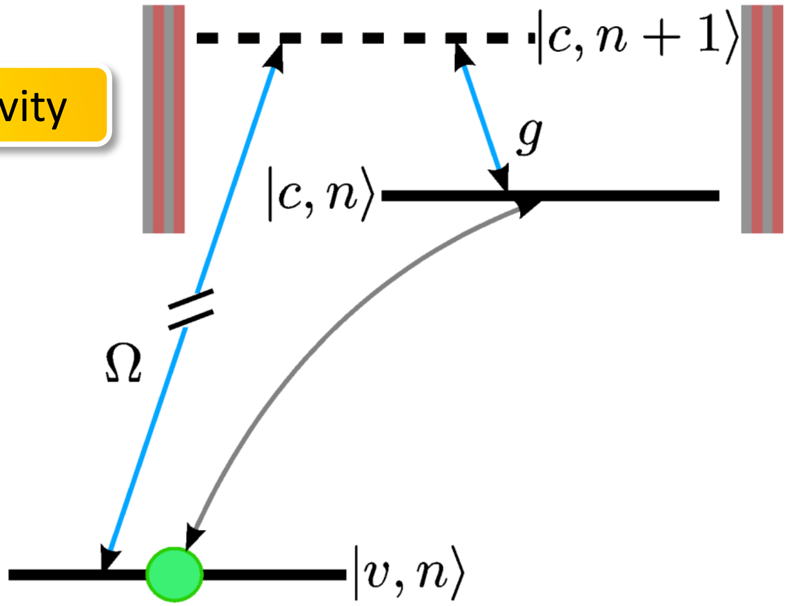
Proposal for a phonon laser

Non-resonant excited 2-level system in an acoustic cavity



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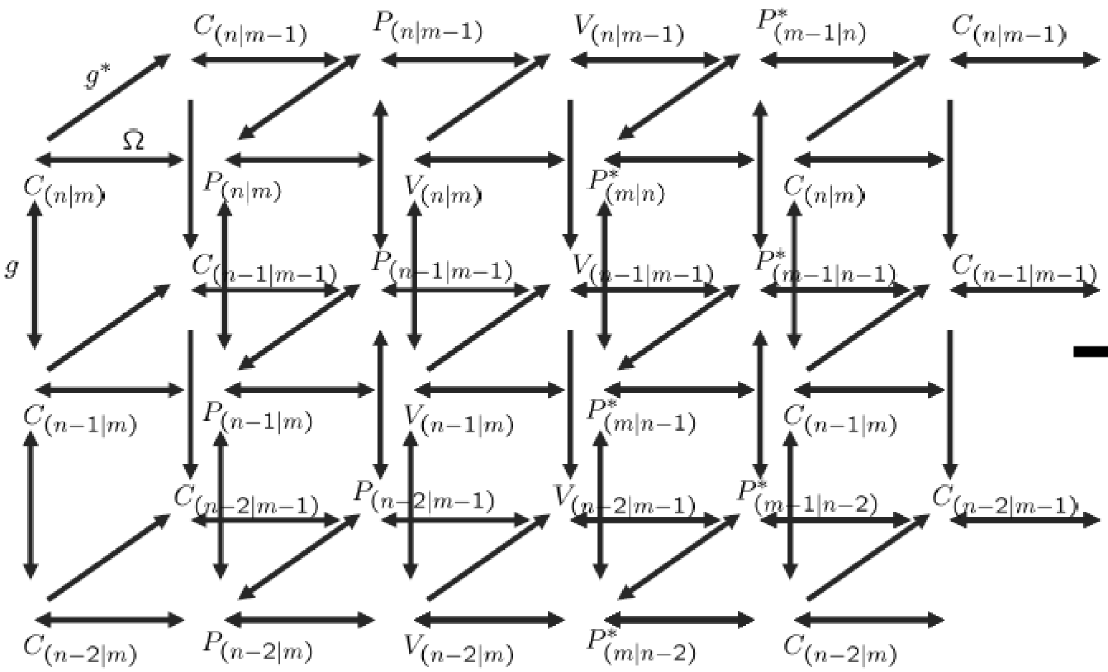
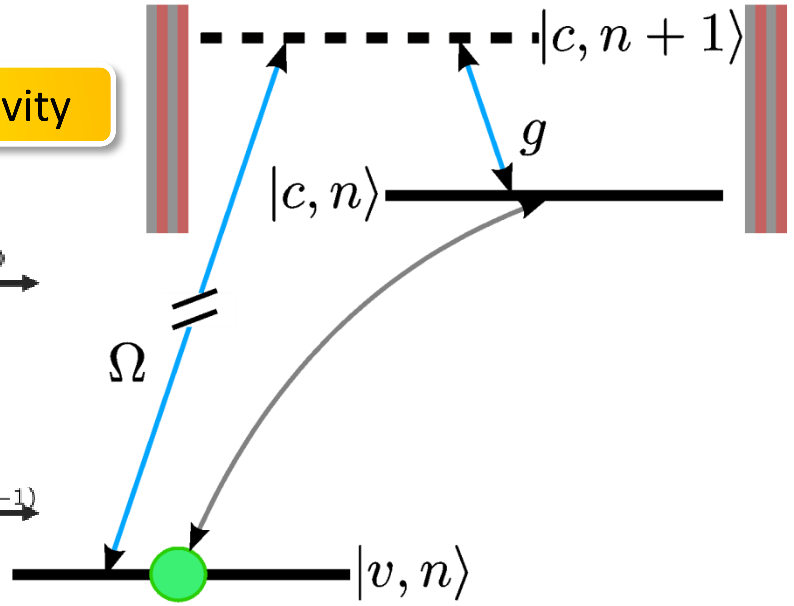
Non-resonant excited 2-level system in an acoustic cavity

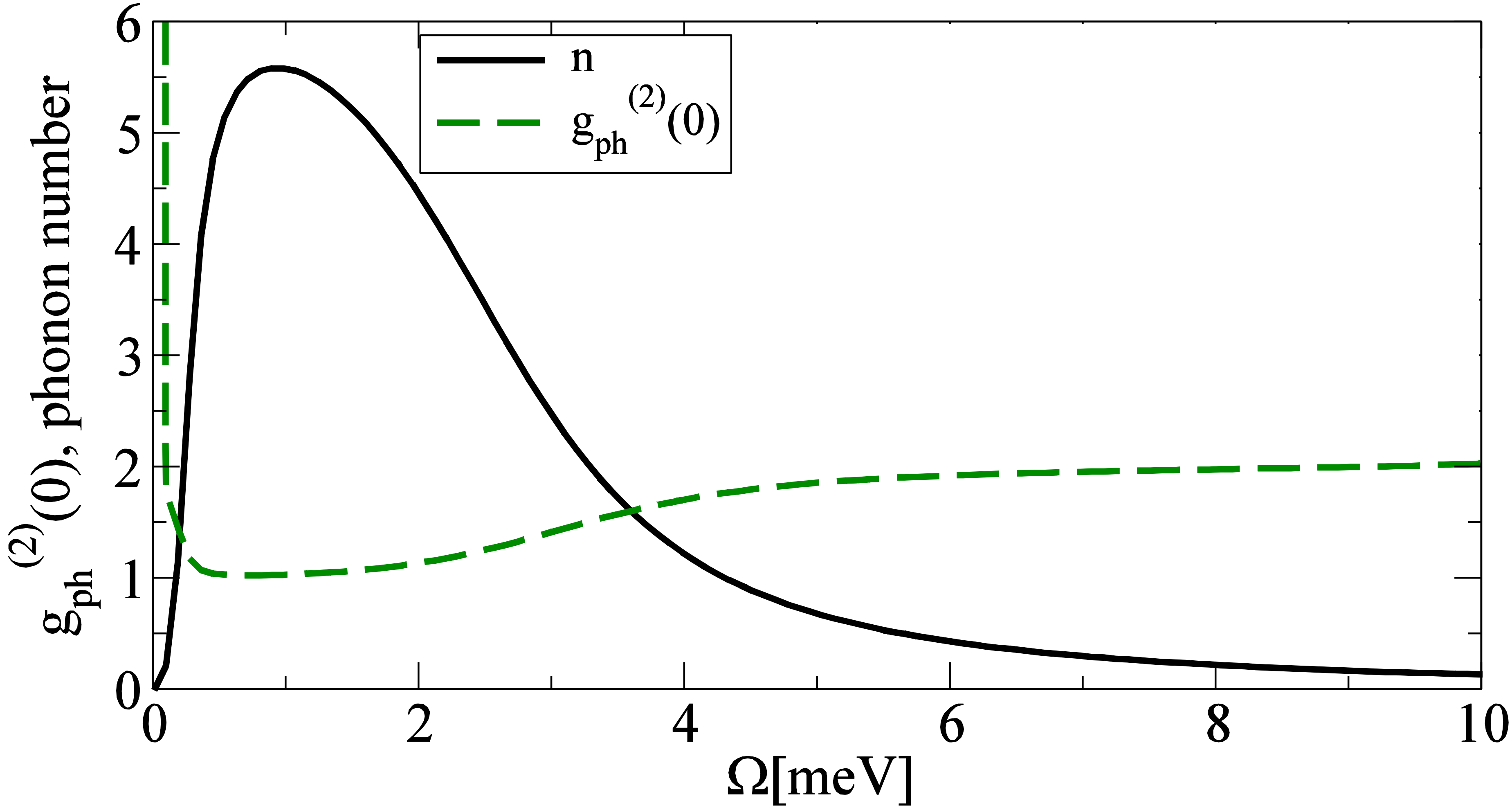




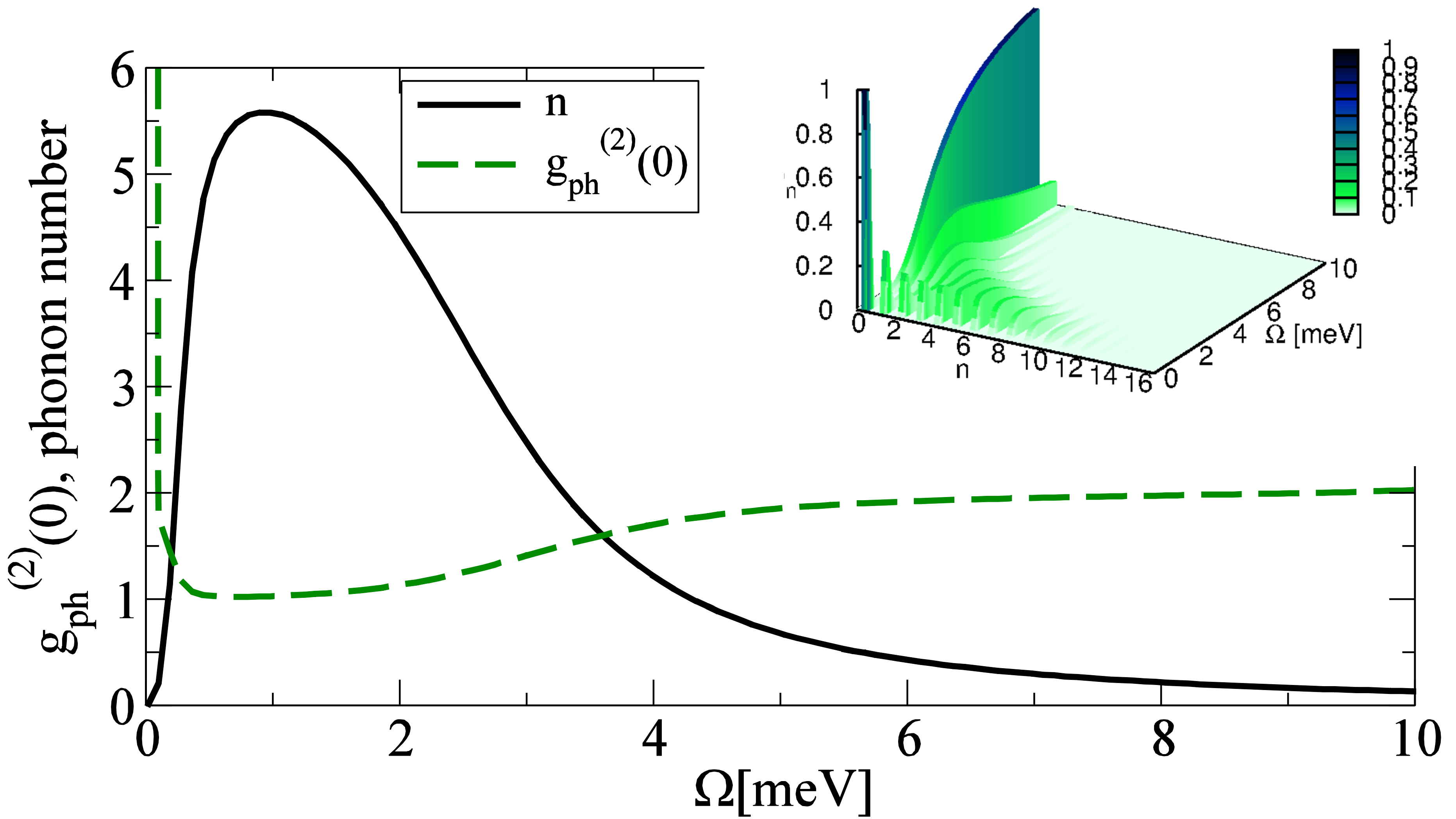
High orders of phonon operators become important [1,2,3]

[1]PRL **104**, 156801 (2010), [2]PSS(b) 248, **872** (2011), [3] submitted (2012)

Generation of coherent phonons



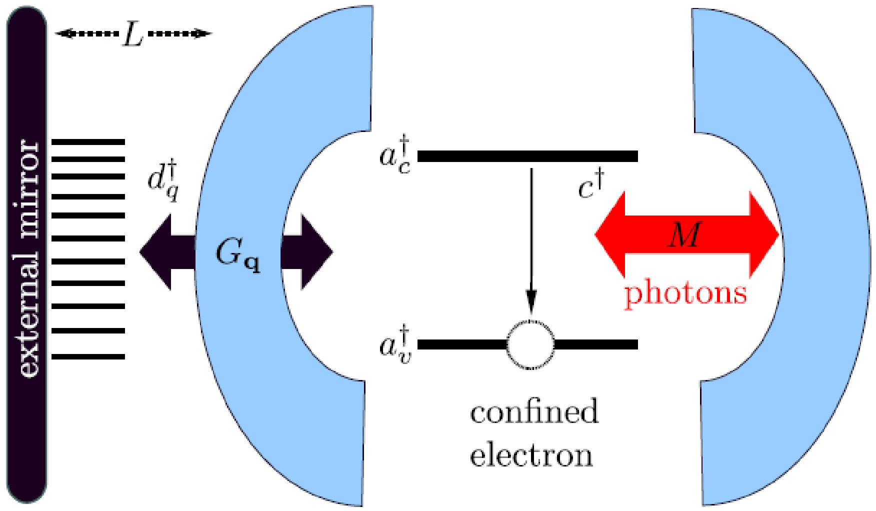
Generation of coherent phonons



(ii) quantum dot as a two-level system

- ❑ Time-resolved phonon-assisted Mollow triplet
- ❑ Proposal for a phonon laser
- ❑ Photon-loss and induced quantum feedback

Photon-loss induced quantum feedback



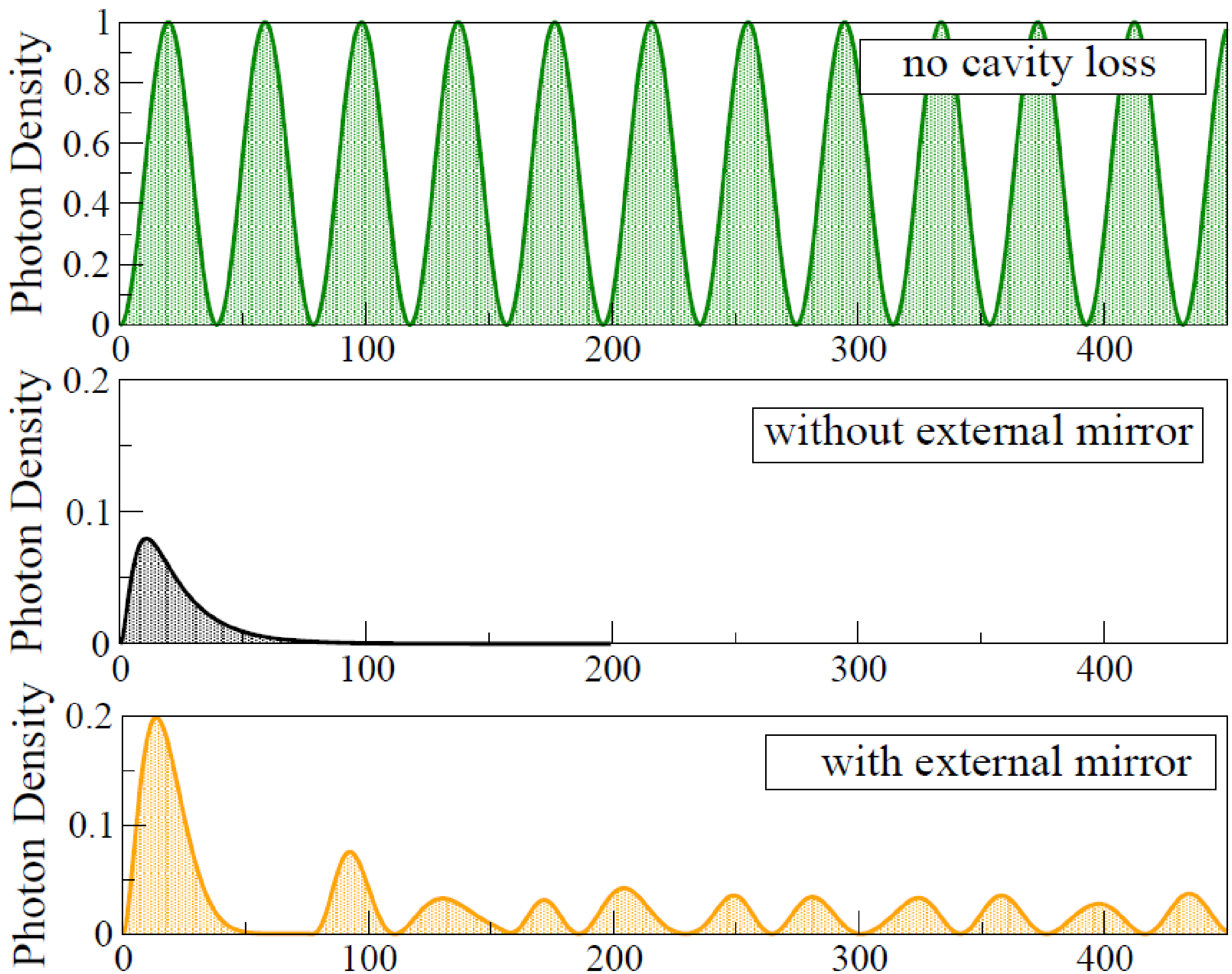
Set-up for quantum feedback

$$G_q = G \sin(q\, L)$$

External mirror shapes the mode continuum in front of the mirror to introduce a delay effect[1]

[1] in preparation (2012)

Photon-loss induced quantum feedback



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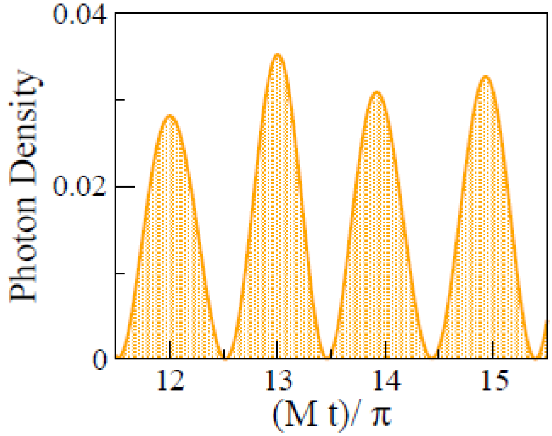
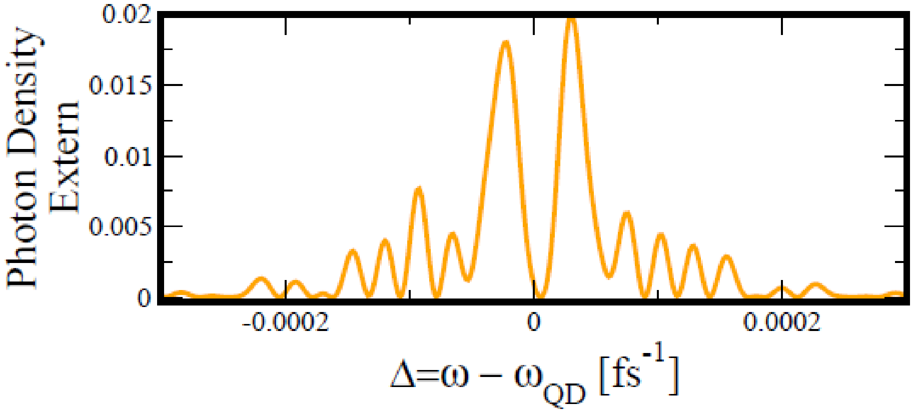


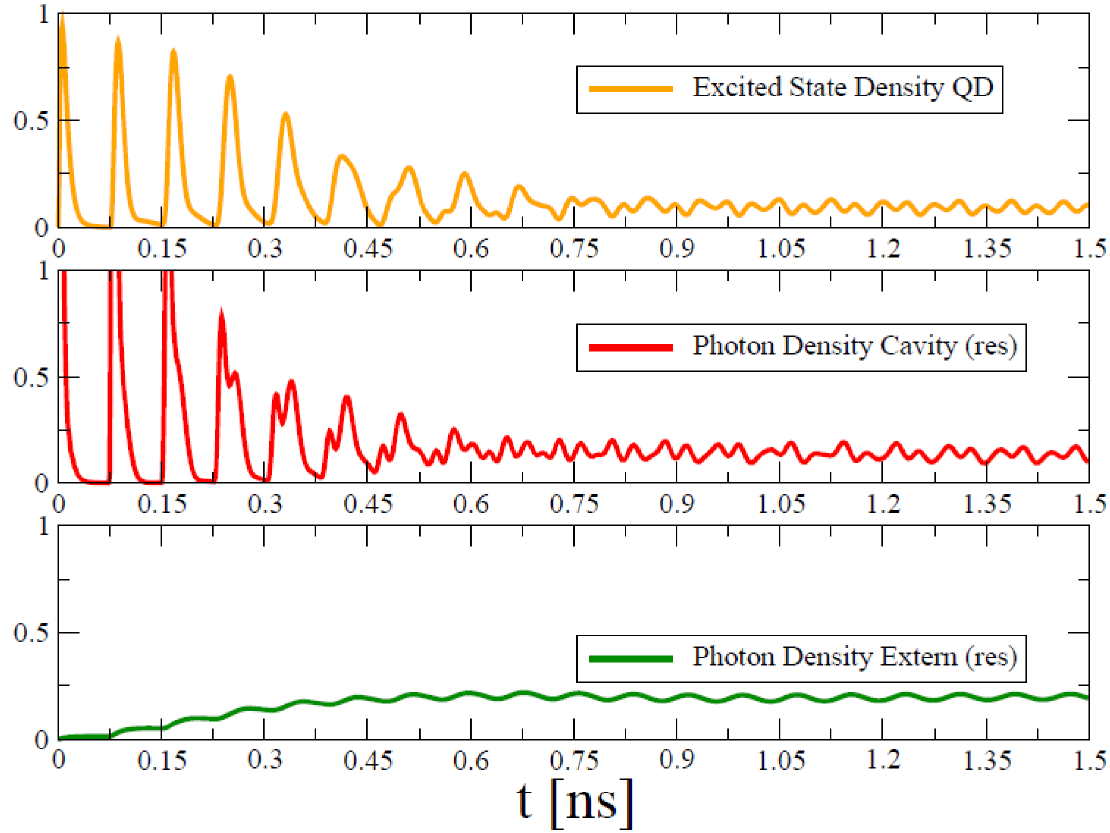


[1] in preparation (2012)

Photon-loss induced quantum feedback

Classical limit[1]:

$$\langle a_c^\dagger a_c c^\dagger c \rangle \approx \langle a_c^\dagger a_c \rangle \langle c^\dagger c \rangle$$

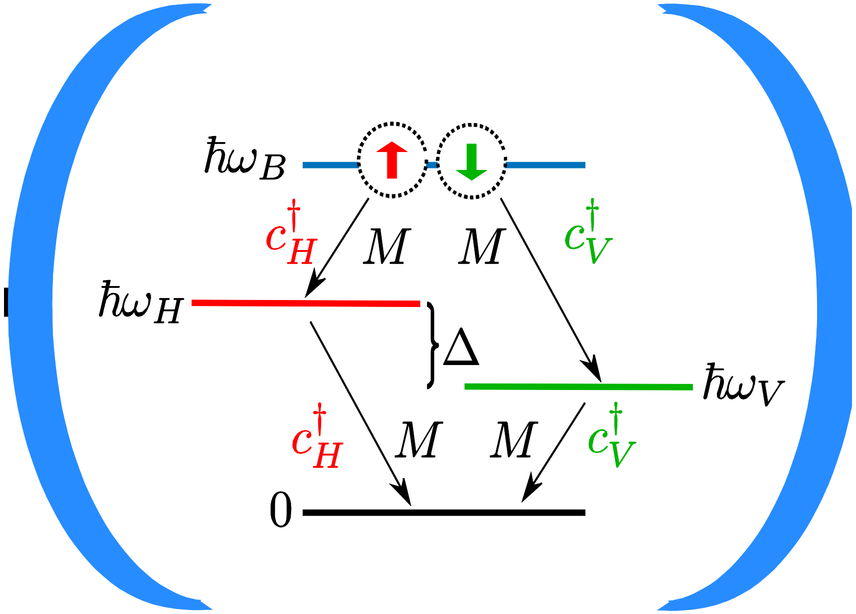


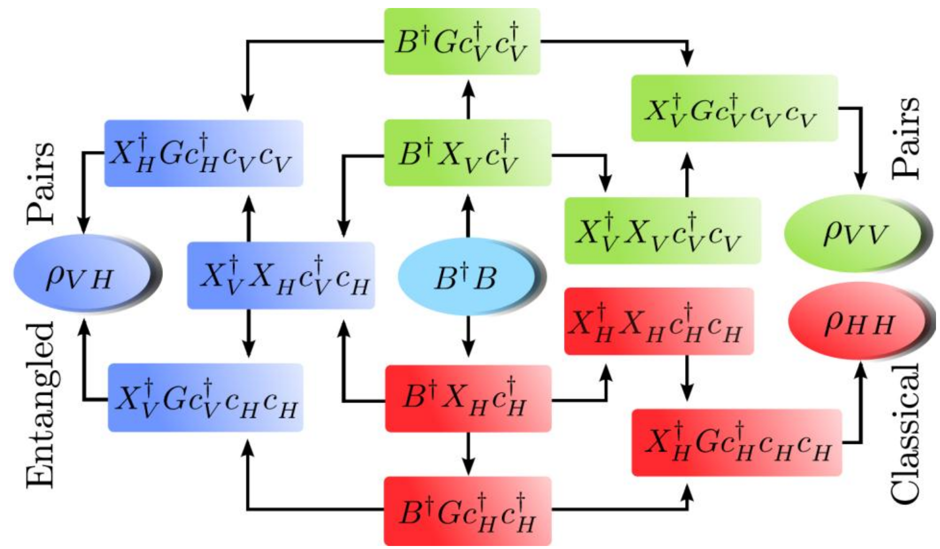
[1] in preparation (2012)

(iii) quantum dot as a four-level system

- ❑ Entangled photons: Pure dephasing does not matter
- ❑ Entangled photons: Multi-phonon scattering beyond 70K
- ❑ Strongly coupled cavity-QED: Crosscorrelation

Generation of entangled photon pairs via biexciton cascade





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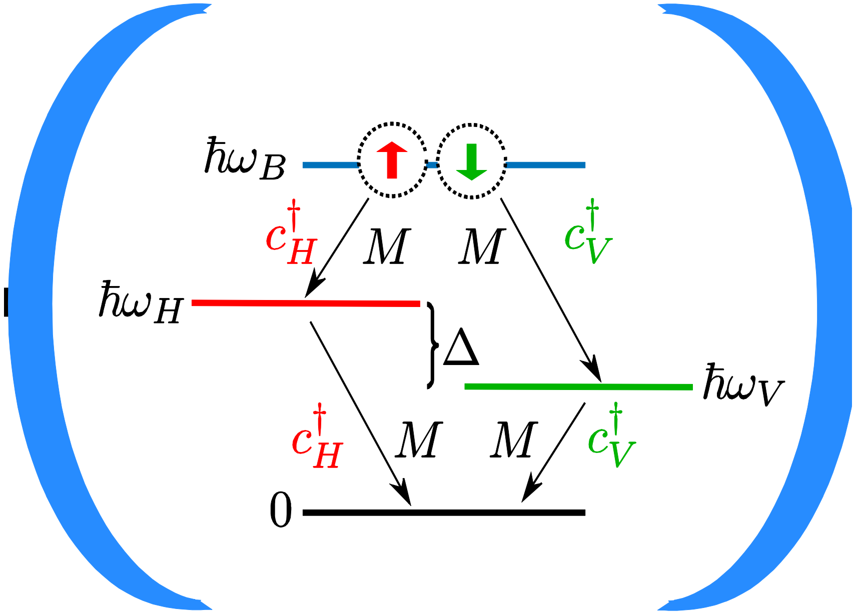
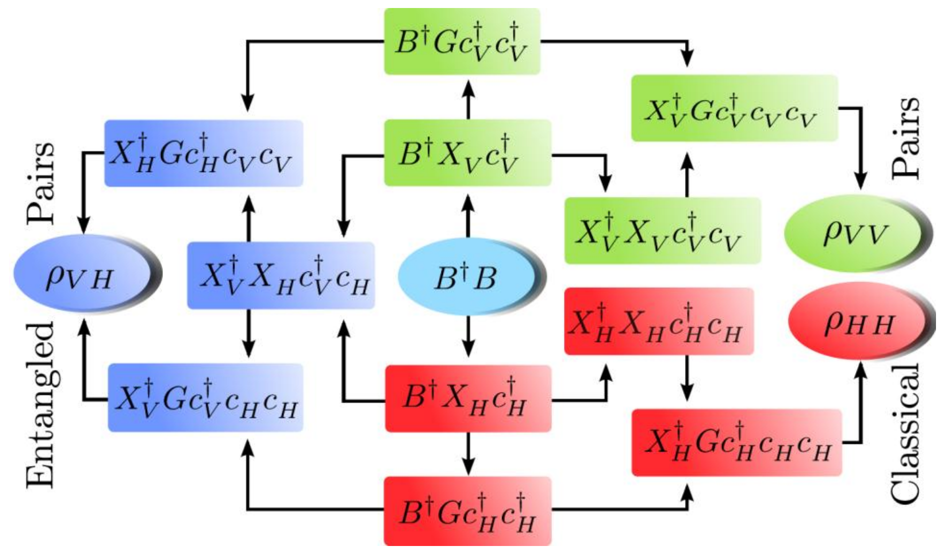




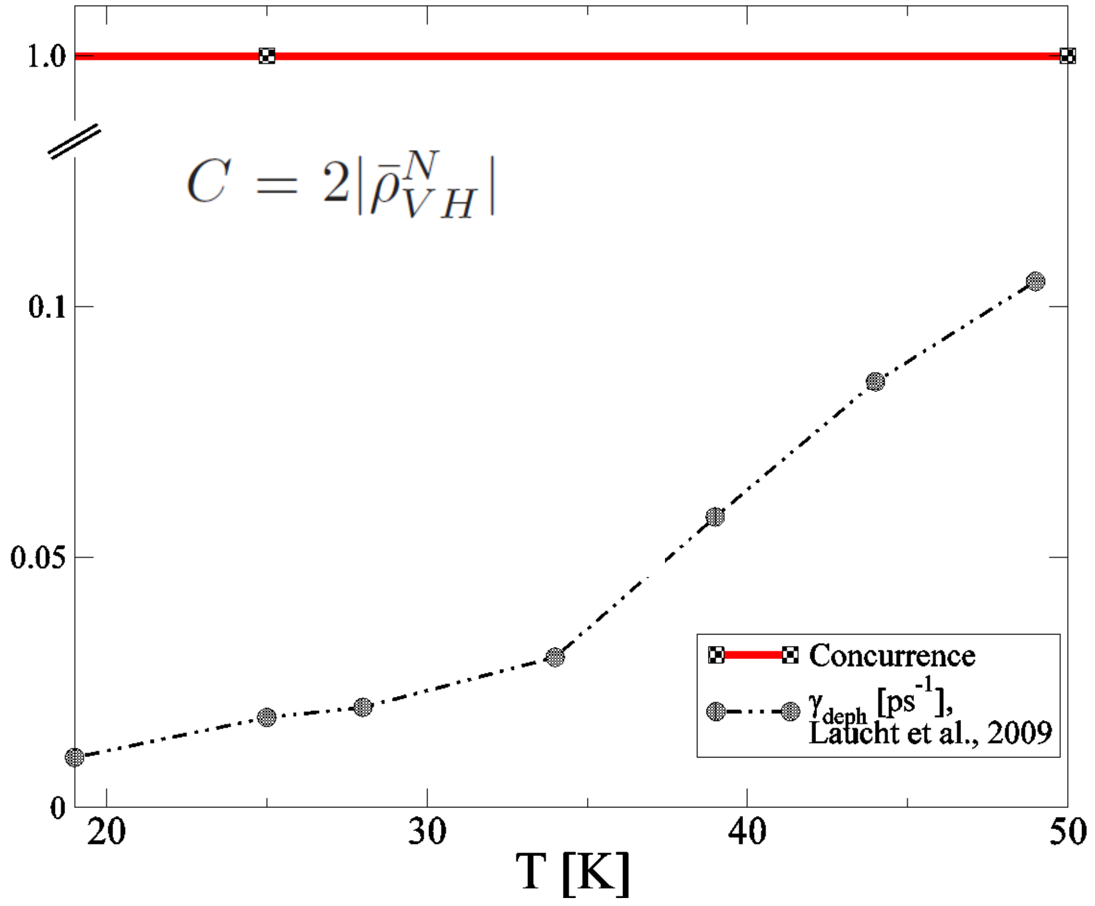
Analytical solvable in the adiabatic limit[1]:

$$C = 2|\bar{\rho}^N_{VH}|$$

$$\bar{\rho}_{VH} = \frac{8M^4\,[2/(\kappa_X - 2i\Delta) + 1/(\kappa_{GB} - i\Delta)]}{(\kappa_{GX} - 2i\Delta)(\kappa_{XB} - i\Delta)(\kappa_G - 2i\Delta)(\kappa_B - 2i\Delta)}$$

[1]PRB **84**, 075328 (2011)

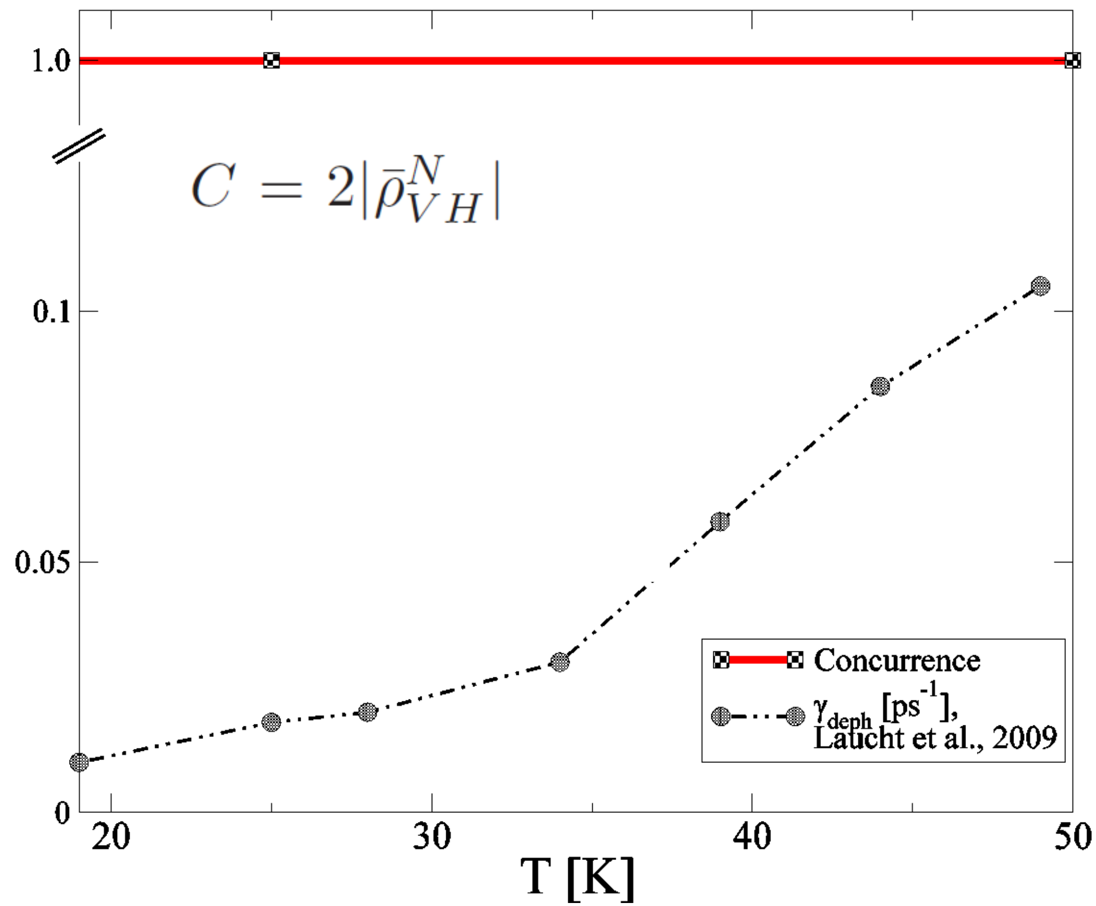
Generation of entangled photon pairs via biexciton cascade



Generation of entangled photon pairs via biexciton cascade

For typical dephasing constants, in semiconductor quantum dot cavity-QED, the degree is not affected by pure dephasing mechanism[1]

No impact if the detuning is zero



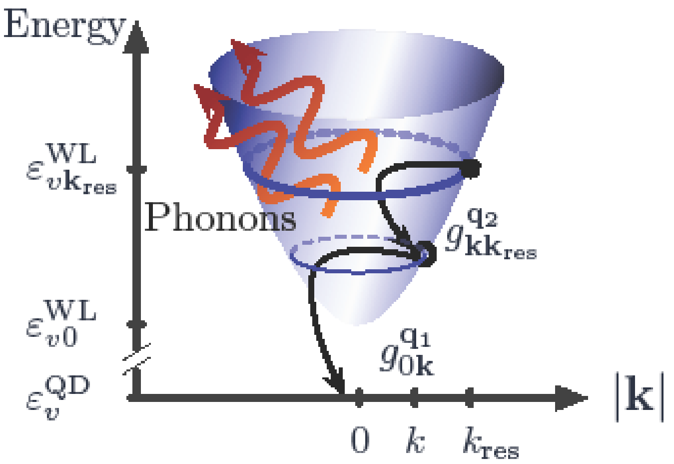
[1]PRB **84**, 075328 (2011)

(iii) quantum dot as a four-level system

- ❑ Entangled photons: Pure dephasing does not matter
- ❑ Entangled photons: Multi-phonon scattering beyond 70K
- ❑ Strongly coupled cavity-QED: Crosscorrelation

Temperature dependent degree of entanglement

- In an effective Hamiltonian approach a higher order Markovian process is assumed[1]
- Multiphonon processes attack the generation of polarization entangled photons [2]:



[1]PSSB **247**, 809 (2010), [2]PRB **81**, 195319 (2010)

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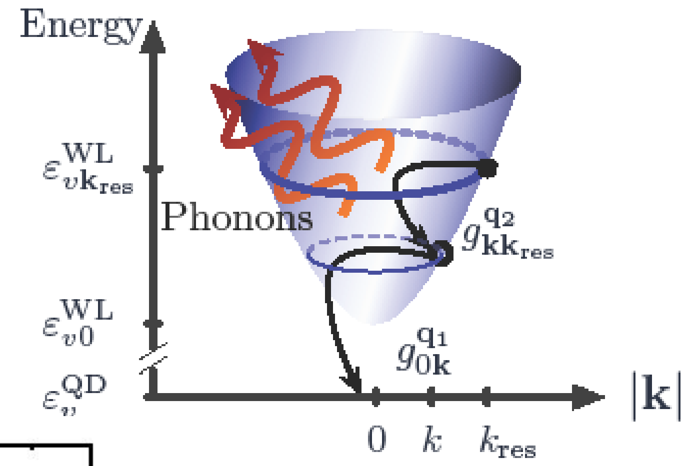

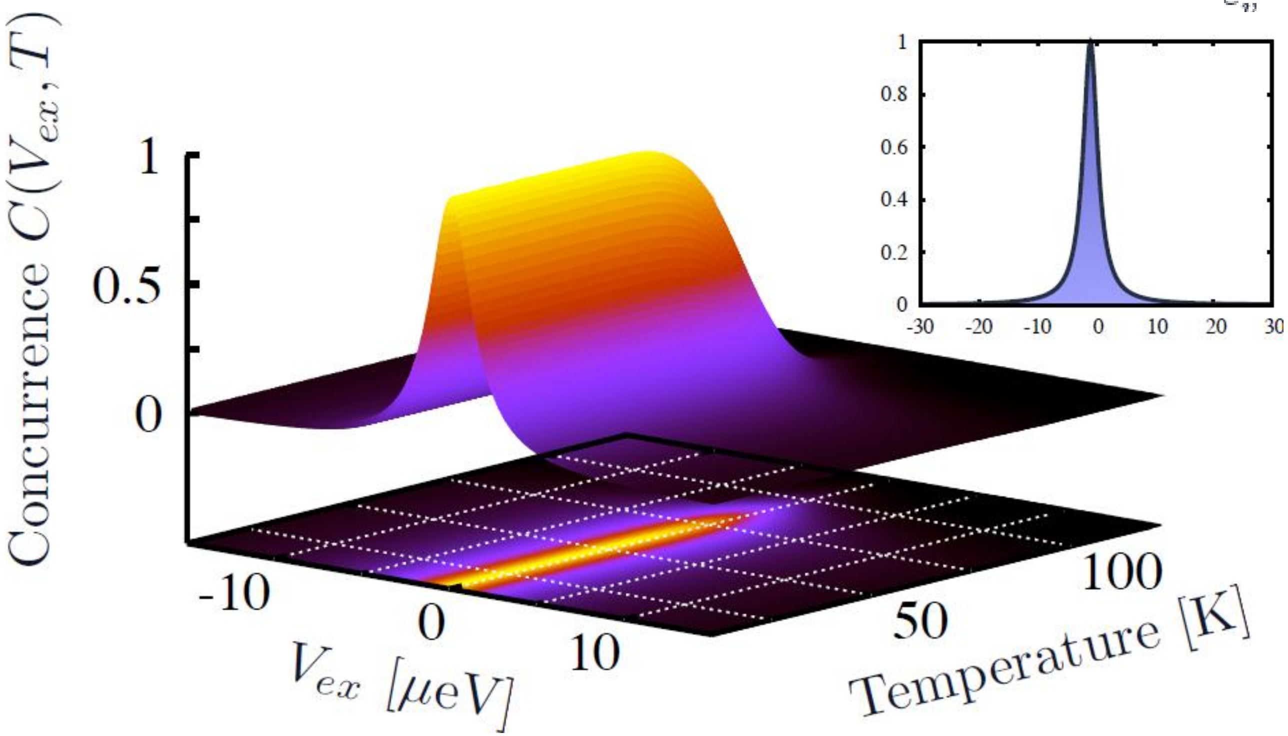

[1]PSSB **247**, 809 (2010), [2]PRB **81**, 195319 (2010)

(iii) quantum dot as a four-level system

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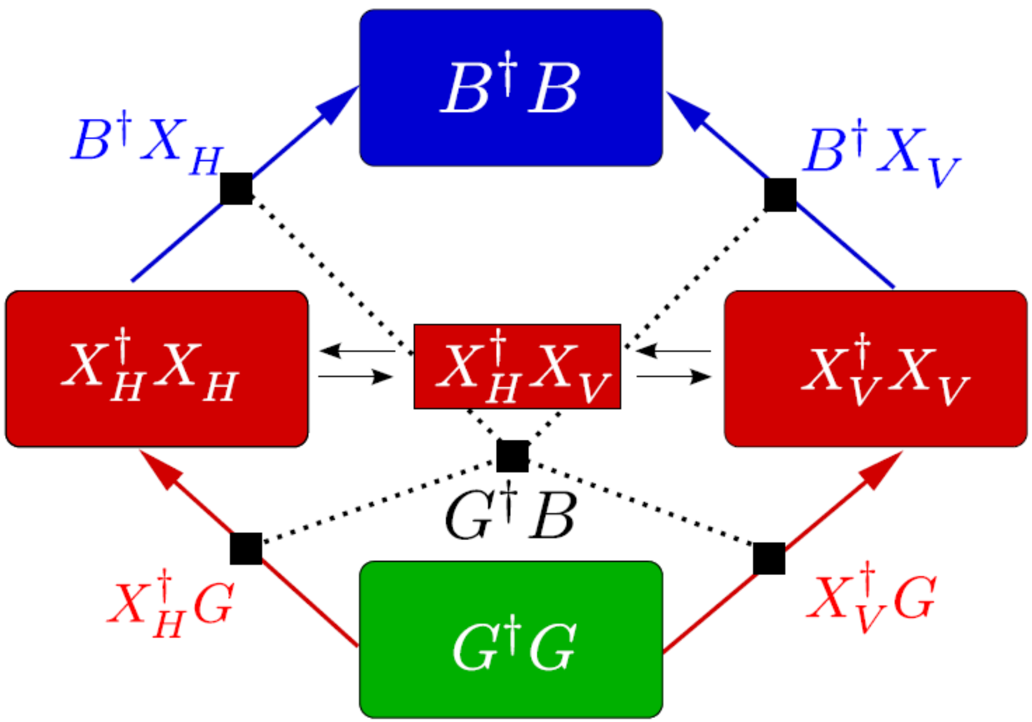
- ❑ Entangled photons: Multi-phonon scattering beyond 70K
- ❑ Strongly coupled cavity-QED: Crosscorrelation

Strongly coupled cavity-QED:

- Four-level system strongly coupled to cavity mode[1]
- Investigation of entanglement dynamics and cross-correlation dynamics for exciton-lifetimes



[1]in preparation (2012)

Strongly coupled cavity-QED:

- Example for an equation of motion[1]
- Inclusion of coherent and incoherent excitation

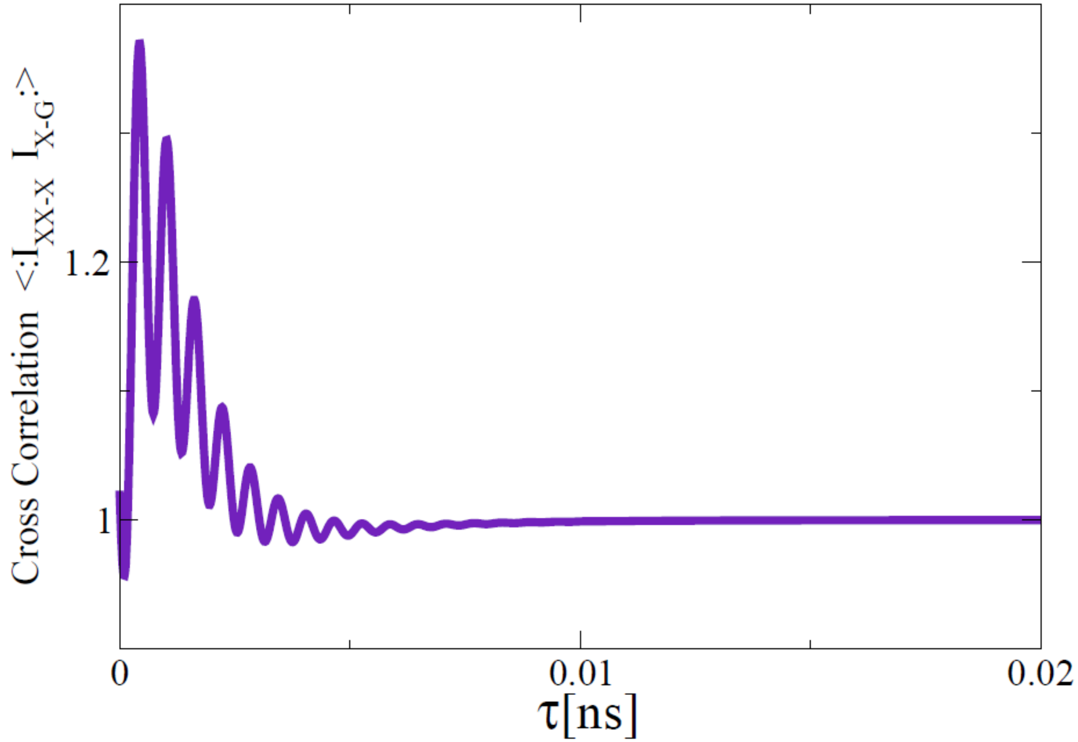
$$
\begin{aligned}
&\partial_t \langle B^\dagger B H^{m,n} V^{p,q} \rangle \\
&= i\left[(m-n)\omega_H^0 + (p-q)\omega_V^0 + i\kappa(m+n+p+q)\right] \langle B^\dagger B H^{m,n} V^{p,q} \rangle \\
&\quad - iM \langle X_H^\dagger B H^{m+1,n} V^{p,q} \rangle + iM \langle X_V^\dagger B H^{m,n} V^{p+1,q} \rangle \\
&\quad + iM \langle B^\dagger X_H H^{m,n+1} V^{p,q} \rangle - iM \langle B^\dagger X_V H^{m,n} V^{p,q+1} \rangle \\
&\quad - i\Omega_H(t) \langle X_H^\dagger B H^{m,n} V^{p,q} \rangle + i\Omega_V(t) \langle X_V^\dagger B H^{m,n} V^{p,q} \rangle \\
&\quad + i\Omega_H^*(t) \langle B^\dagger X_H H^{m,n} V^{p,q} \rangle - i\Omega_V^*(t) \langle B^\dagger X_V H^{m,n} V^{p,q} \rangle
\end{aligned}
$$

[1] in preparation (2012)

Strongly coupled cavity-QED:

- Investigation of entanglement dynamics and cross-correlation dynamics for exciton-lifetimes
- Tau-dependence of cross-correlations

$$g^{(2)}(t,\tau) = \frac{\langle c^{\dagger}_{XX}(t) c^{\dagger}_{X}(t+\tau) c_{X}(t+\tau) c_{XX}(t) \rangle}{\langle c^{\dagger}_{XX} c_{XX} \rangle(t) \ \ \langle c^{\dagger}_{X} c_{X} \rangle(t+\tau)}$$



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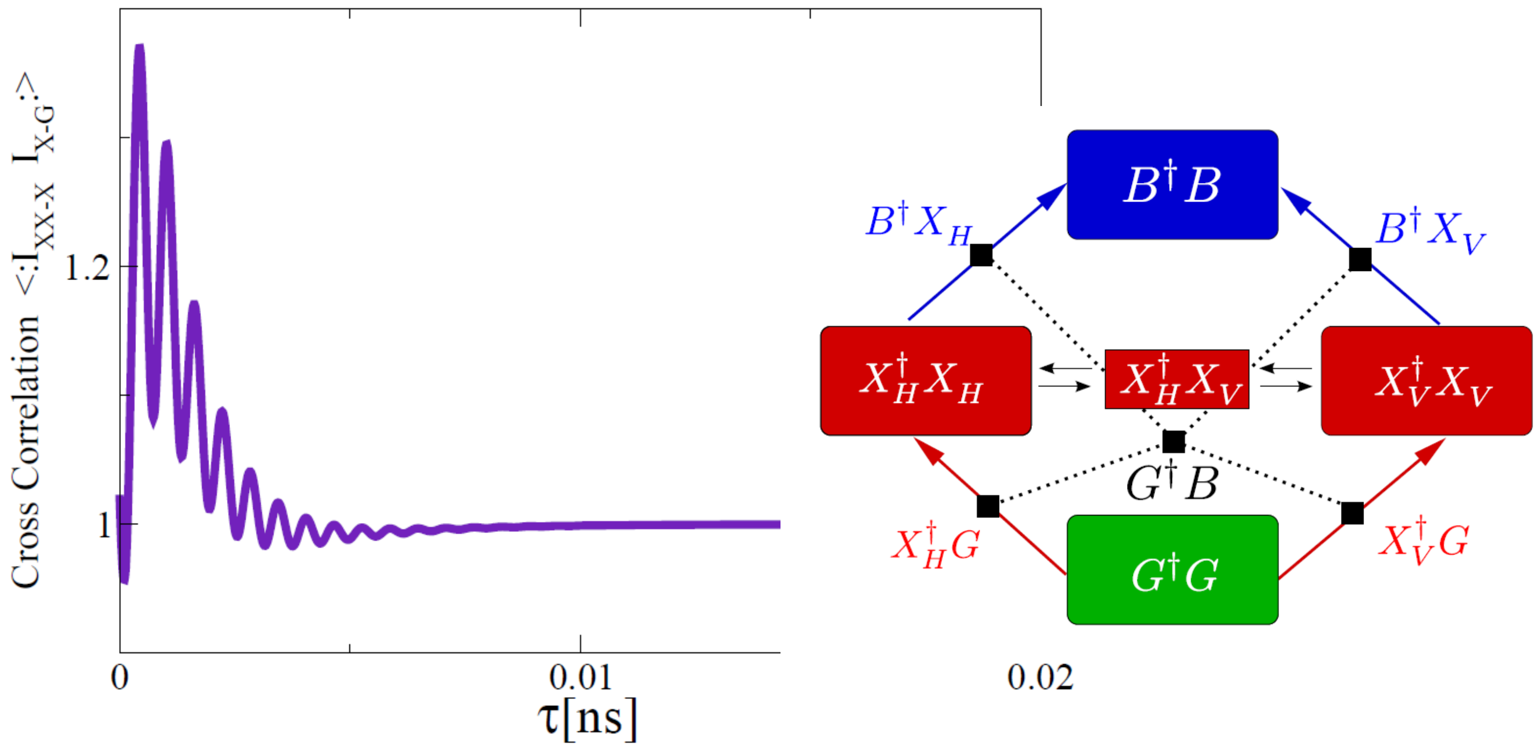

[1]in preparation (2012)

(iv) conclusion

Conclusion

$$
\begin{aligned}
H &= H_{\rm el} + H_{\rm phonon} + H_{\rm photon} + H_{\rm laser} \\
H_{\rm el} &= \hbar \sum_i \omega_i a_i^\dagger a_i + \hbar \sum_{ijlm} V_{lm}^{ij} a_i^\dagger a_j^\dagger a_l a_m \\
H_{\rm phonon} &= \hbar \sum_{\mathbf{q}} \omega_q b_{\mathbf{q}}^\dagger b_{\mathbf{q}} + \hbar \sum_{i,j,\mathbf{q}} g_{\mathbf{q}}^{ij} \; a_i^\dagger a_j b_{\mathbf{q}}^\dagger + \mathrm{H.c.} \\
H_{\rm photon} &= \hbar \sum_{\mathbf{k}} \omega_k c_{\mathbf{k}}^\dagger c_{\mathbf{k}} + \hbar \sum_{i,j,\mathbf{q}} M_{\mathbf{k}}^{ij} \; a_i^\dagger a_j c_{\mathbf{k}}^\dagger + \mathrm{H.c.} \\
H_{\rm laser} &= \hbar \sum_{i,j} \Omega^{ij} \; a_i^\dagger a_j + \mathrm{H.c.}
\end{aligned}
$$

- ❑ Electron-electron interaction: enhanced Pauli-blocking
- ❑ Electrical pumping
- ❑ Förster coupled QD: solid-state qbits

Conclusion

$$
\begin{aligned}
H &= H_{\rm el} + H_{\rm phonon} + H_{\rm photon} + H_{\rm laser} \\
H_{\rm el} &= \hbar \sum_i \omega_i a_i^\dagger a_i + \hbar \sum_{ijlm} V_{lm}^{ij} a_i^\dagger a_j^\dagger a_l a_m \\
H_{\rm phonon} &= \hbar \sum_{\mathbf{q}} \omega_q b_{\mathbf{q}}^\dagger b_{\mathbf{q}} + \hbar \sum_{i,j,\mathbf{q}} g_{\mathbf{q}}^{ij} \; a_i^\dagger a_j b_{\mathbf{q}}^\dagger + \mathrm{H.c.} \\
H_{\rm photon} &= \hbar \sum_{\mathbf{k}} \omega_k c_{\mathbf{k}}^\dagger c_{\mathbf{k}} + \hbar \sum_{i,j,\mathbf{q}} M_{\mathbf{k}}^{ij} \; a_i^\dagger a_j c_{\mathbf{k}}^\dagger + \mathrm{H.c.} \\
H_{\rm laser} &= \hbar \sum_{i,j} \Omega^{ij} \; a_i^\dagger a_j + \mathrm{H.c.}
\end{aligned}
$$

- ❑ LO-phonon enhanced anti-bunching
- ❑ LA-phonon enhanced collapse and revival phenomenon
- ❑ Generation of coherent phonons in acoustic cavities

Thank you for your attention !!