Non-Markovian Quantum Control of solid-state based Qubits

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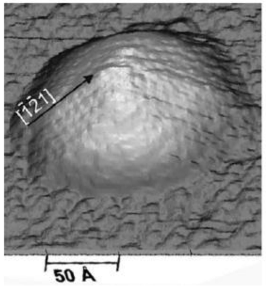
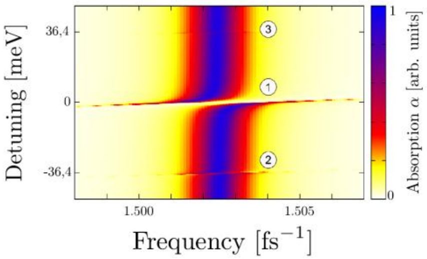
Technische Universität Berlin

OUTLINE



I: Semiconductor Quantum Dot

- Semiconductor physics and heterostructures
- Quantum Dot Hamiltonian and equation of motion approach



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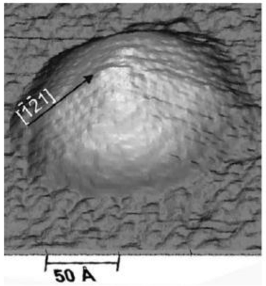

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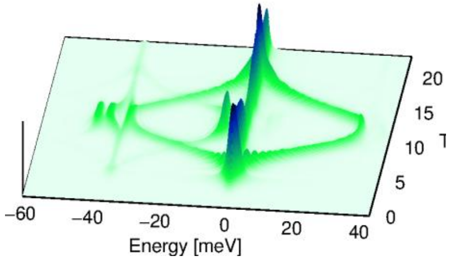
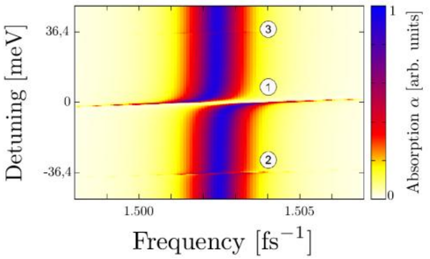


II: Laser-driven Quantum Dot

- ❑ Time-resolved phonon-assisted Mollow triplet
- ❑ Proposal for a phonon laser



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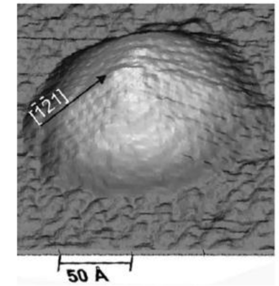

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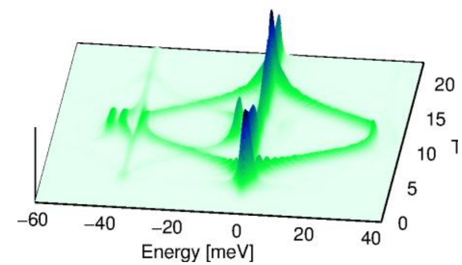

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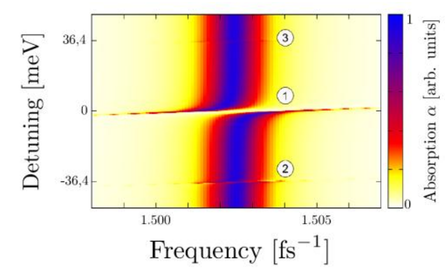
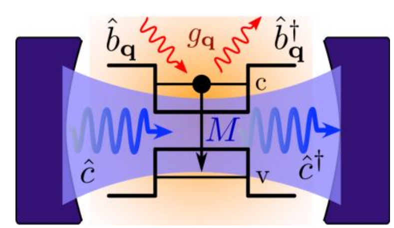


III: Quantum Dot – cavity QED

- ❑ Enhancement of collapse and revival phenomenon
- ❑ Photon-loss induced quantum feedback



(i) semiconductor quantum dot

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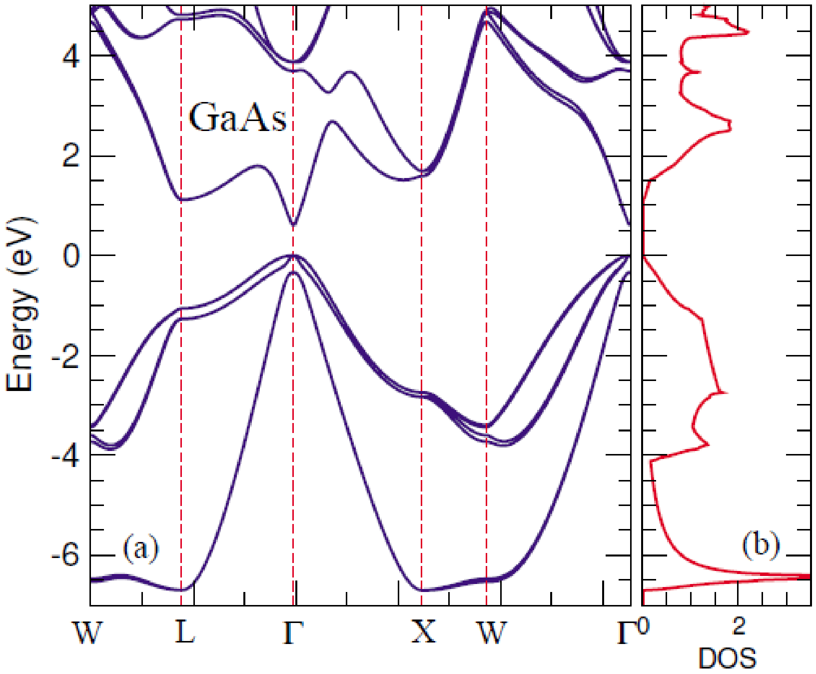
- Quantum Dot Hamiltonian and equation of motion approach

Semiconductor structures – effective mass approximation

$$\mathscr{H}_0 = -\frac{\hbar^2}{2m_0}\Delta + V_{\text{lat}}(\mathbf{r})$$

Bandstructure of GaAs:
Parabolic structure of lowest conduction and highest valence band

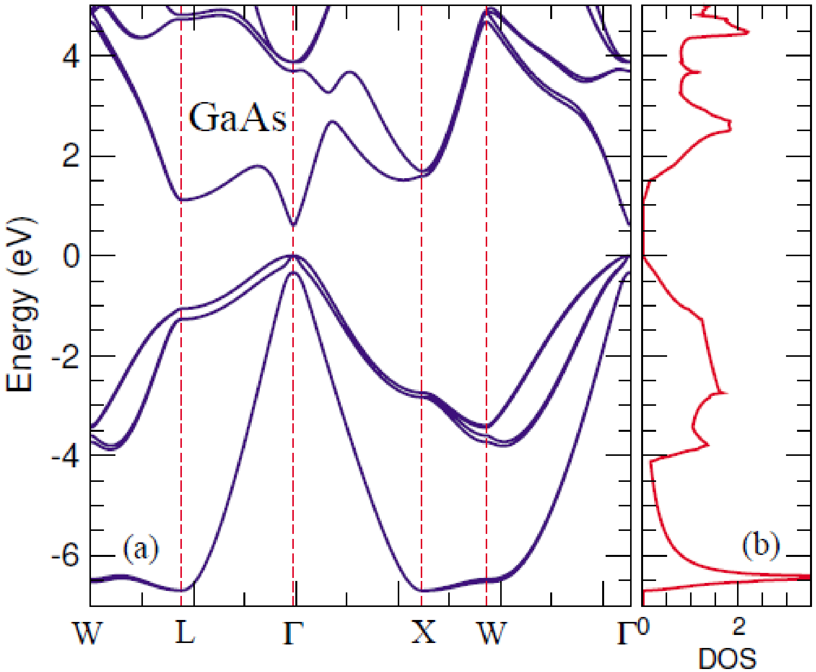


Semiconductor structures – effective mass approximation

$$\mathcal{H}_0 = -\frac{\hbar^2}{2m_0}\Delta + V_{\text{lat}}(\mathbf{r})$$

Bandstructure of GaAs:
Parabolic structure of lowest conduction and highest valence band

$$E_n(\mathbf{k}) \approx E_n(0) + \frac{\hbar^2 k^2}{2m_n^*} \quad \text{with} \quad \frac{1}{m_n^*} = \frac{1}{\hbar^2}\frac{\partial^2 E_n(\mathbf{k})}{\partial k^2}\bigg|_{\mathbf{k}_\Gamma=0}$$

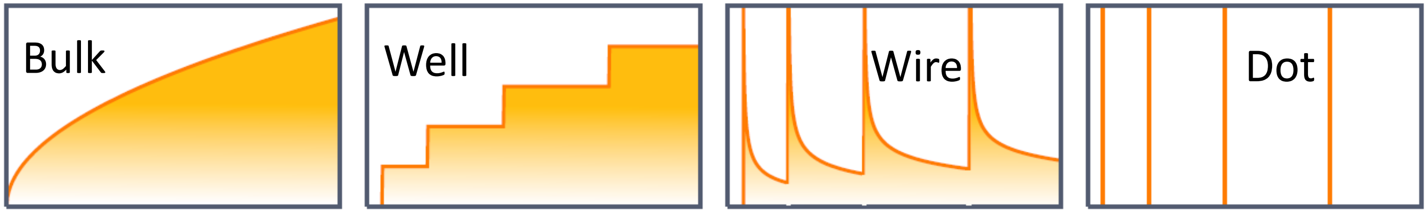


Mixing semiconductors with different band gaps: nanostructures

Semiconductor Nanostructures and second quantization

Confinement potential:
Geometry, Size,
Material specifics

$$\left[-\frac{\hbar^2}{2m_n^*}\Delta + V_{\text{conf}}(\mathbf{r})\right]\xi_n(\mathbf{r}) = \varepsilon_n \xi_n(\mathbf{r}) \quad \text{with} \quad \varepsilon_n = E - E_n(0)$$



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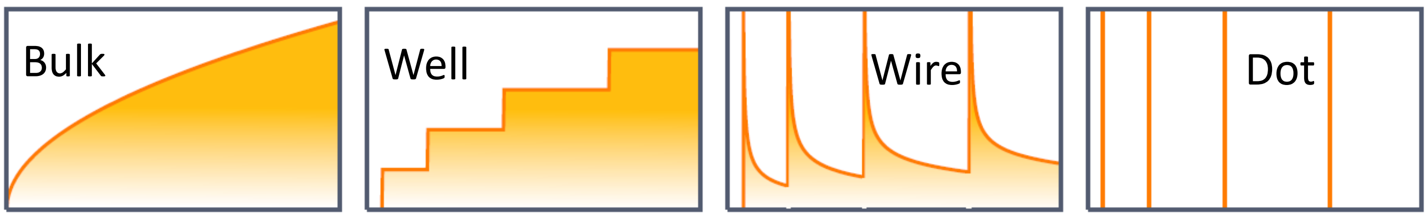

$$\varphi_n(\mathbf{r}) = u_{n,\mathbf{k}\approx 0}(\mathbf{r})\xi_n(\mathbf{r}) \qquad \hat{\Psi}(\mathbf{r}) = \sum_n \varphi_n(\mathbf{r})\hat{a}_n, \qquad \hat{\Psi}^\dagger(\mathbf{r}) = \sum_n \varphi_n^*(\mathbf{r})\hat{a}_n^\dagger$$

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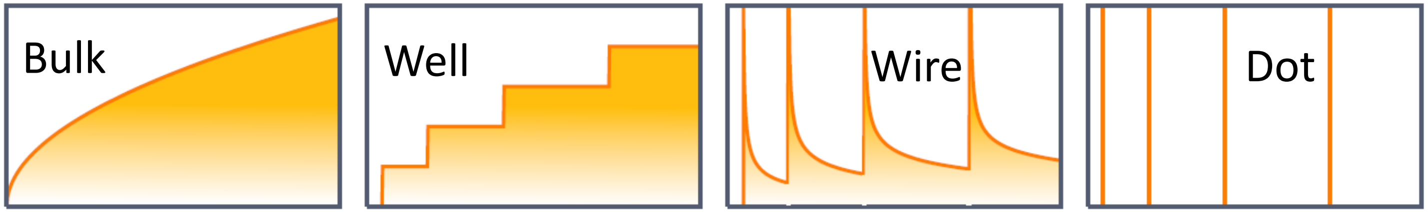

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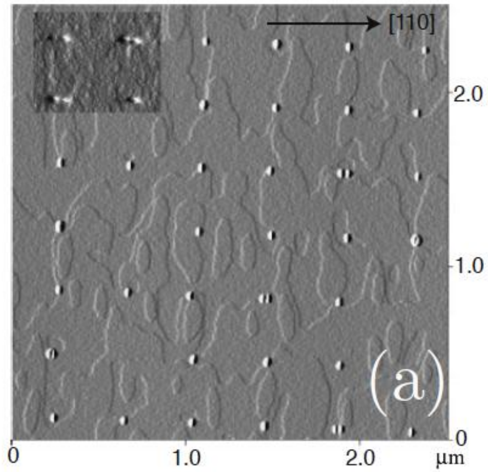
Microscopic calculated
Wave function for
2nd quantization

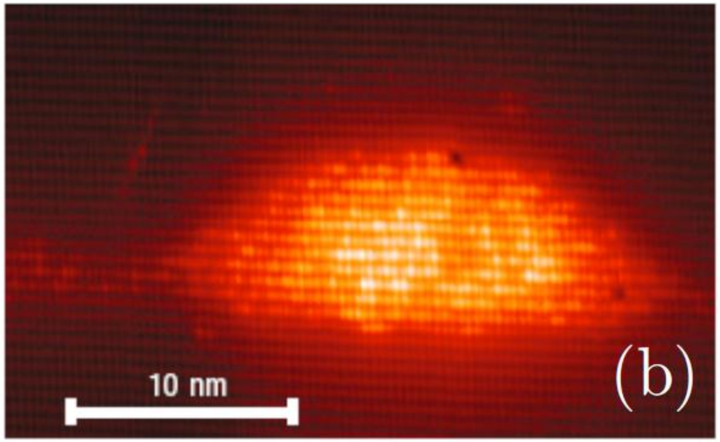
$$\hat{H}_0^c = \int d^3r\ \hat{\Psi}^\dagger(\mathbf{r})\left(-\frac{\hbar^2}{2m_0}\Delta + V_{\mathrm{lat}}(\mathbf{r}) + V_{\mathrm{conf}}(\mathbf{r})\right)\hat{\Psi}(\mathbf{r})$$
$$= \sum_n \varepsilon_n \hat{a}_n^\dagger \hat{a}_n.$$

Semiconductor quantum dots

Quantum dots:
An artifical atom



Atkinson et al.,Jpn. J. Appl. Phys. 45 (2006)



Shields, Nat. Photonics, 221 (2007)

Discrete energy levels
→ Optical properties by design
→ Electrical pumping possible

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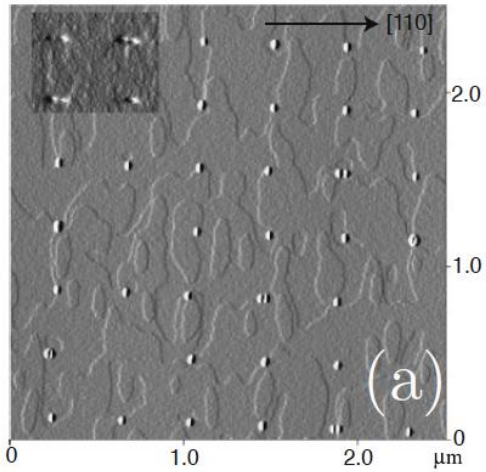

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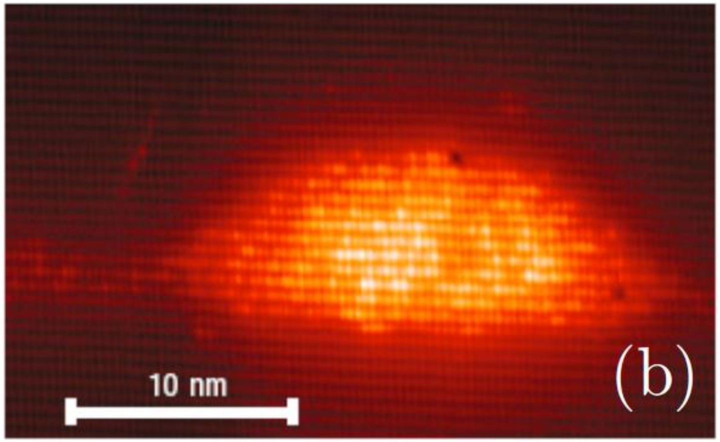

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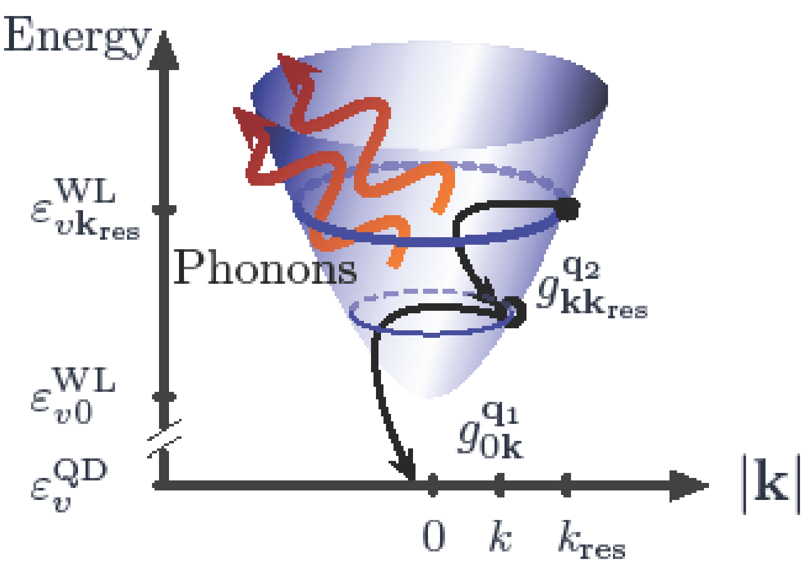
Discrete energy levels
→ Optical properties by design
→ Electrical pumping possible
But:
Semiconductor environment (wetting layer, phonons) leads to dephasing!

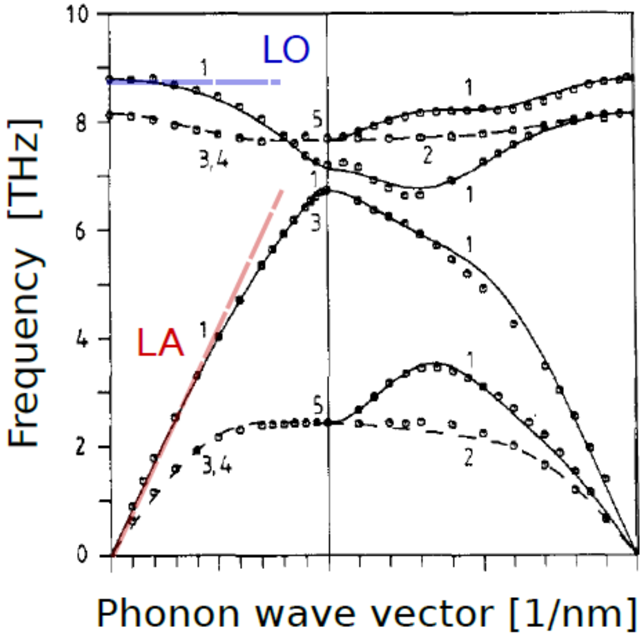


Dephasing mechanism in a semiconductor QD

Pure dephasing:

→ Deformation (LA) potential[1]

$$g^{\lambda\mu,3D}_{LA,\mathbf{q}} = \delta_{\lambda,\mu}\sqrt{\frac{\hbar q}{2\rho c_s V}}D_\lambda$$



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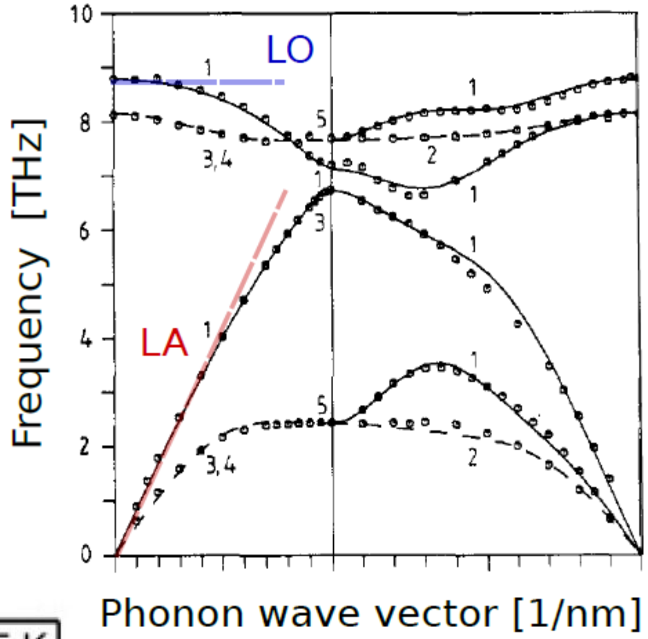

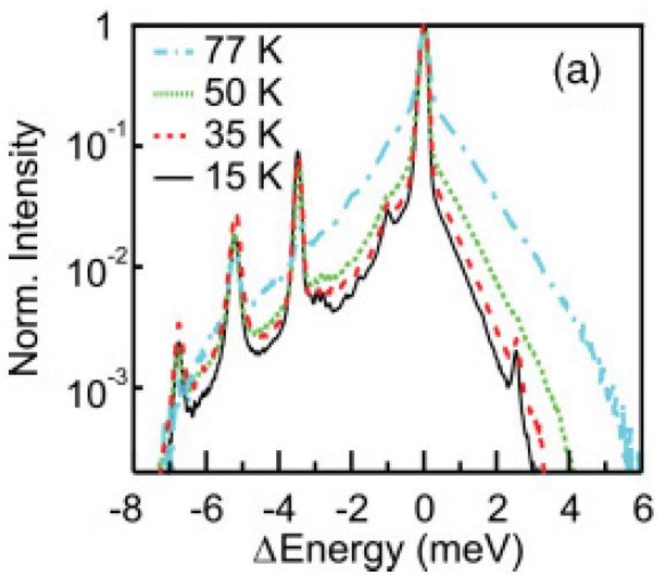

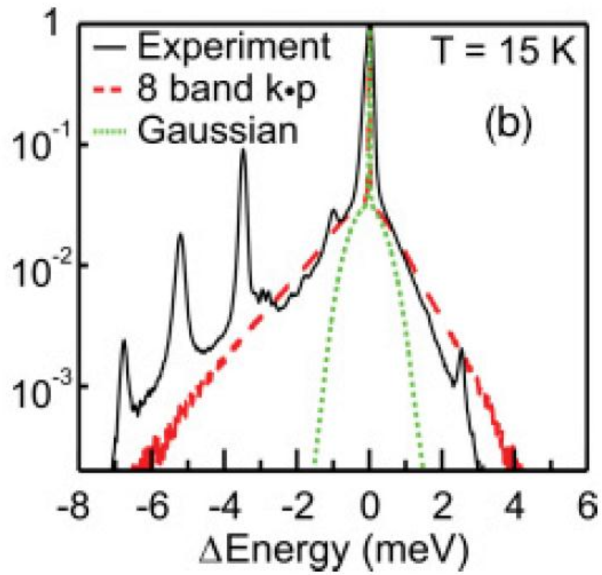

[1]PRB **83**, 041304(R) (2011)

Dephasing mechanism in a semiconductor QD

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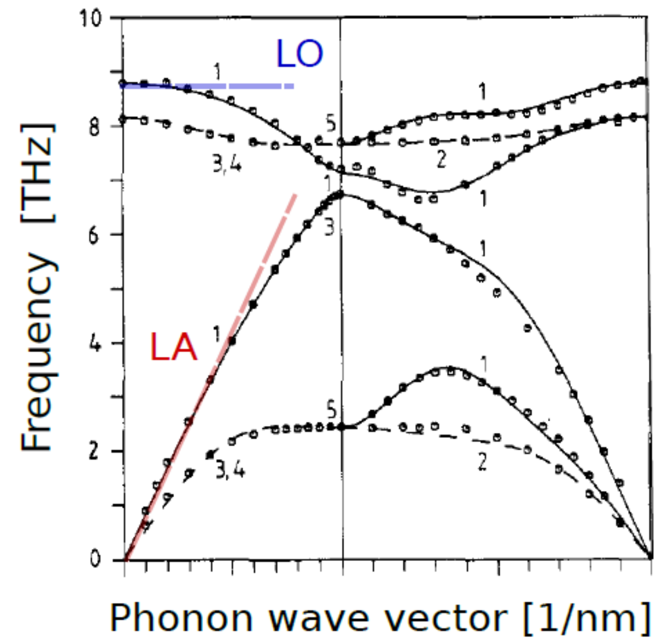
→ Fröhlich (LO) potential[1]

$$g_{\mathrm{LO},q}^{\lambda\mu,3\mathrm{D}} = \frac{1}{q}\sqrt{\frac{e_0^2\hbar\omega_{\mathrm{LO}}}{2\varepsilon_0 V}\left(\frac{1}{\varepsilon_\infty}-\frac{1}{\varepsilon_{\mathrm{st}}}\right)}$$



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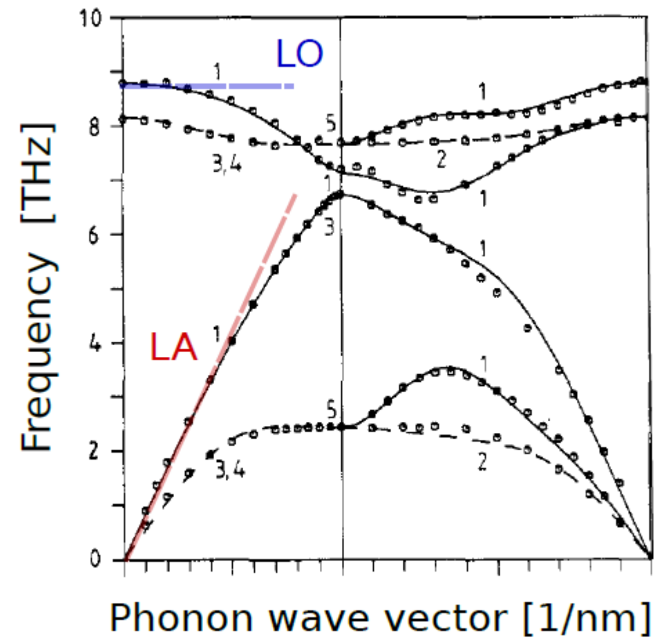

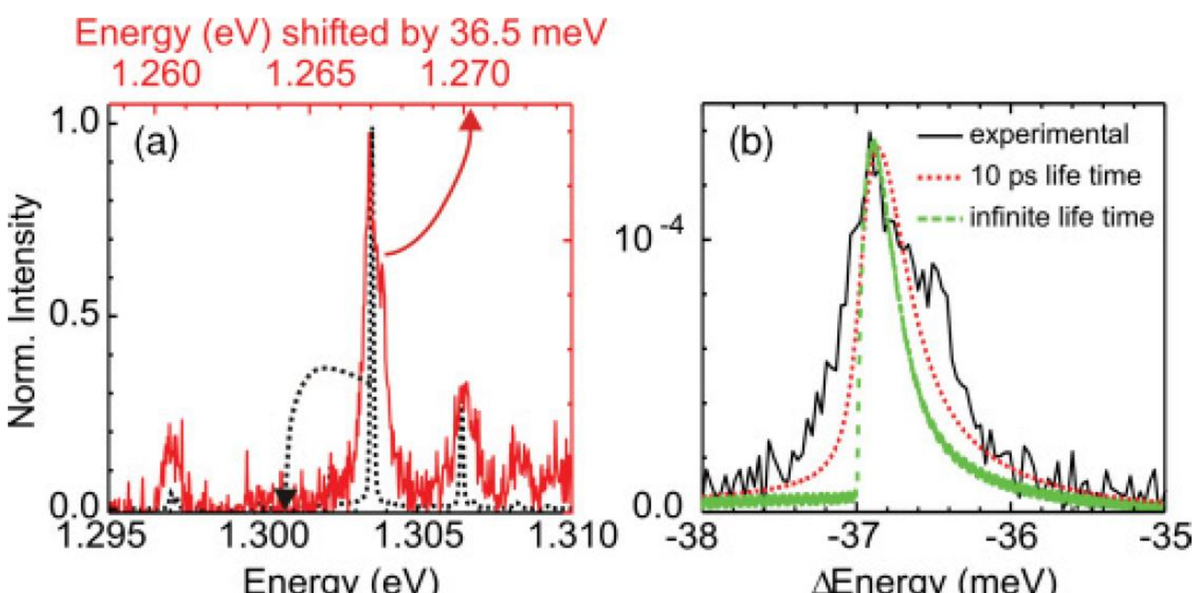

[1]PRB **83**, 041304(R) (2011)

Goal

GOAL

Developing a theoretical framework to find advantageous features of the semiconductor environment

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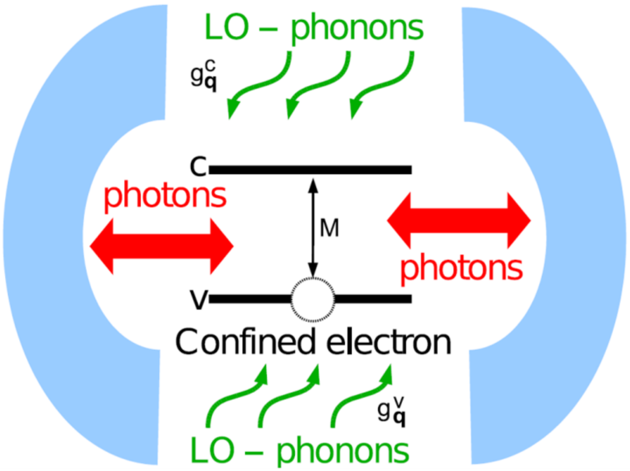
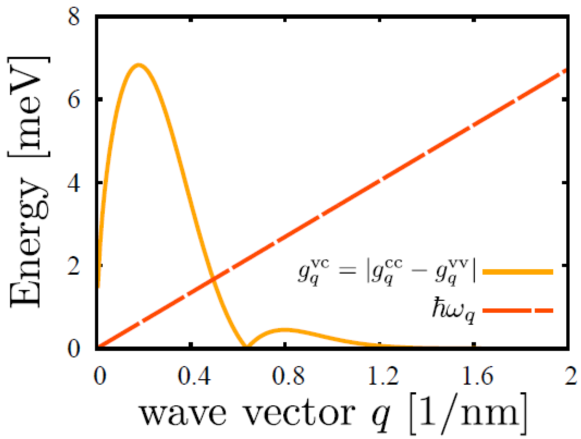
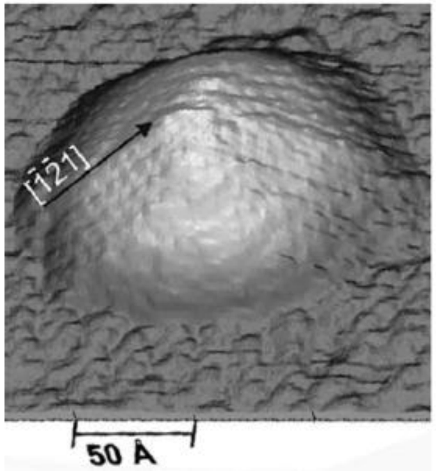
- Quantum Dot Hamiltonian and equation of motion approach

Semiconductor QD cavity-QED Hamiltonian







Marquez et al., Appl.Phys.Lett. 78 (2001)

$$H = H_{\text{el}} + H_{\text{phonon}} + H_{\text{photon}} + H_{\text{laser}}$$

$$H_{\text{el}} = \hbar\sum_i \omega_i a_i^\dagger a_i + \hbar \sum_{ijlm} V^{ij}_{lm} a_i^\dagger a_j^\dagger a_l a_m$$

$$H_{\text{phonon}} = \hbar\sum_{\mathbf{q}} \omega_q b^\dagger_{\mathbf{q}} b_{\mathbf{q}} + \hbar\sum_{i,j,\mathbf{q}} g^{ij}_{\mathbf{q}}\ a_i^\dagger a_j b^\dagger_{\mathbf{q}} + \text{H.c.}$$

$$H_{\text{photon}} = \hbar\sum_{\mathbf{k}} \omega_k c^\dagger_{\mathbf{k}} c_{\mathbf{k}} + \hbar\sum_{i,j,\mathbf{q}} M^{ij}_{\mathbf{k}}\ a_i^\dagger a_j c^\dagger_{\mathbf{k}} + \text{H.c.}$$

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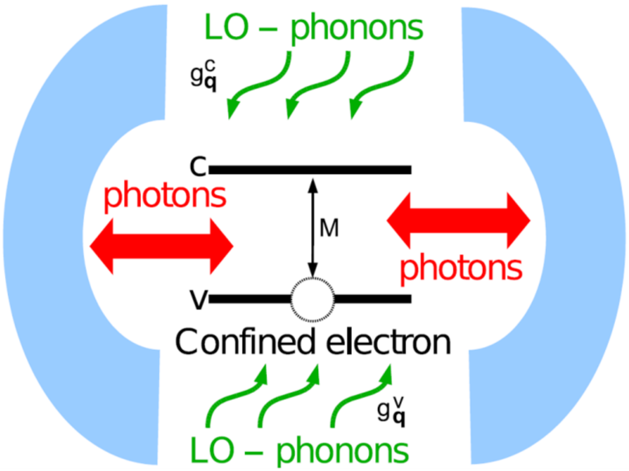
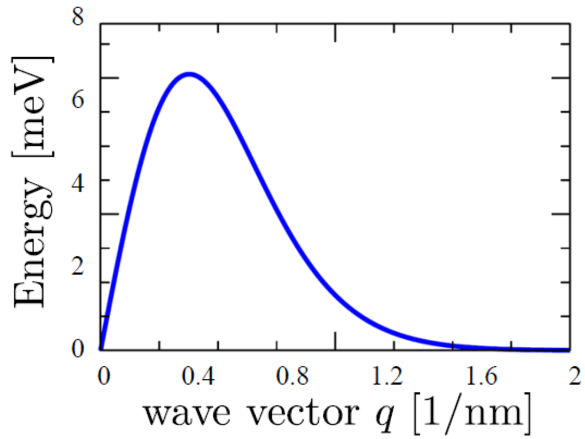
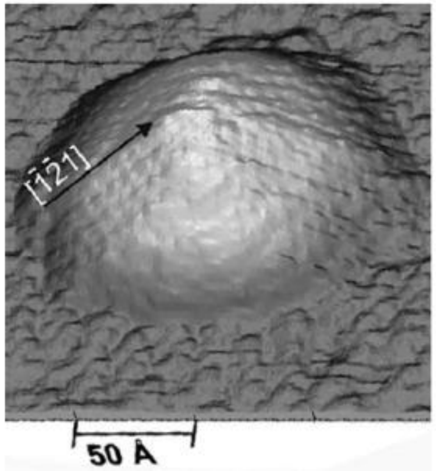
Semiconductor QD cavity-QED Hamiltonian







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Solving LO-phonon QD cavity-QED without factorization

for every possible combination of phonon, photon, and electron operators for example a two-level system:

$$G^{p,s}_{m,n} := a^\dagger_v a_v c^{\dagger p} c^s \bar{b}^{\dagger m} \bar{b}^n$$
$$E^{p,s}_{m,n} := a^\dagger_c a_c c^{\dagger p} c^s \bar{b}^{\dagger m} \bar{b}^n$$
$$T^{p,s}_{m,n} := a^\dagger_v a_c c^{\dagger p} c^s \bar{b}^{\dagger m} \bar{b}^n$$

Solving LO-phonon QD cavity-QED without factorization

Using product rule for operators: $\partial_t \left(a_c^\dagger a_c c^\dagger c b_q^\dagger b_q\right) = \left(\partial_t\ a_c^\dagger a_c c^\dagger c\right) b_q^\dagger b_q + c^\dagger c \left(\partial_t\ a_c^\dagger a_c b_q^\dagger b_q\right)$

and generalized commutation relations:

$$[A, F(B)] = [A, B]F'(B)$$

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and

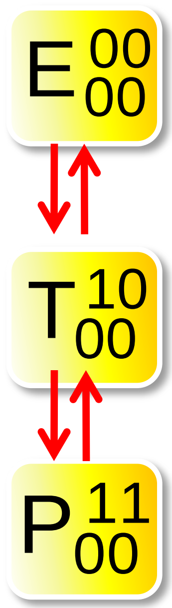
$$T^{p,s}_{m,n} := a_v^\dagger a_c c^{\dagger p} c^s \bar{b}^{\dagger m} \bar{b}^n$$

their dynamics, e.g.

$$\begin{aligned}
&\partial_t \langle T^{p,s}_{m,n} \rangle = \\
&= -i \left[\omega_{cv} - (p-s)\omega_0 - (m-n)\omega_{LO} - i(p+s)\kappa - i\gamma\right] \langle T^{p,s}_{m,n} \rangle \\
&\quad - ip\, M \langle E^{p-1,s}_{m,n} \rangle - iM(\langle E^{p,s+1}_{m,n} \rangle - \langle G^{p,s+1}_{m,n} \rangle) - i\Omega(t) \left(\langle E^{p,s}_{m,n} \rangle - \langle G^{p,s}_{m,n} \rangle\right) \\
&\quad - i \, \langle T^{p,s}_{m,n+1} \rangle - i \, \langle T^{p,s}_{m+1,n} \rangle + i \; m \; g_v \, \langle T^{p,s}_{m-1,n} \rangle - i \; n \; g_c \, \langle T^{p,s}_{m,n-1} \rangle,
\end{aligned}$$

General set of equations of motion

For example, in the case of LO-phonon assisted vacuum Rabi oscillations ($E^{11}_{00} = 0$) :



Photon interaction: ←

numerically solvable up to arbitrary accuracy, reproducing analytical solutions of the IBM and JCM

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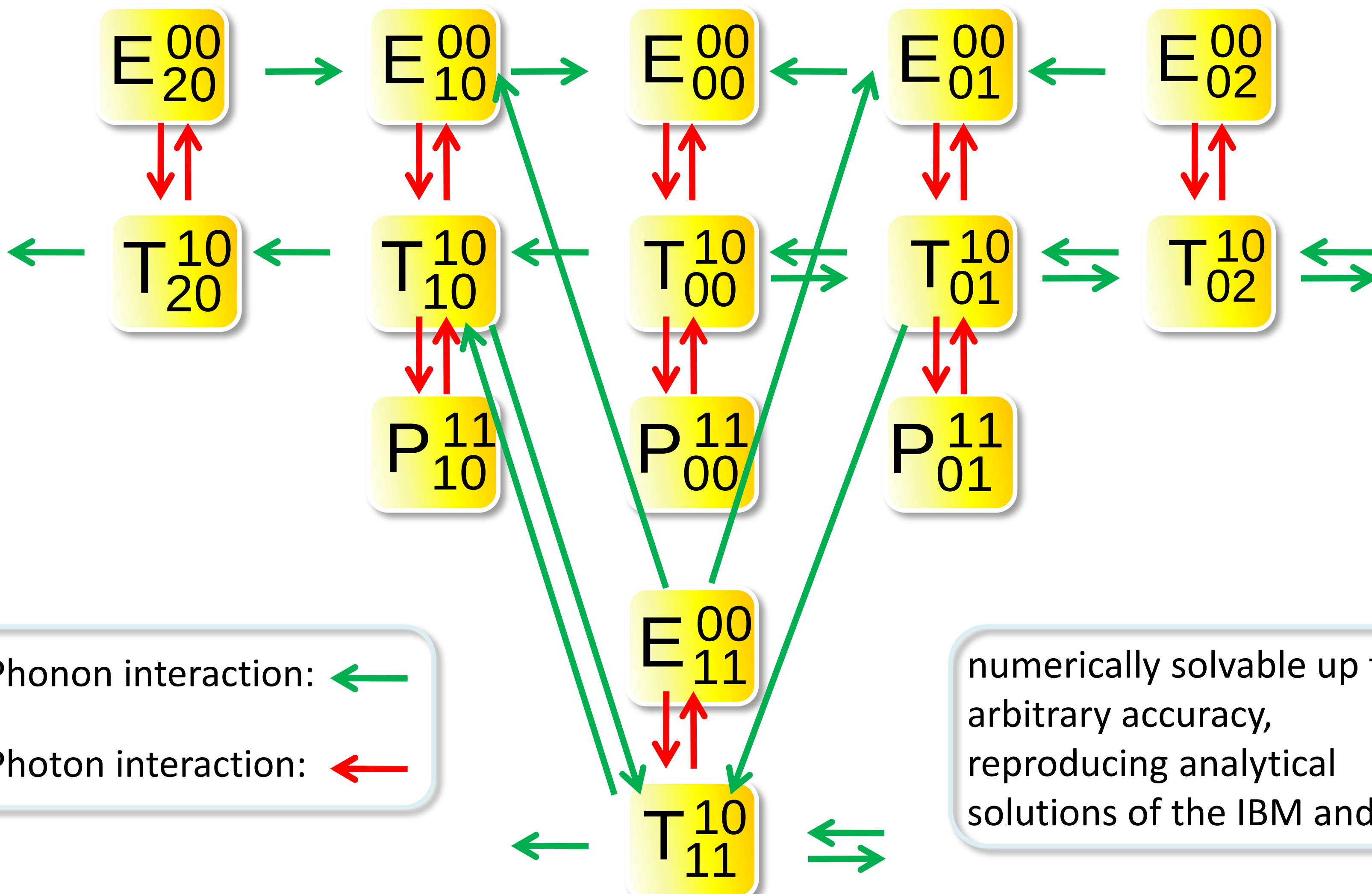

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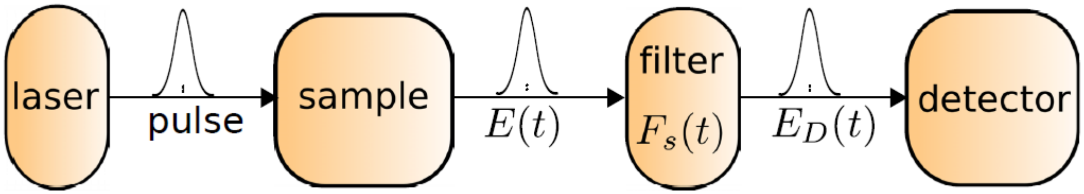
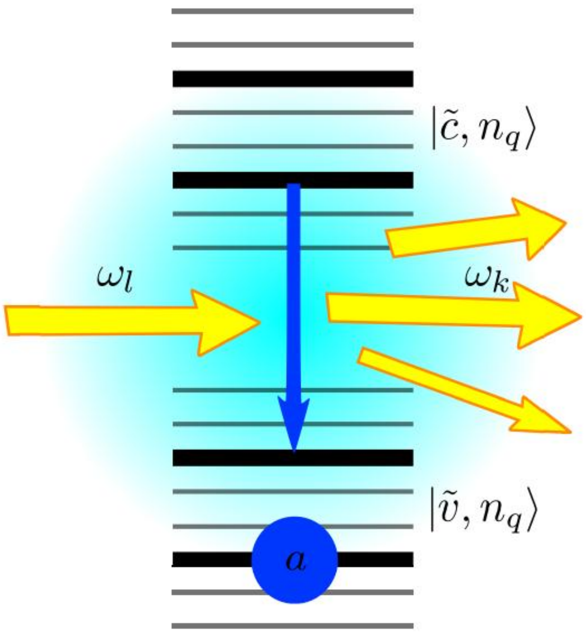
(ii) Laser-driven quantum dot

- ❑ Time-resolved phonon-assisted Mollow triplet
- ❑ Proposal for a phonon laser

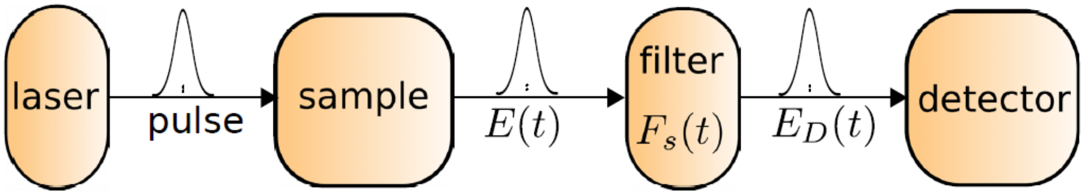
Strong excitation:

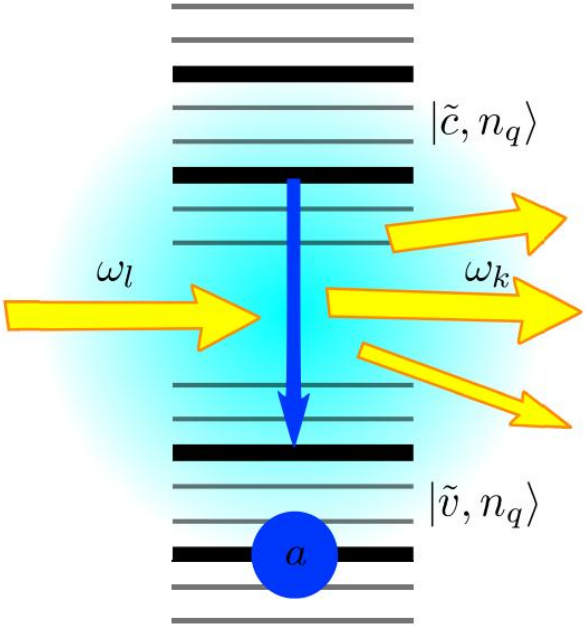




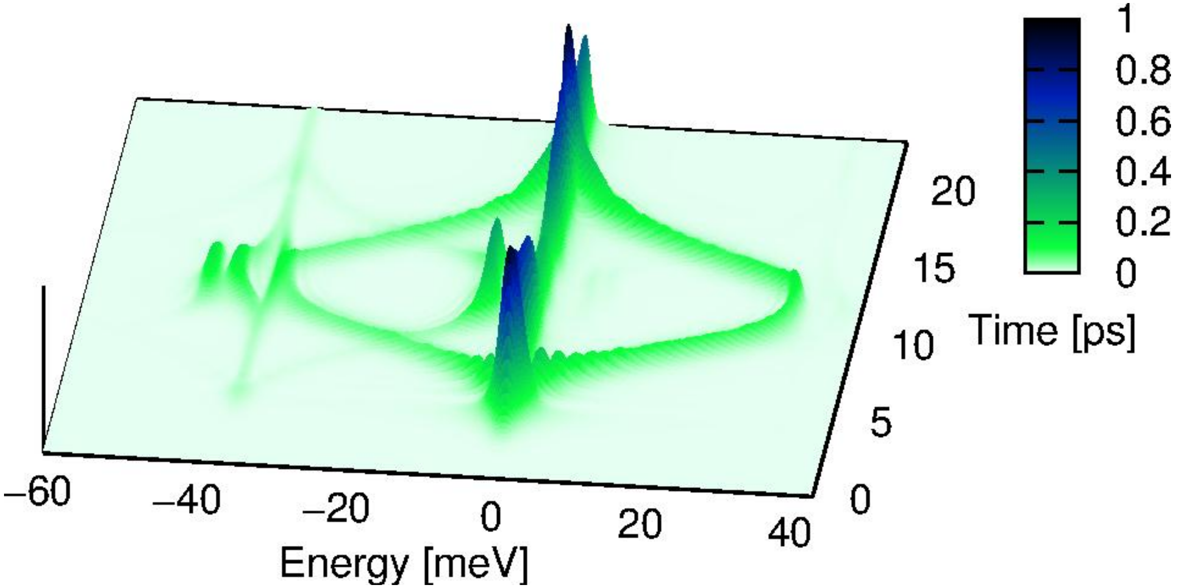
(b)strong excitation

Strong excitation:



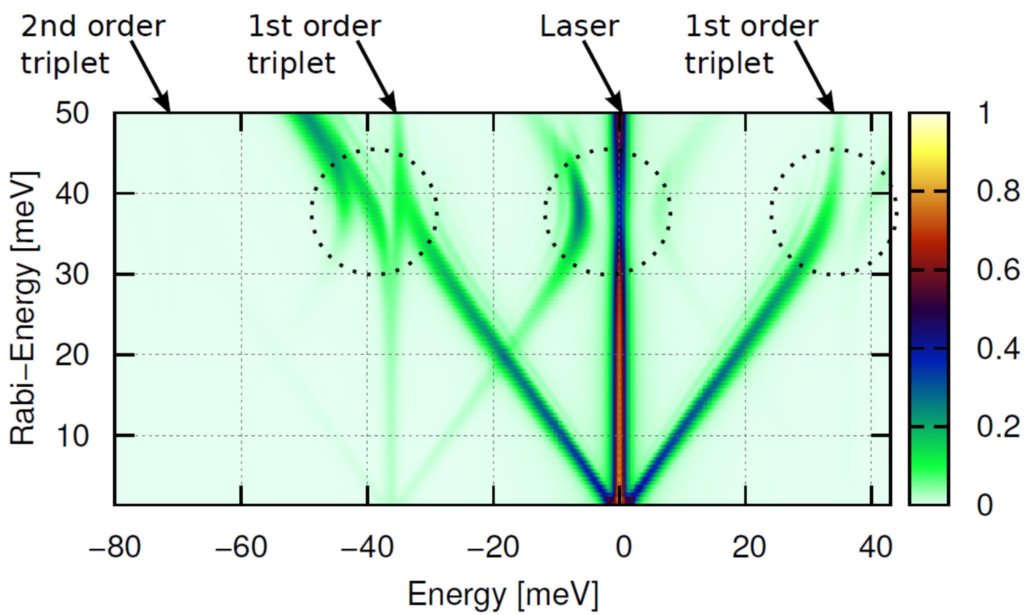


(b)strong excitation



[1]PRB **84**, 075314 (2011)

Phonon coupling strength via anti-crossing



- Spectrum shows the usual Mollow triplet and phonon-assisted Mollow triplets
- Additional anticrossings, when the Rabi-energy matches the phonon energy (Here 36.4 meV for InGaAs/GaAs-QD)
- These anti-crossings scale with the electron-phonon coupling strength

[1]PRB **84**, 125324 (2011)

(ii) Laser-driven quantum dot

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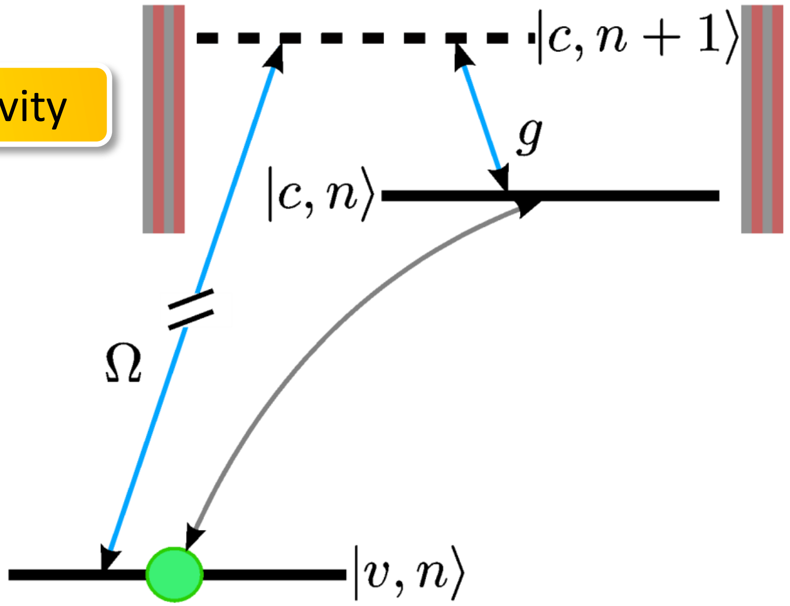
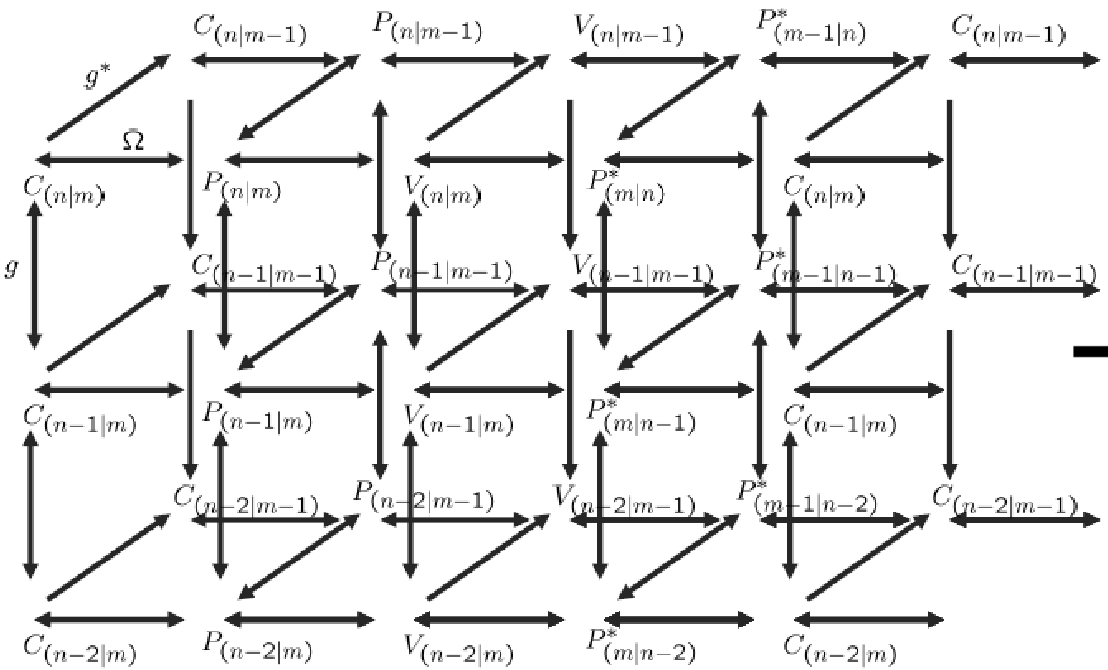
Proposal for a phonon laser

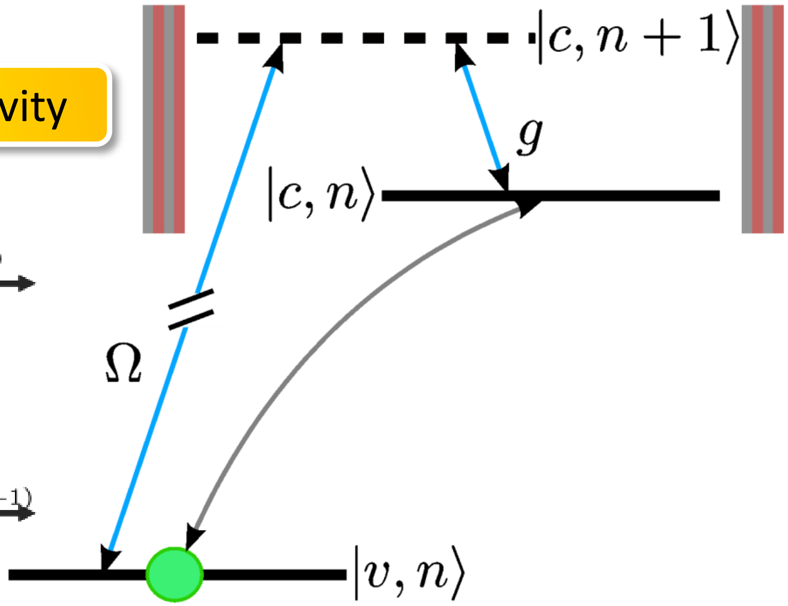
Non-resonant excited 2-level system in an acoustic cavity



Proposal for a phonon laser

Non-resonant excited 2-level system in an acoustic cavity

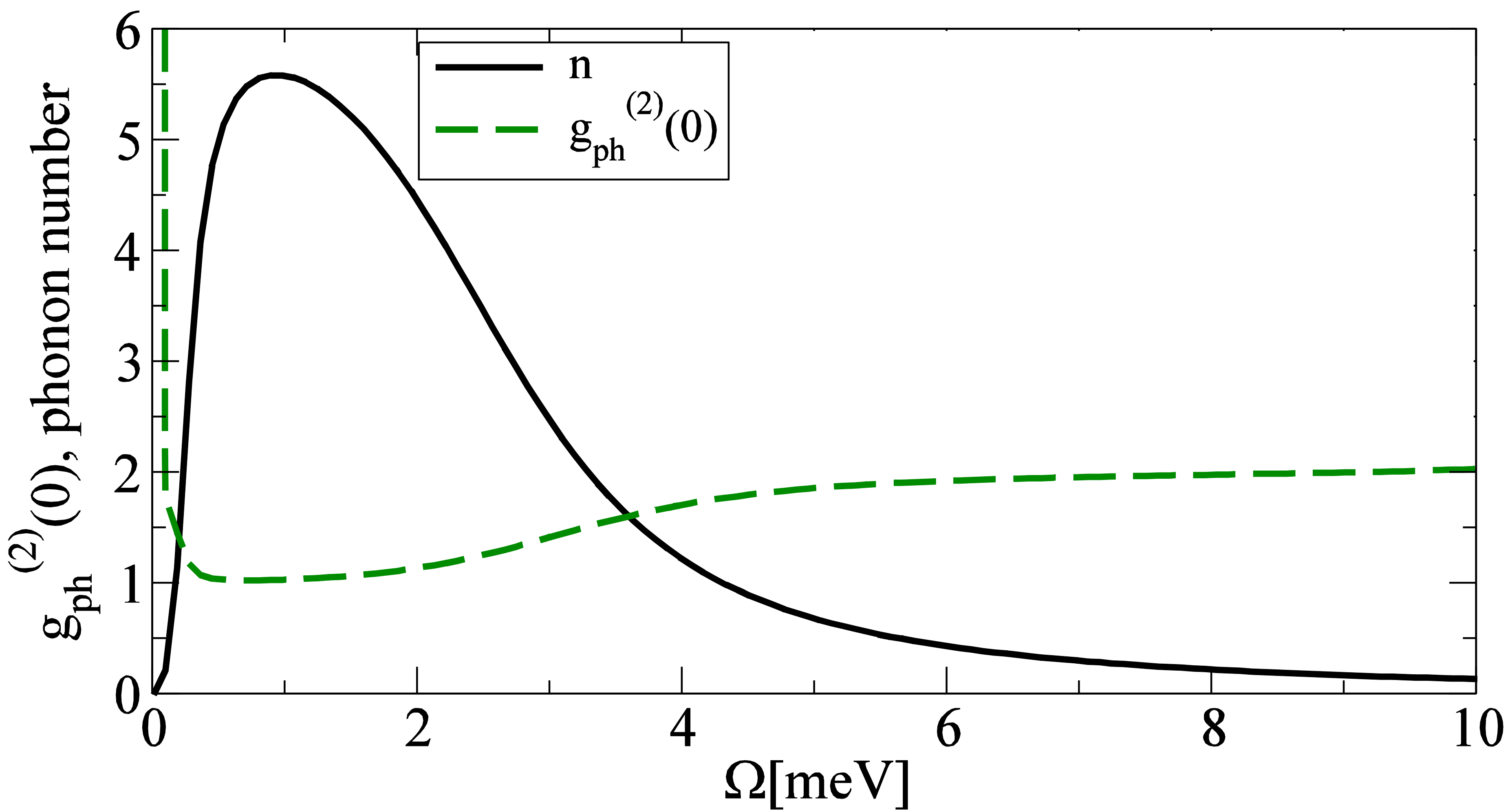




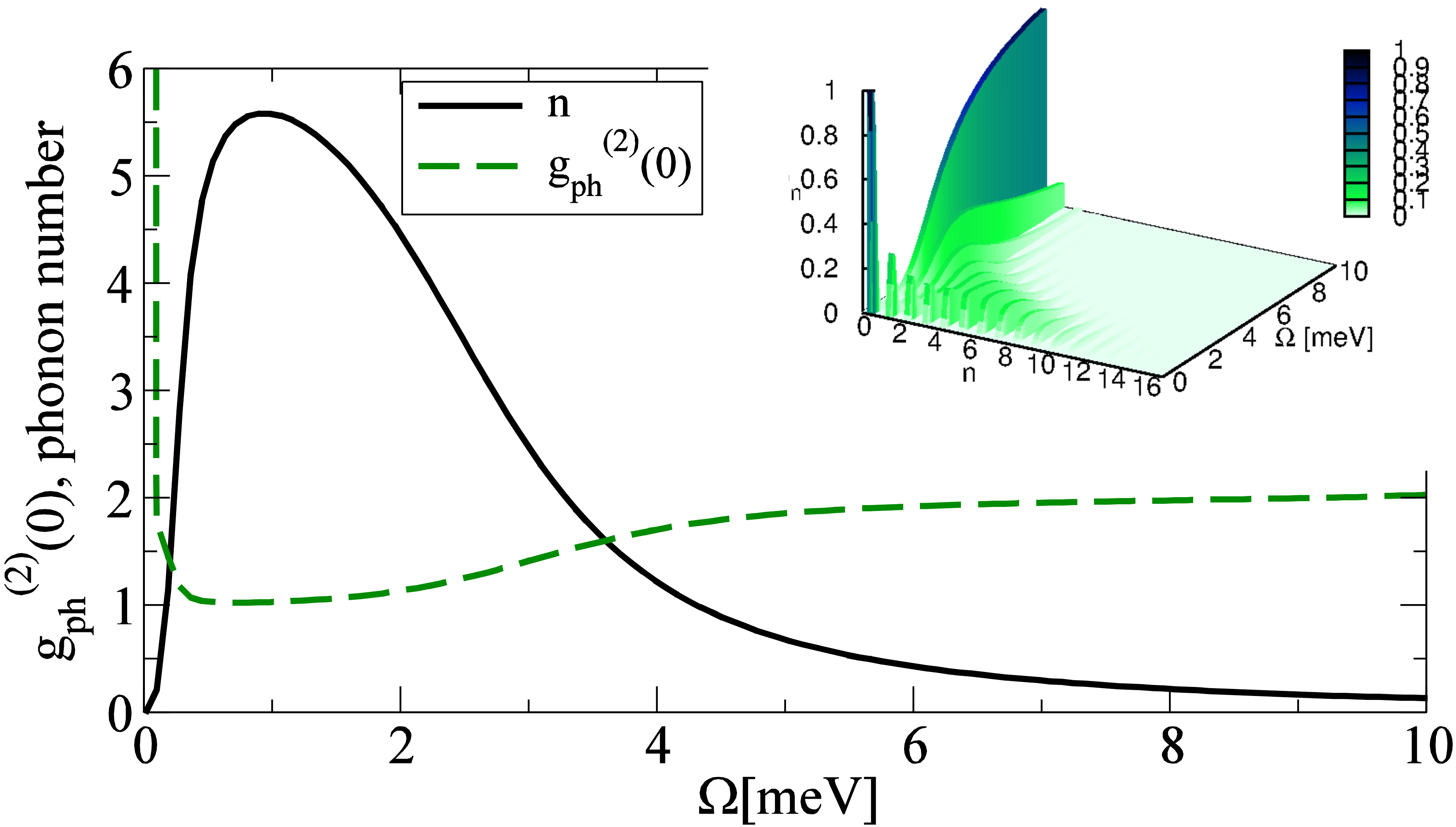
High orders of phonon operators become important [1,2,3]

[1]PRL **104**, 156801 (2010), [2]PSS(b) 248, **872** (2011), [3] submitted (2012)

Generation of coherent phonons



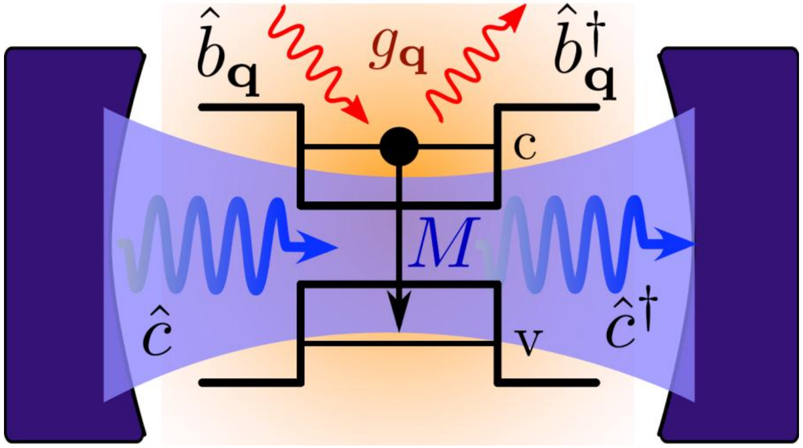
Generation of coherent phonons



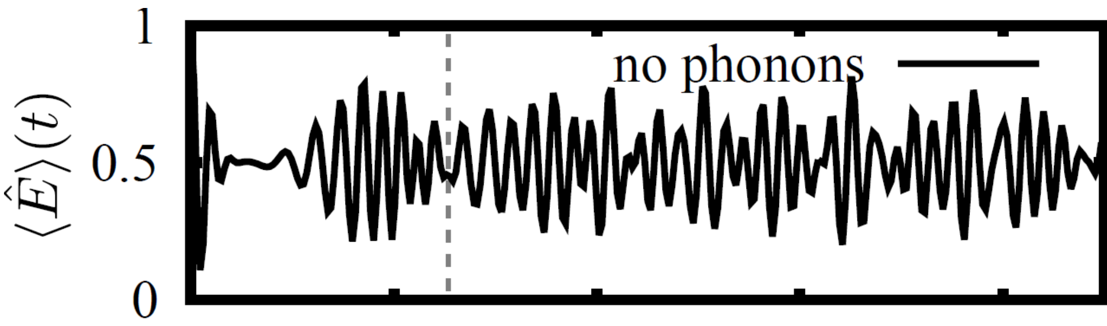
(iii) quantum dot cavity-QED

- ❑Enhancement of collapse and revival phenomenon
- ❑ Photon-loss and induced quantum feedback

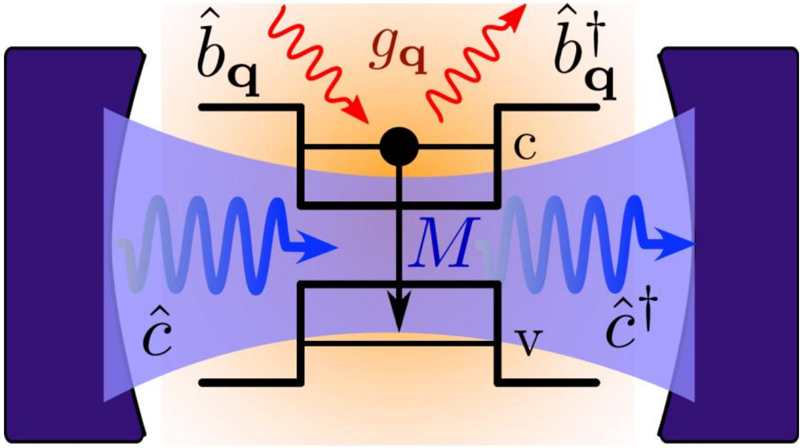
LA-phonon assisted collapse and revival



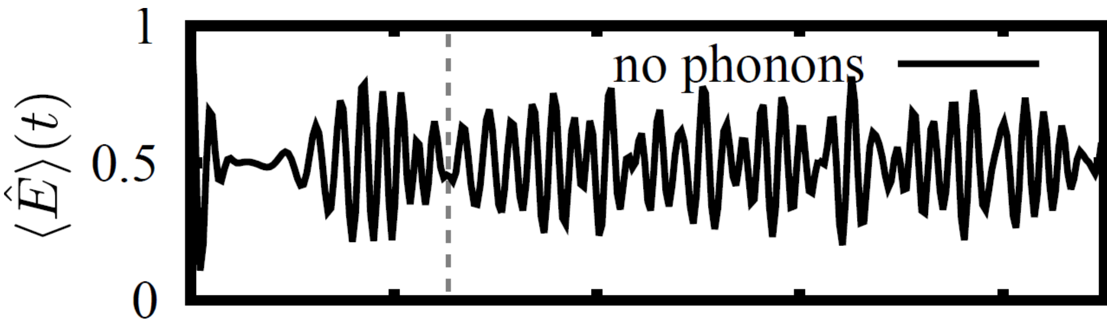
Coherent state: cavity field prepared initially in Glauber states



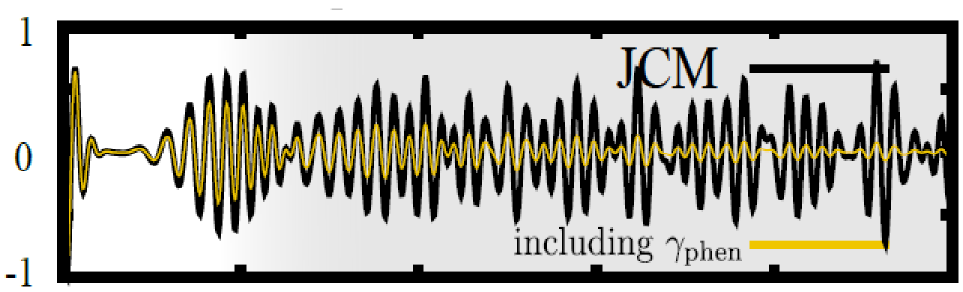
LA-phonon assisted collapse and revival



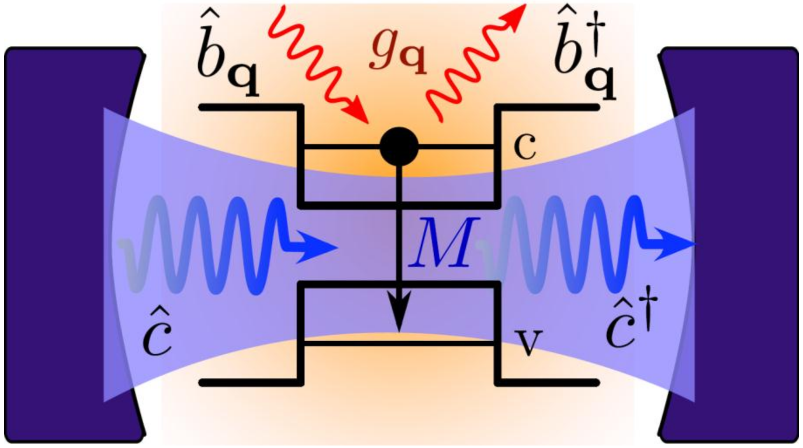
Coherent state: cavity field prepared initially in Glauber states



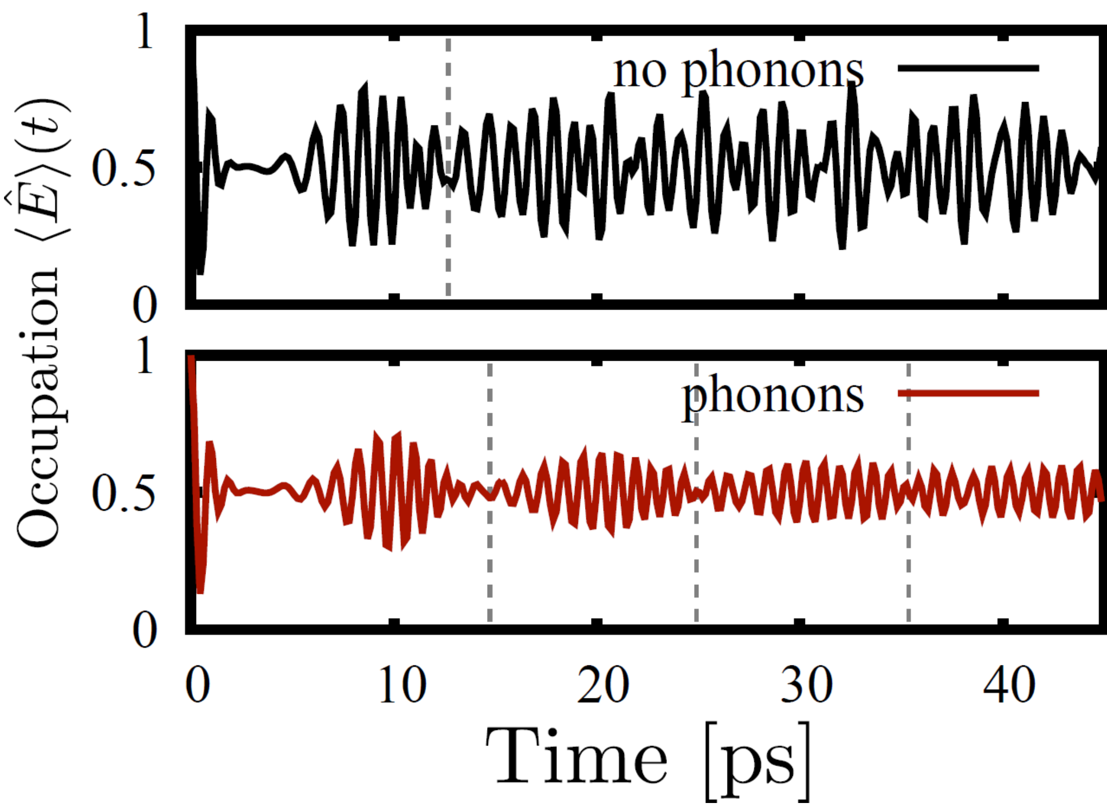
Markovian theory



LA-phonon assisted collapse and revival



Coherent state: cavity field prepared initially in Glauber states



Non-Markovian theory

Collapse and revival phenomenon is enhanced due to LA-phonon induced dephasing [1]

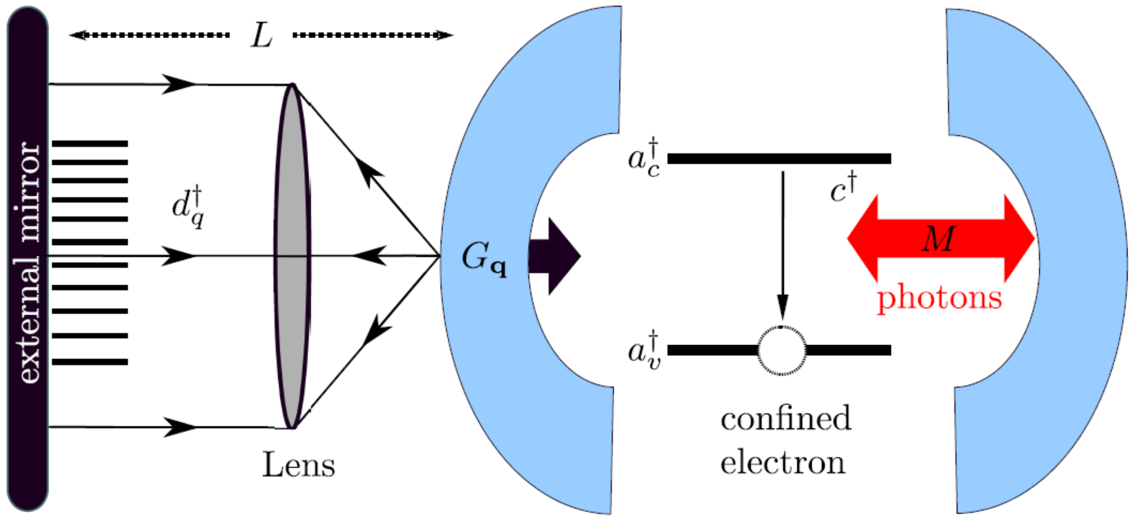
[1] submitted (2012)

(iii) quantum dot cavity-QED

- Enhancement of collapse and revival phenomenon
- Photon-loss and induced quantum feedback

Photon-loss induced quantum feedback

Set-up for quantum feedback

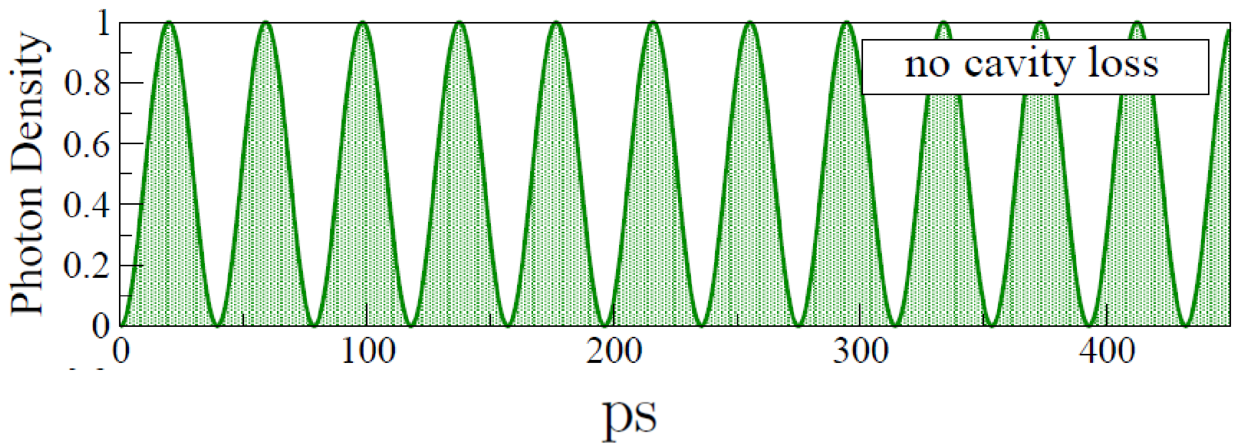


External mirror shapes the mode continuum in front of the mirror to introduce a delay effect[1]

$$H = \sum_q \left(G_q c^\dagger d_q + G_q^* d_q^\dagger c \right)$$

[1] in preparation (2012)

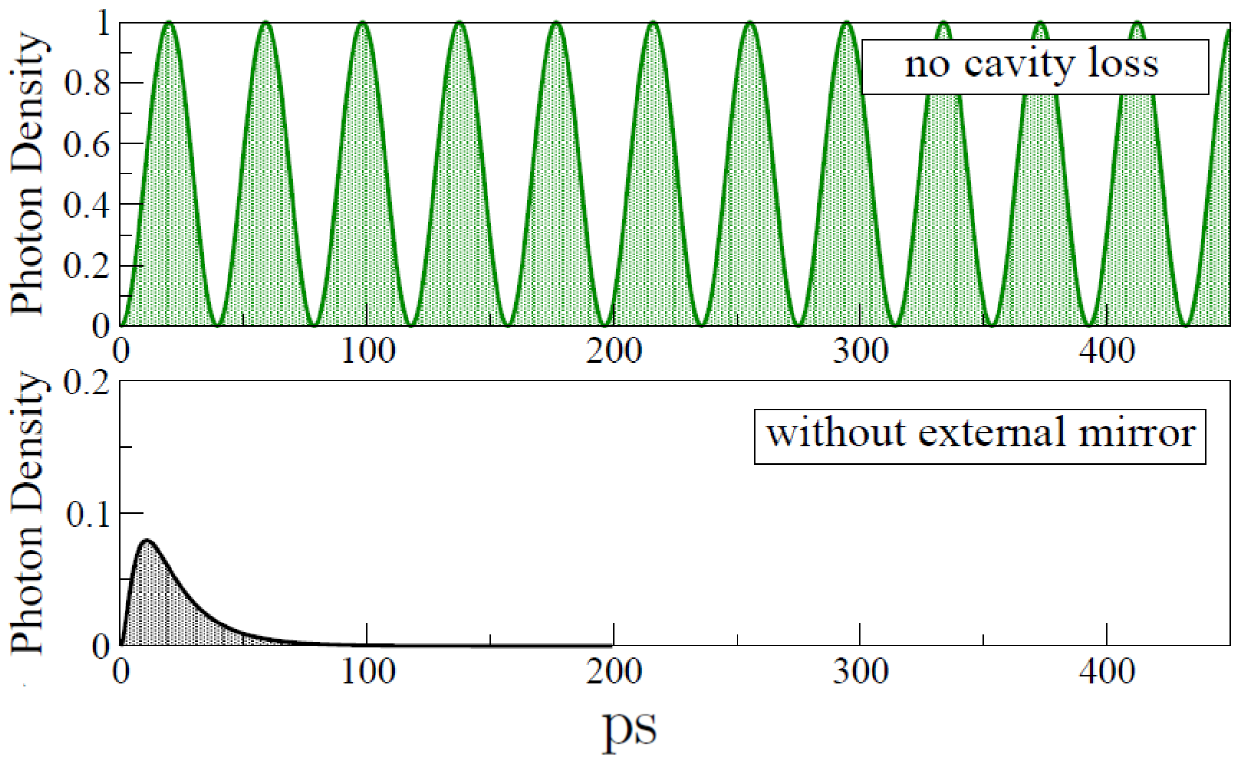
Photon-loss induced quantum feedback



$$G_q = 0$$

Jaynes-Cummings model solution,
if no outcoupling is present

Photon-loss induced quantum feedback

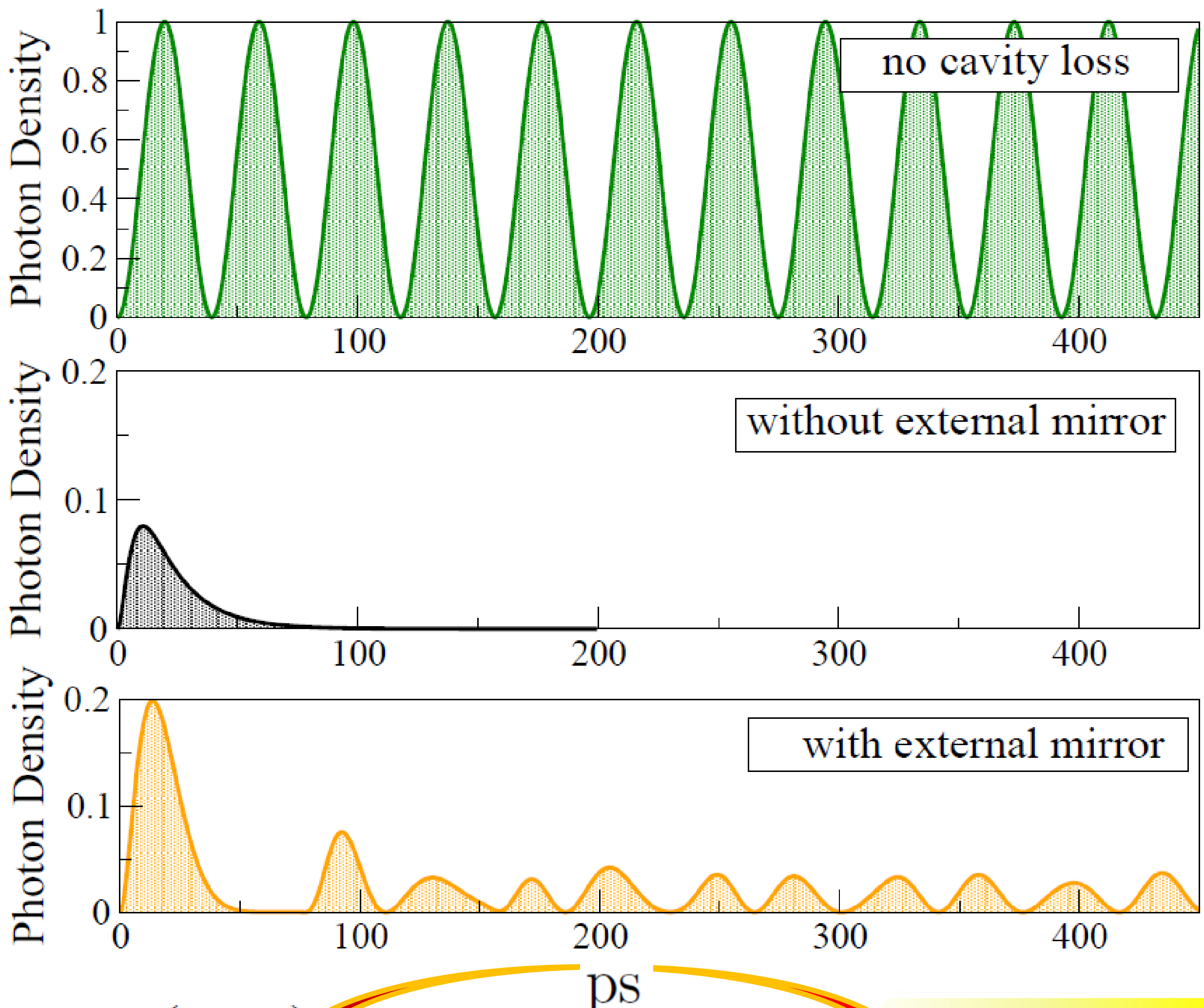


$$G_q = G_0 \neq 0$$

$$G_0 \gg M$$

Weak coupling solution, if outcoupling is present and stronger than light coupling

Photon-loss induced quantum feedback

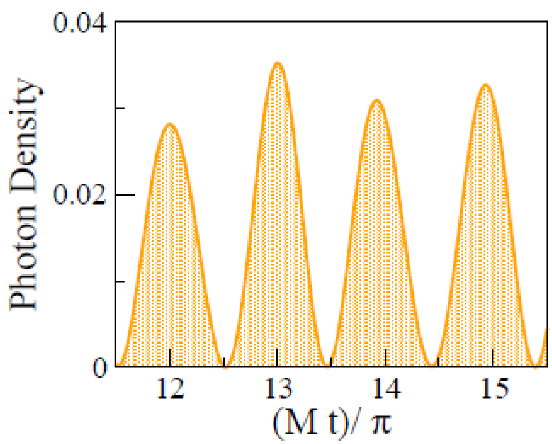


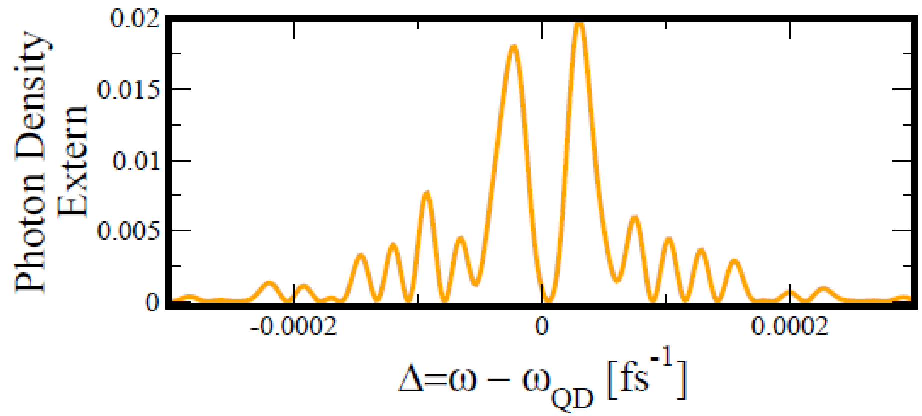
$$G_q = G_0 \sin(qL)$$

Non-Markovian feedback

Photon-loss induced quantum feedback

External mirror shapes the mode continuum in front of the mirror to introduce a delay effect[1]



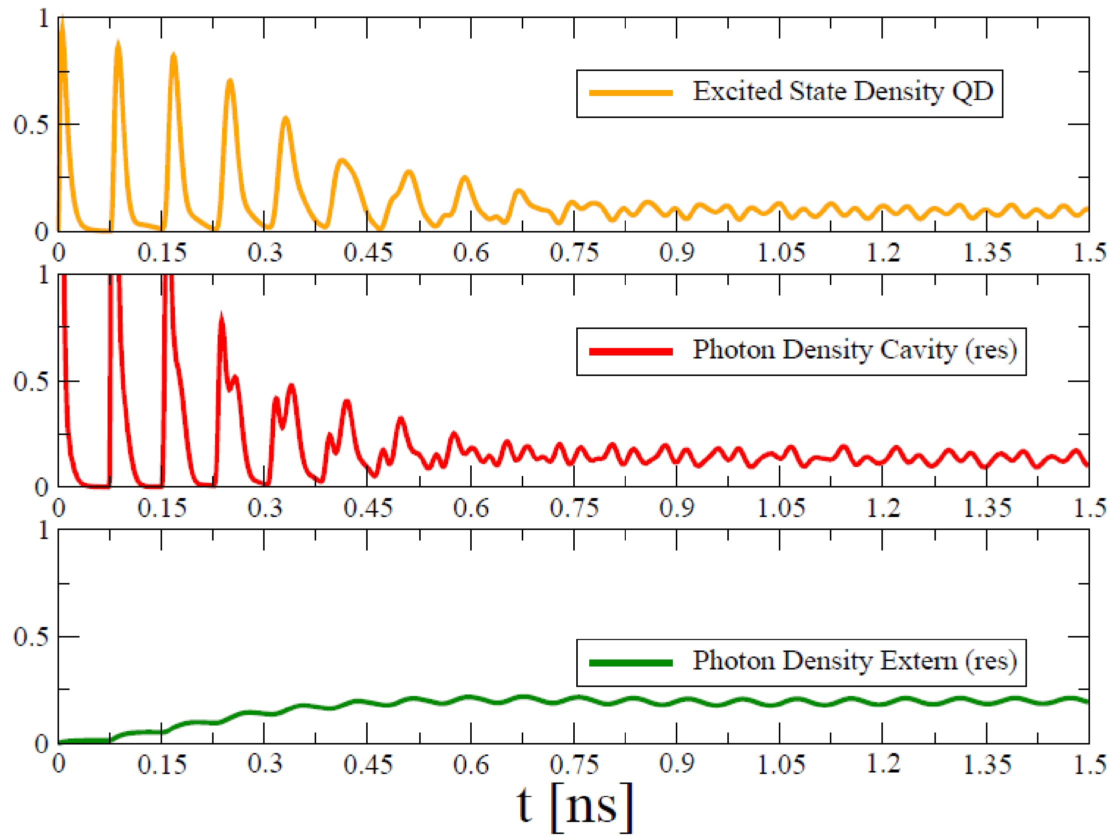


[1] in preparation (2012)

Photon-loss induced quantum feedback

Classical limit[1]:

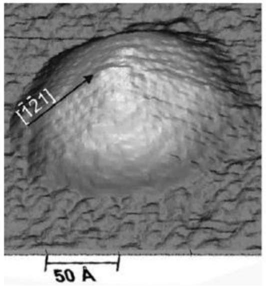
$$\langle a_c^\dagger a_c c^\dagger c \rangle \approx \langle a_c^\dagger a_c \rangle \langle c^\dagger c \rangle$$



[1] in preparation (2012)

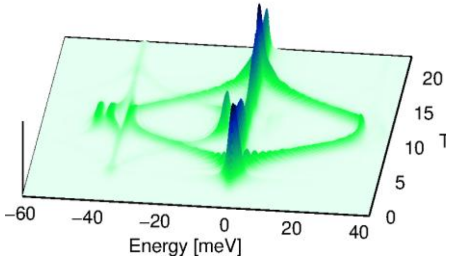
(iv) conclusion

Conclusion



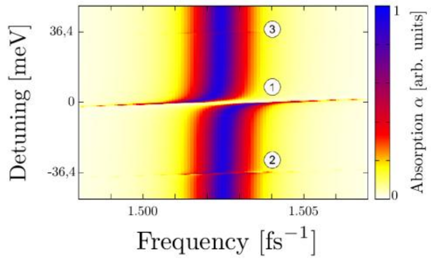
I: Semiconductor Quantum Dot

- ❑ Semiconductor physics and heterostructures
- ❑ Quantum Dot Hamiltonian and equation of motion approach



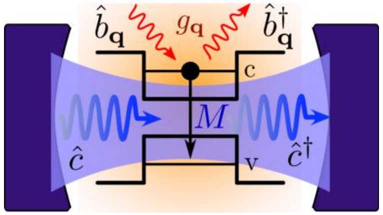
II: Laser-driven Quantum Dot

- ❑ Time-resolved phonon-assisted Mollow triplet
- ❑ Proposal for a phonon laser



III: Quantum Dot – cavity QED

- ❑ Enhancement of collapse and revival phenomenon
- ❑ Photon-loss induced quantum feedback



Thank you for your attention !!