Technische Universität Berlin

Theory of an optically driven quantum dot phonon laser

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Outline

I: Introduction and motivation

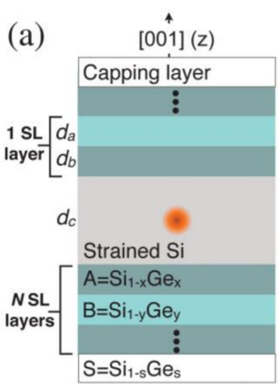
II: Quantum dot phonon laser

- model system
- phonon assisted Rabi-oscillations & phonon lasing
- effective semiclassical treatment

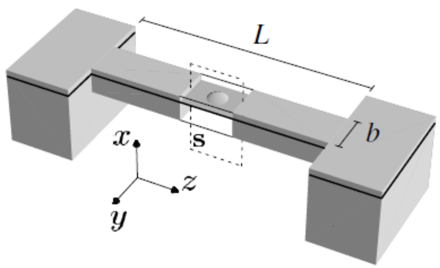
III: Conclusion and goals

phonon-cavities

- Designed to confine a single acoustic phonon mode
- Control of electron-phonon interaction
- Realized in ions, nanomechanical oscillators, semiconductors



[1]Soykal et al, Phys. Rev. Lett. (2011)



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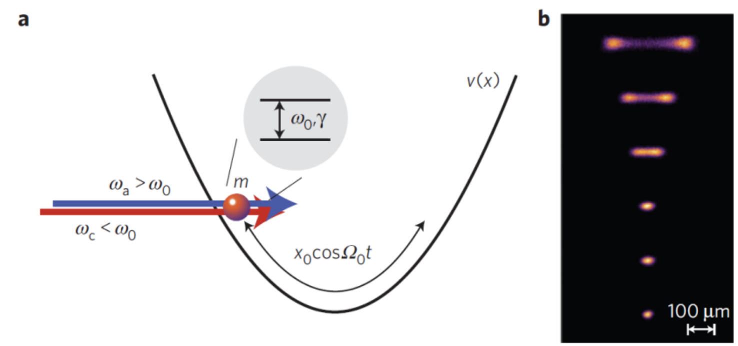
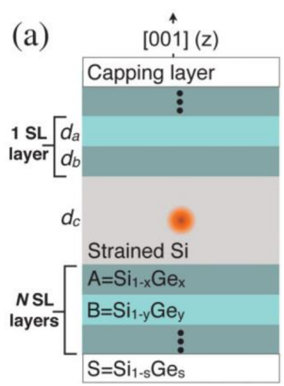


[1] Vahala et al, Nature Phys. (2009)

phonon-cavities

Impose the laser mechanism to phononic systems



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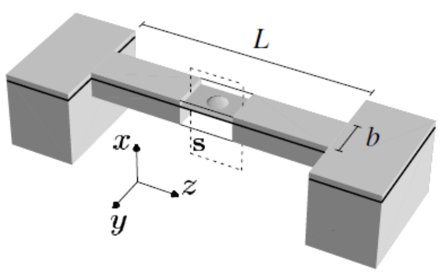

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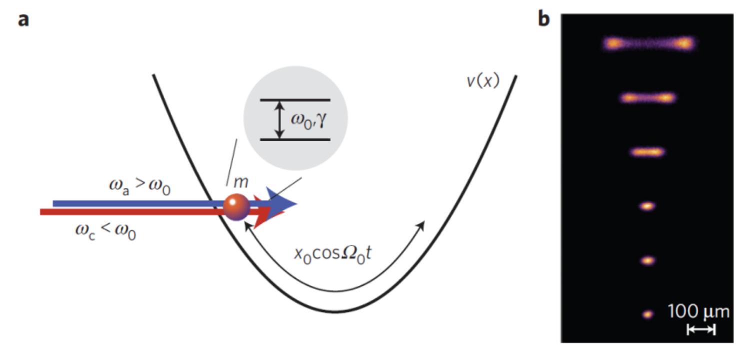

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Quantum dot phonon laser: model

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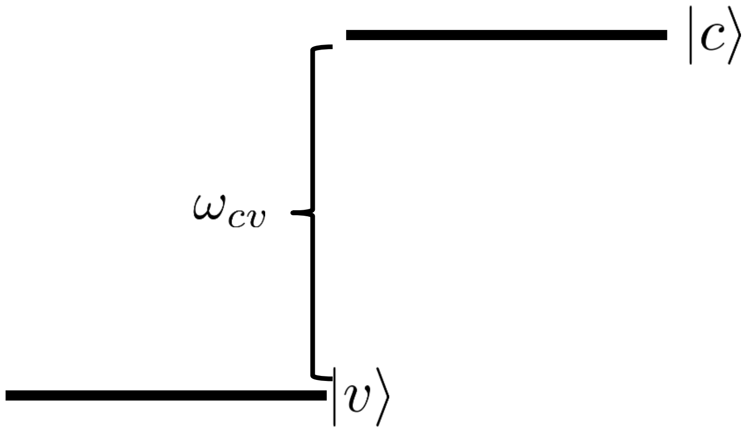
- Quantum dot (two levels) coupled to a singel acoustic phonon mode
- External laser pumps the system at the anti-Stokes resonance



$$\mathcal{H}(t) = \frac{\hbar\omega_{cv}}{2}\sigma_z$$

Kabuss et al., Phys. Rev. Lett. (2012)

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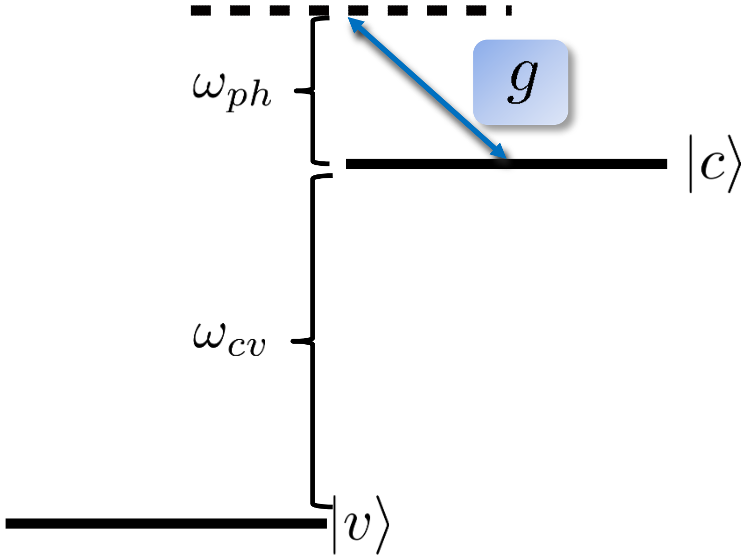

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$$\mathcal{H}(t) = \frac{\hbar\omega_{cv}}{2}\sigma_z + \hbar\omega_{ph} b^\dagger b + g\sigma^\dagger\sigma b^\dagger + H.c.$$

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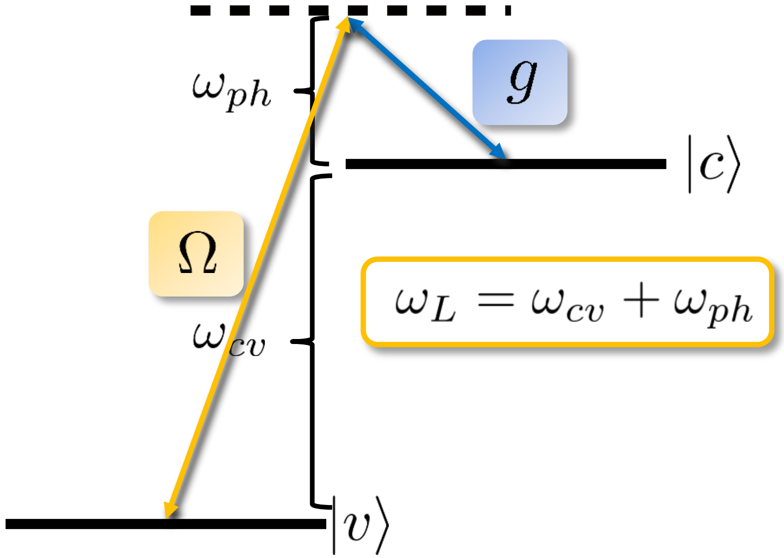

$$\mathcal{H}(t) = \frac{\hbar\omega_{cv}}{2}\sigma_z + \hbar\omega_{ph} b^\dagger b + \Omega(t)\sigma e^{i\omega_L t} + g\sigma^\dagger \sigma b^\dagger + H.c.$$

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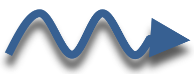
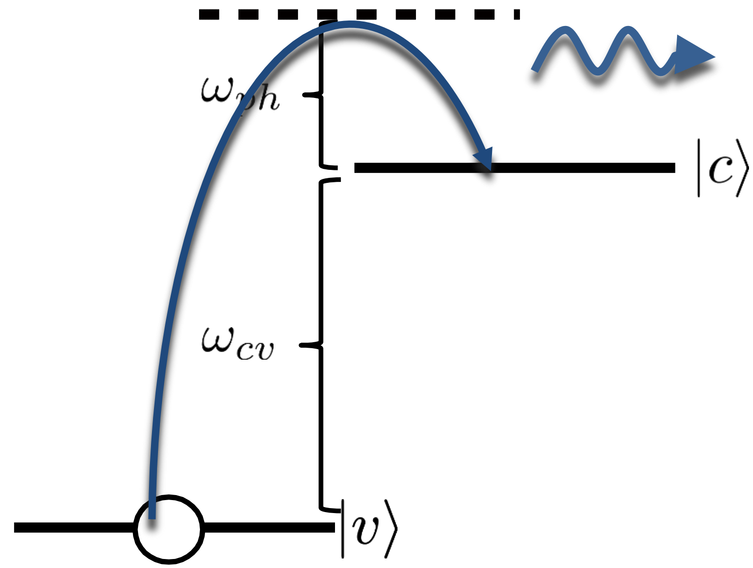


phonon emission



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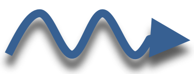
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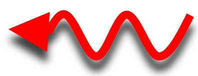
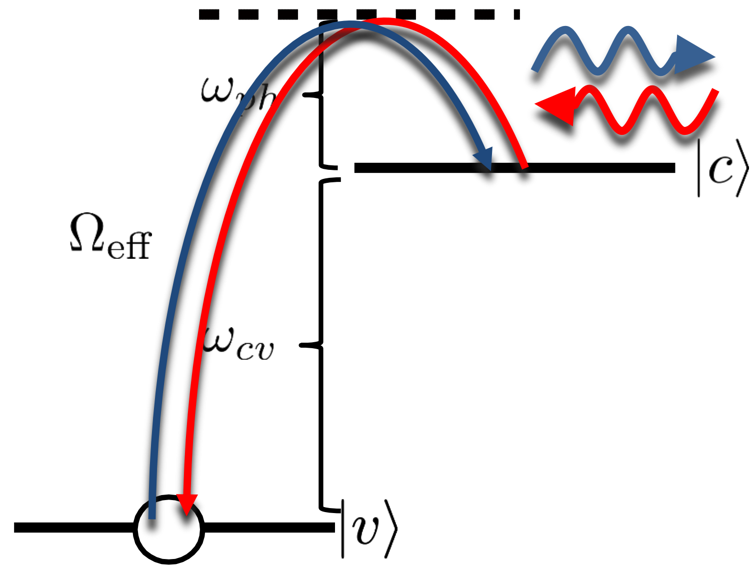

phonon absorption

→ phonon-assisted **Rabi-oscillations**



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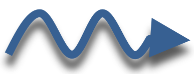

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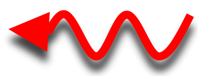
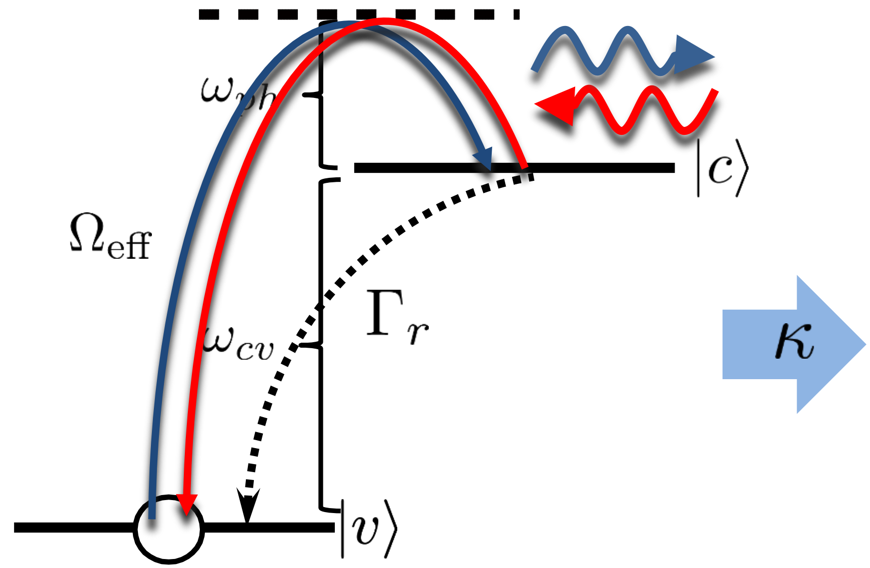


phonon absorption

→ **phonon-assisted**
Rabi-oscillations



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$$\dot{\rho} = -i[\mathcal{H}(t), \rho] + \mathcal{L}\rho$$

Γ_r closes **pumpcycle**
κ phonon loss
γ_{pd} pure dephasing

Kabuss et al., Phys. Rev. Lett. (2012)

Phonon statistics

- Phonon-phonon correlation function

$$g^{(2)}(0) = \frac{\langle b^\dagger b^\dagger b b\rangle}{\langle b^\dagger b\rangle^2}$$

- Phonon probabilities

$$P_n(t) = \frac{1}{n!}\left[\langle b^{\dagger n} b^n\rangle - \sum_{j=1}^{J_{end}} \frac{(n+j)!}{j!} P_{n+j}\right]$$

$$\frac{\dot{N}^{(n|n)}}{n} = -2\kappa N^{(n|n)} - igC^{(n-1|n)} + igC^{(n,n-1)}$$

- Calculation of higher order phonon-phonon correlations

$$\begin{aligned} N^{(n|m)} &= \langle b^{\dagger n} b^m\rangle \\ C^{(n|m)} &= \langle |c\rangle\langle c| b^{\dagger n} b^m\rangle \\ P^{(n|m)} &= \langle |v\rangle\langle c| b^{\dagger n} b^m\rangle \end{aligned}$$

Kabuss et al., Phys. Rev. Lett. (2012), Kabuss et al., Phys. Rev. B (2013), Kabuss et al., Phys. Stat. Sol. (b) (2011)

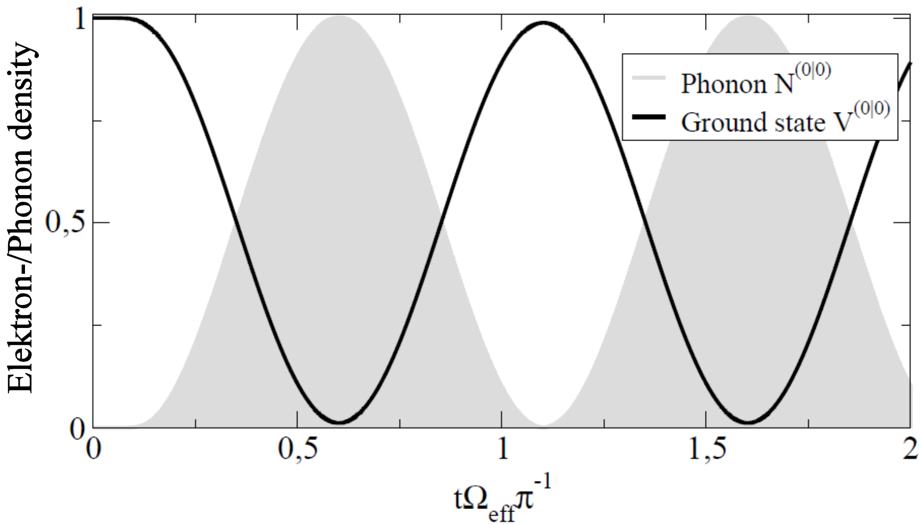
Phonon-assisted Rabi-oscillations & phonon lasing

phonon assisted Rabi-oscillations

$$\kappa = \Gamma_r = \gamma_{pd} = 0$$

$$\Omega_{eff} \approx \frac{\Omega g}{\hbar^2 \omega_{ph}}$$

Carmele et al. Phys. Rev. B(R) (2013),



- **Phonon-assisted Rabi-oscillations**
- Phonon in Fock state
- Fraction of bunched phonons

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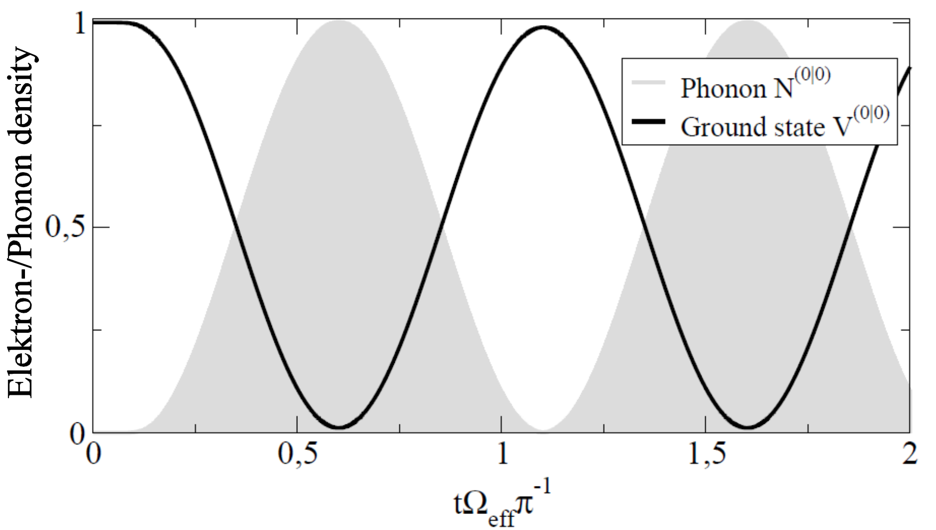

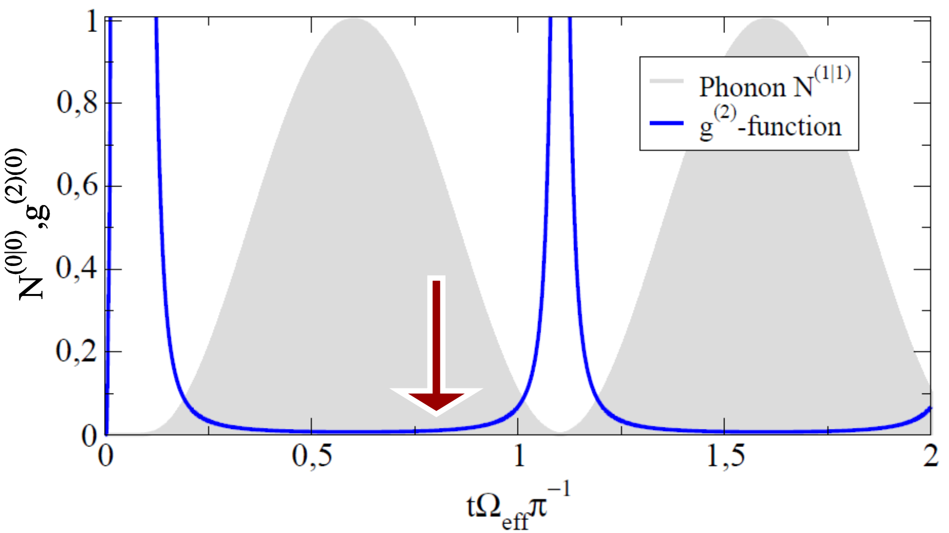

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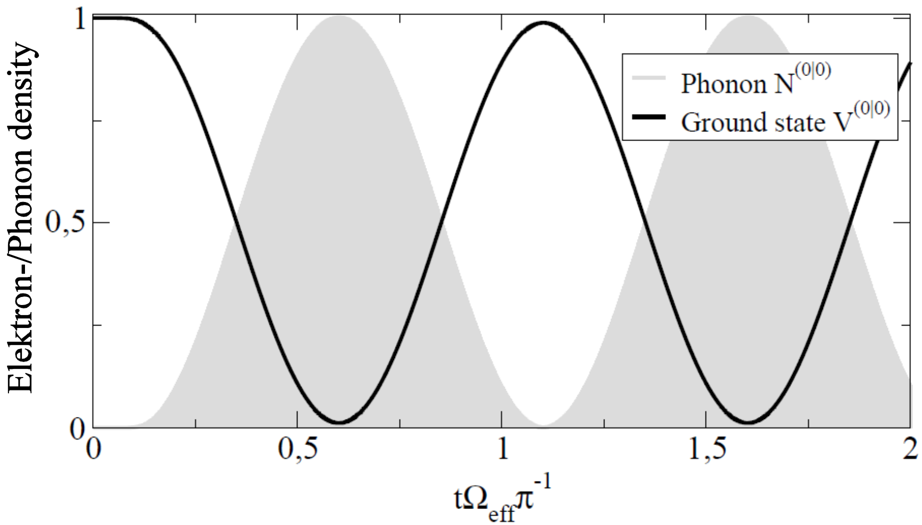

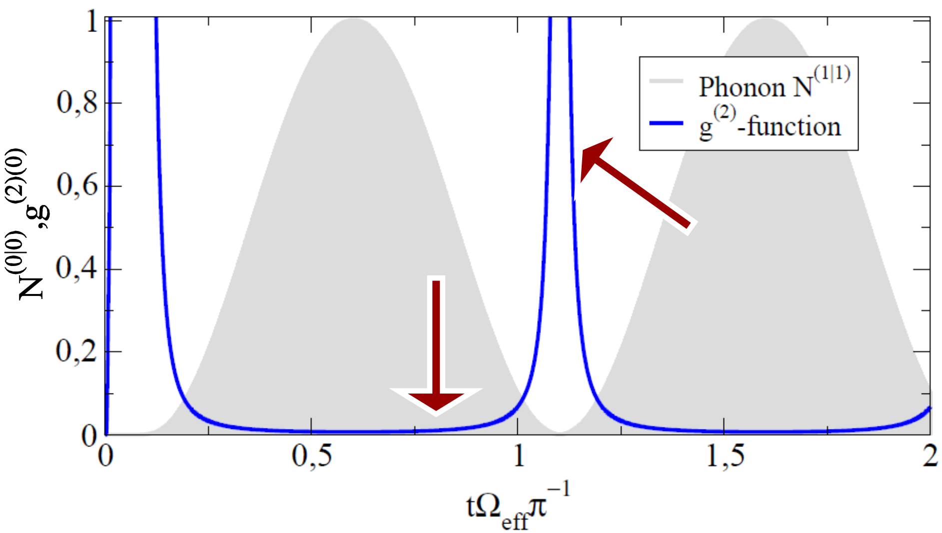

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phonon lasing

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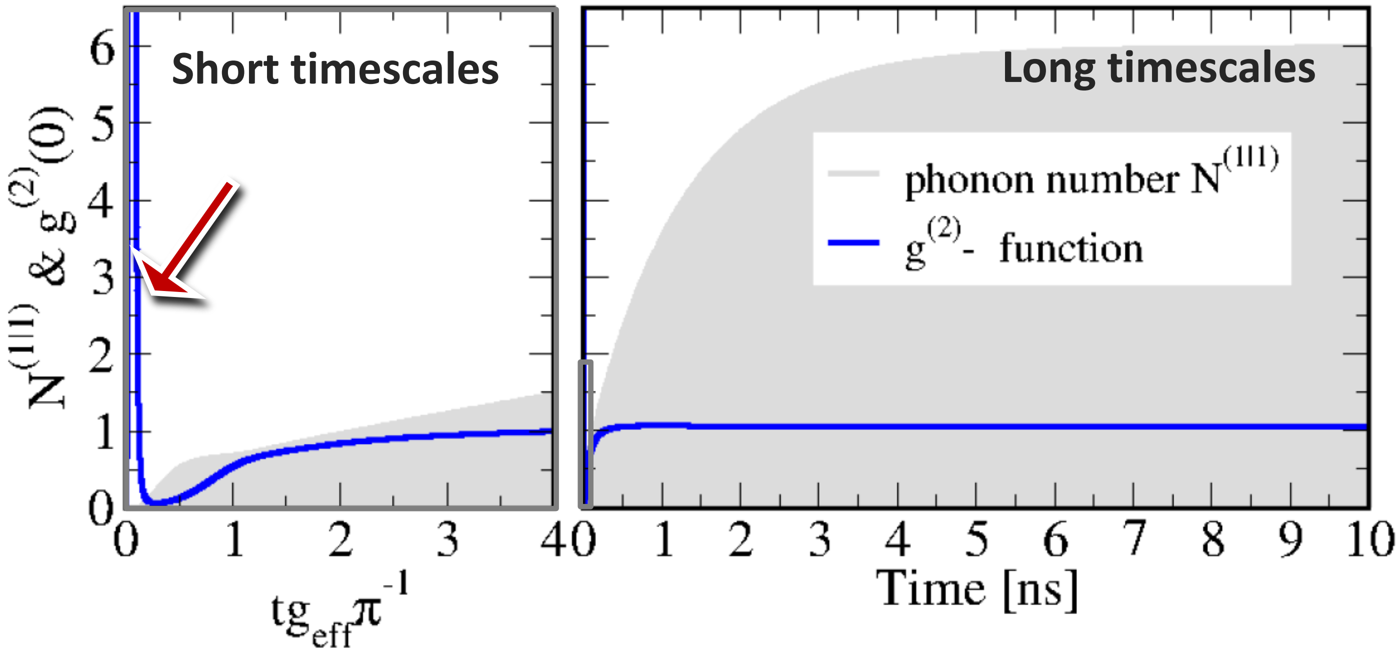

Short timescales

- $t < \Omega_{\text{eff}}^{-1}$ — **phonon fluctuations**
- $t \approx \Omega_{\text{eff}}^{-1}$ — Fock-phonon due to induced Raman process

Long timescales $\longrightarrow$ $t \approx \Gamma_r^{-1}$ — Coherent phonons

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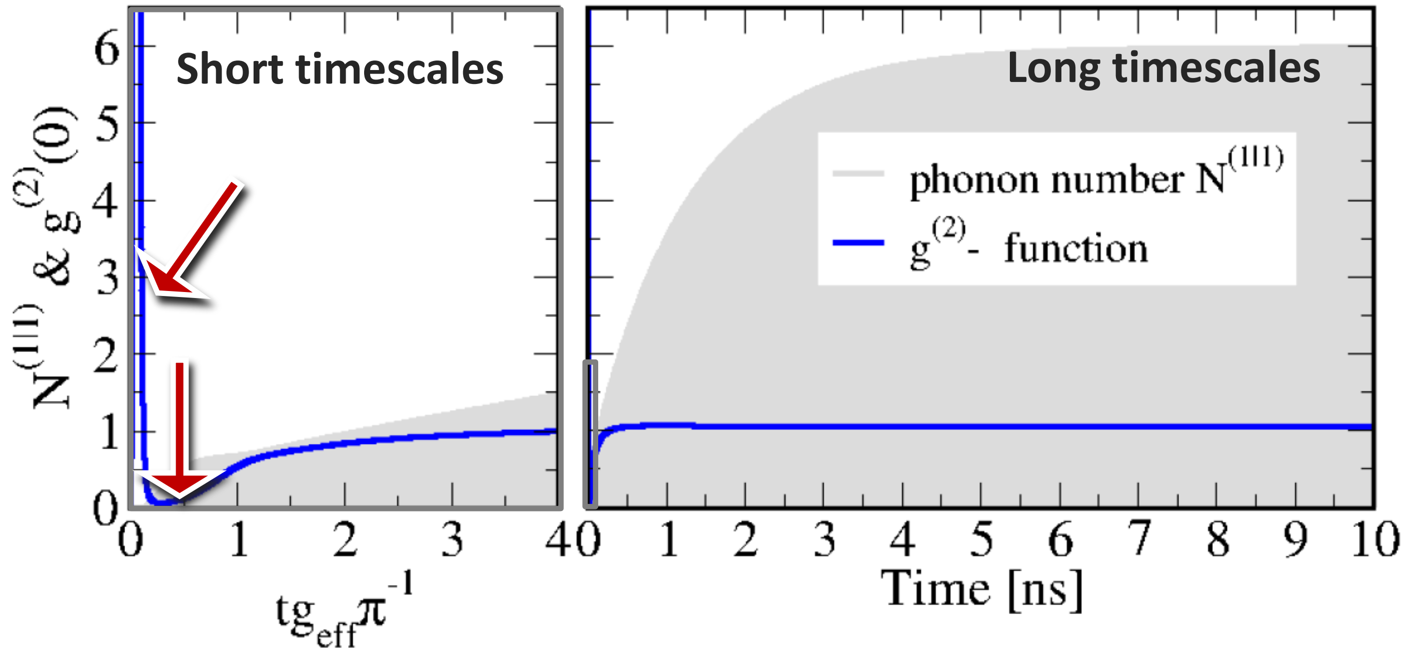

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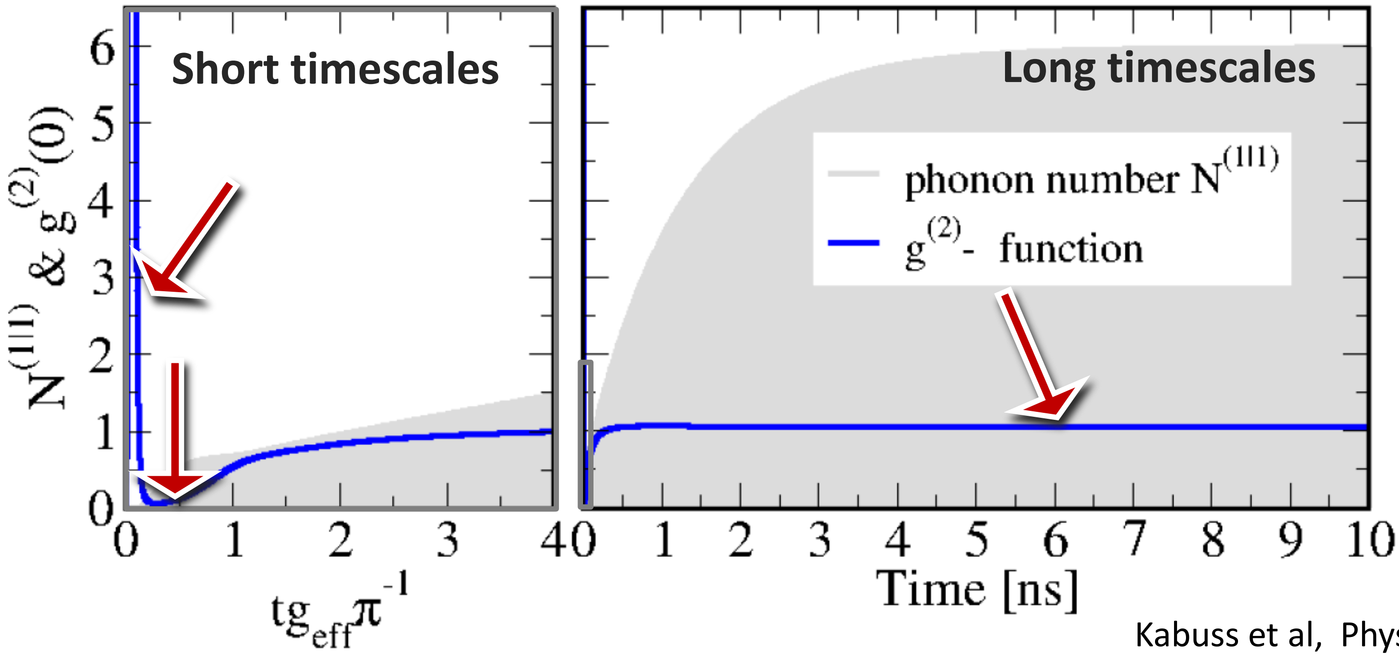

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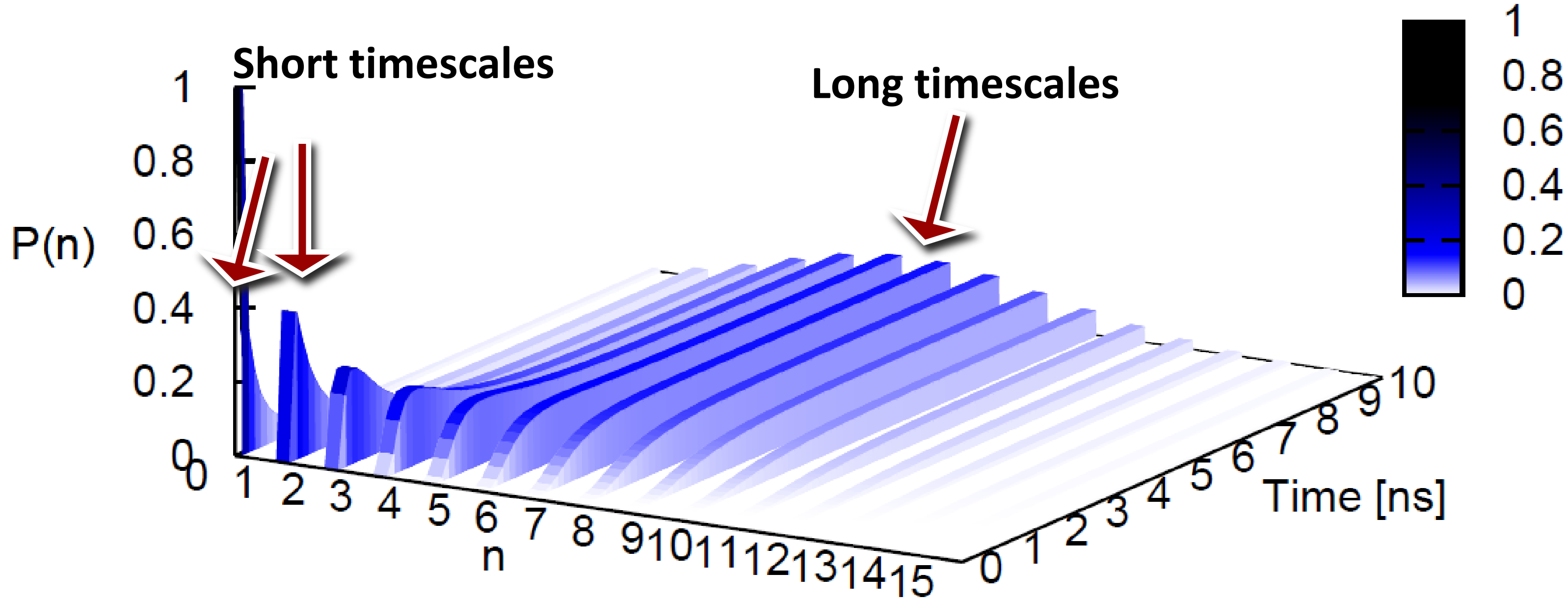

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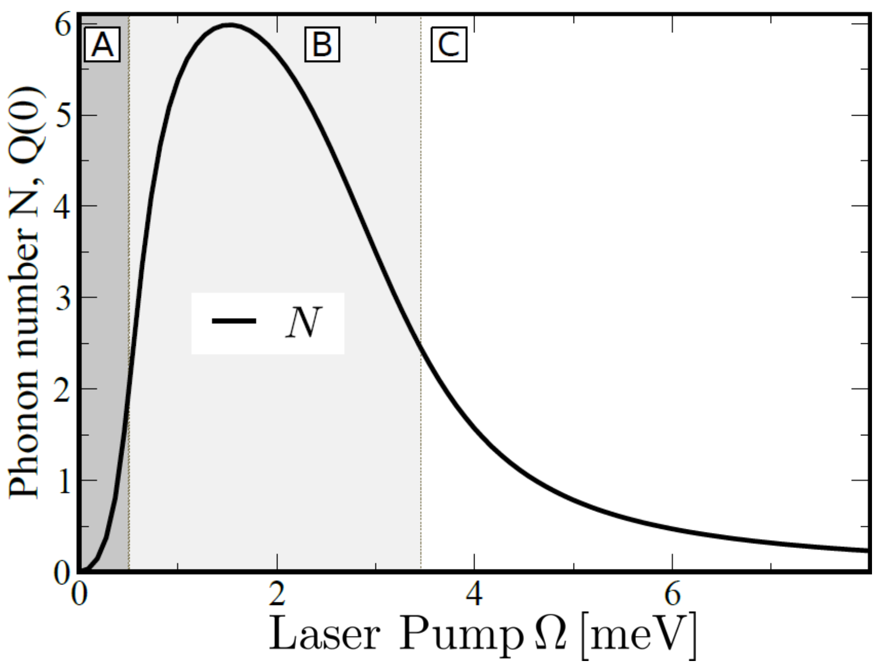
regimes of the QD-phonon laser



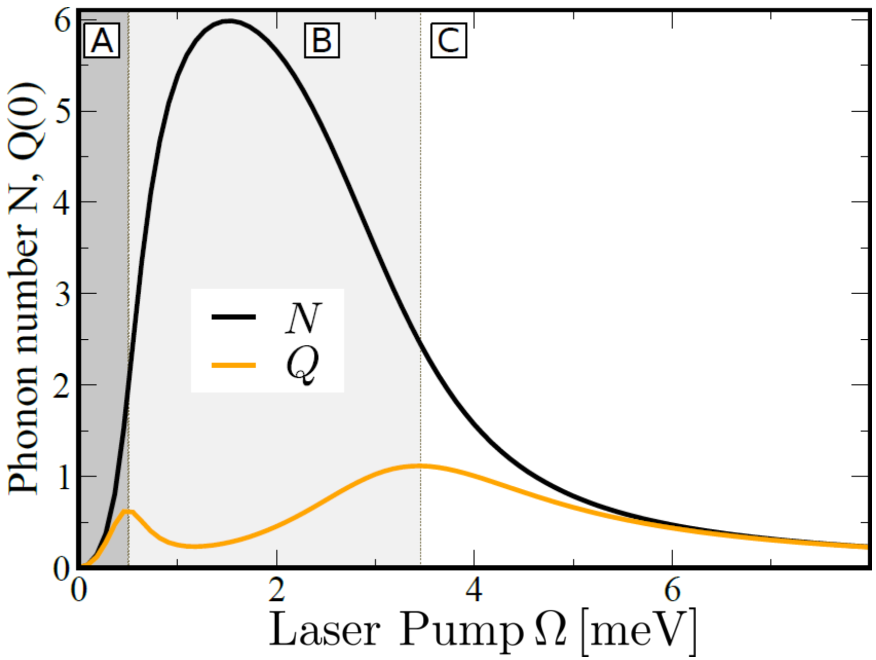
Mandel-Q parameter

$$Q(0) = N(g^{(2)}(0) - 1)$$

$$Q(0) = \begin{cases} N(2-1) = N & \text{thermal} \\ N(1-1) = 0 & \text{coherent} \\ 0 > Q > -1 & \text{Fock} \end{cases}$$

Kabuss et al., Phys. Rev. Lett. (2012)

regimes of the QD-phonon laser

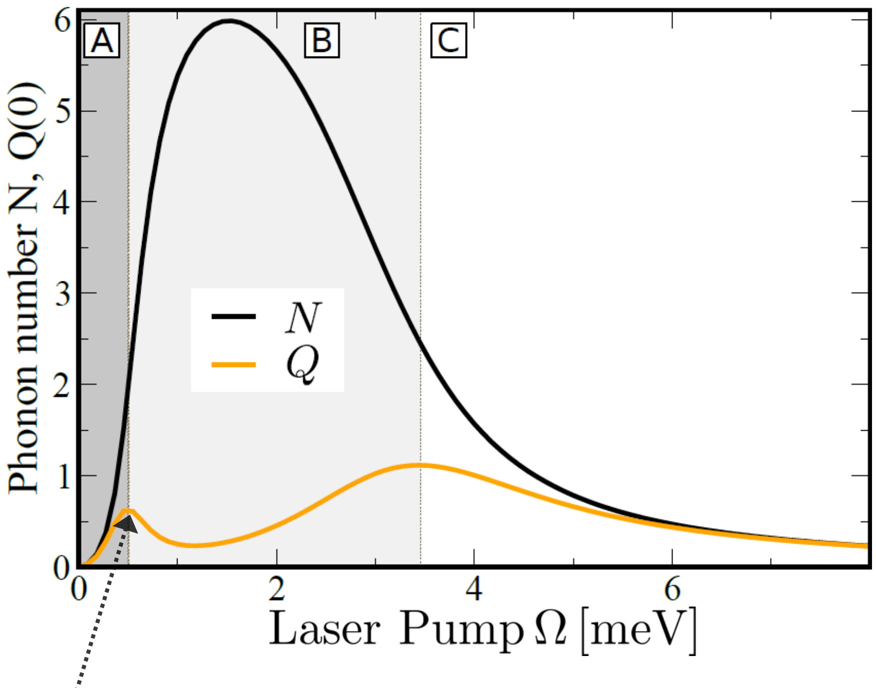


$$Q(0) = \bar{N}(g^{(2)}(0) - 1)$$

Three regimes:

Kabuss et al., Phys. Rev. Lett. (2012)

regimes of the QD-phonon laser

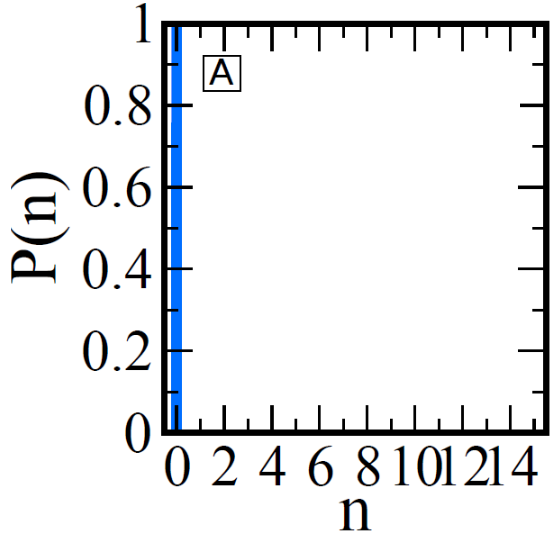


1. threshold:
(onset of lasing)

$$Q(0) = \bar{N}(g^{(2)}(0) - 1)$$

Three regimes:

A. Non-lasing regime



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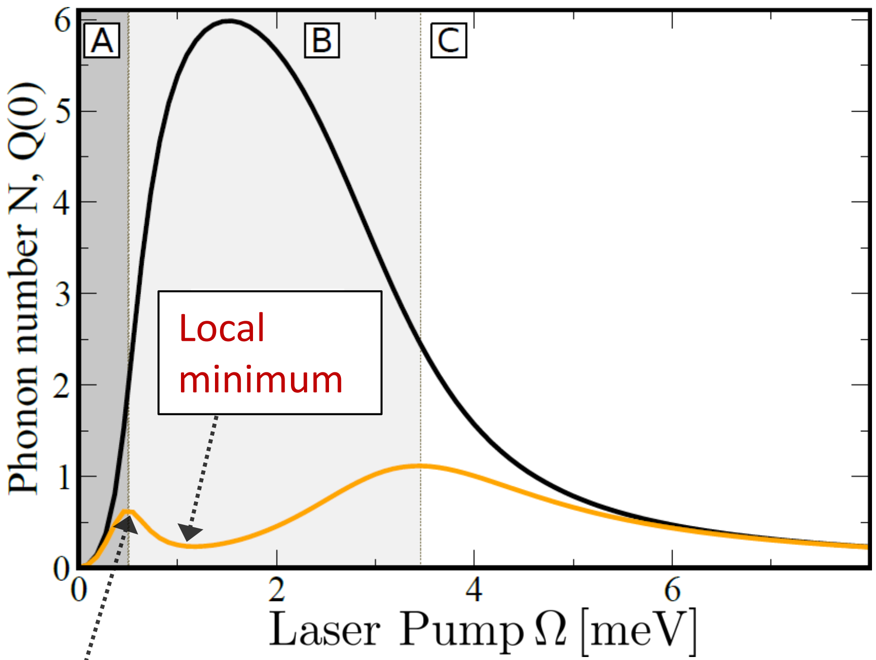

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Three regimes:

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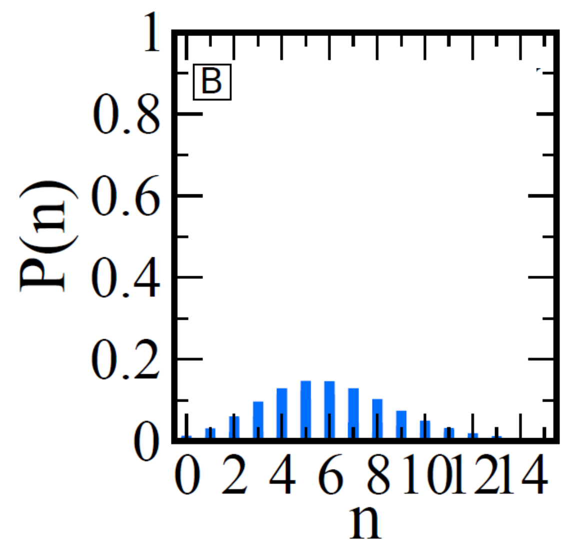
B. Lasing



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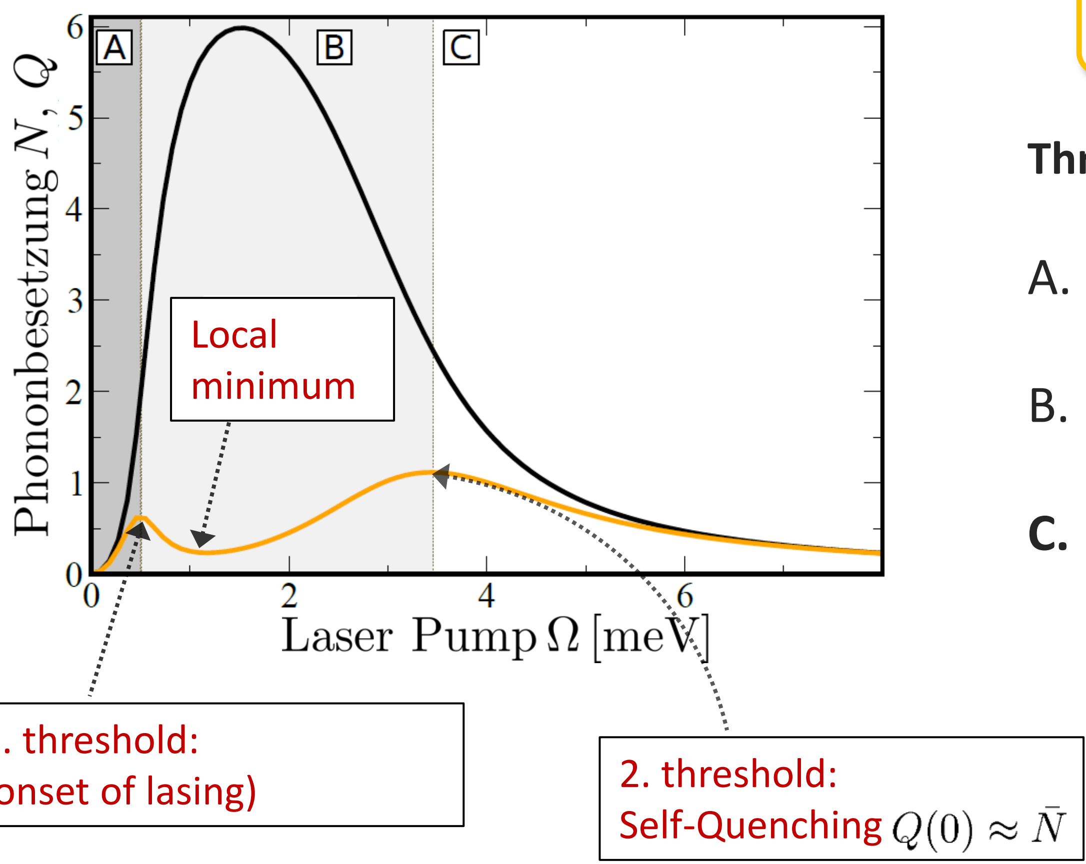


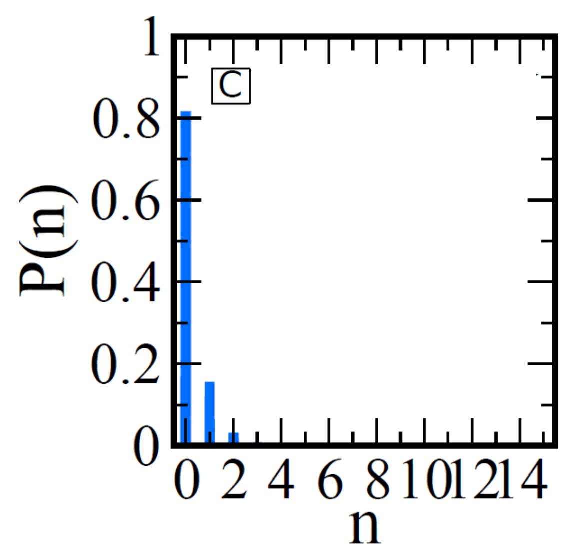
$$Q(0) = \bar{N}(g^{(2)}(0) - 1)$$

Three regimes:

A. Non-lasing regime

B. Lasing

C. Self-Quenching



Kabuss et al., Phys. Rev. Lett. (2012)

Effective semiclassical treatment

effective semiclassical treatment

Reduce description processes relevant for phonon lasing (**induced Raman**)

Effective Hamiltonian

$$H_{\mathrm{eff}} = \hbar\tilde{\omega}|c\rangle\langle c| + \hbar\omega_{ph} b^{\dagger} b \;\; + \hbar g_{\mathrm{eff}}\big(|c\rangle\langle v| b^{\dagger} + |v\rangle\langle c| b\big)$$

Kabuss et. al., Phys. Rev. B (2013)

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Almost similar to
Jaynes-Cummings model

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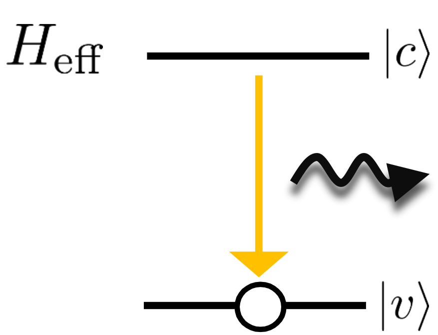
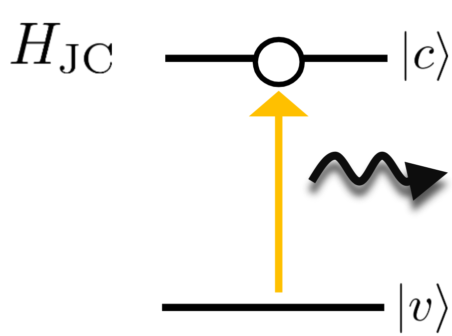
$$H_{\text{JC}} = \hbar\omega|c\rangle\langle c| + \hbar\omega_{pt} c^\dagger c + \hbar g_{\text{cav}}\big(|c\rangle\langle v| c \;\; + |v\rangle\langle c| c^\dagger\big)$$





Kabuss et. al., Phys. Rev. B (2013)

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Validity

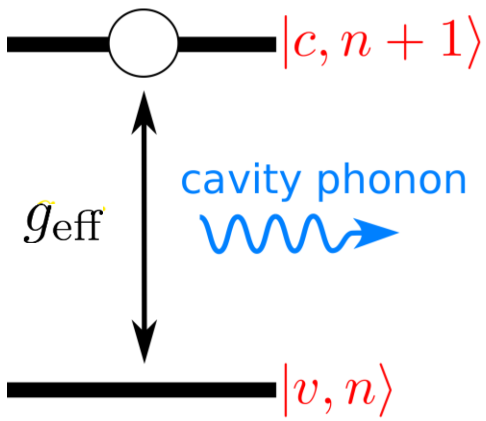
$$\Omega_R \ll \omega_{ph}$$

$$\omega_{ph} \gg \gamma_{pd}, \Gamma_r$$

(no resonant excitation)

Kabuss et. al., Phys. Rev. B (2013)

effective semiclassical treatment



A. Effektive spontaneous emission of phonons at transition into the excited state

B. Radiative decay pumps the ground state (upper laser level)

Kabuss et. al., Phys. Rev. B (2013)

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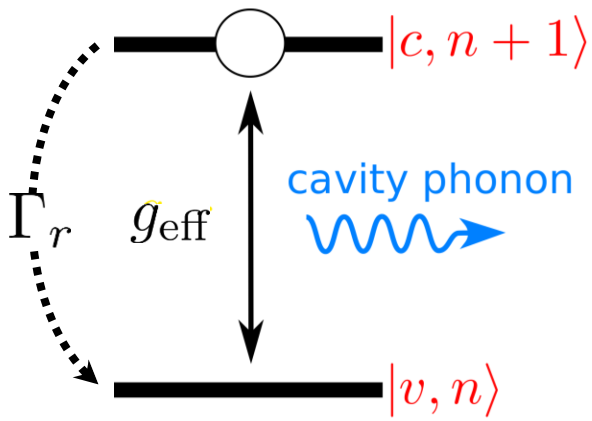

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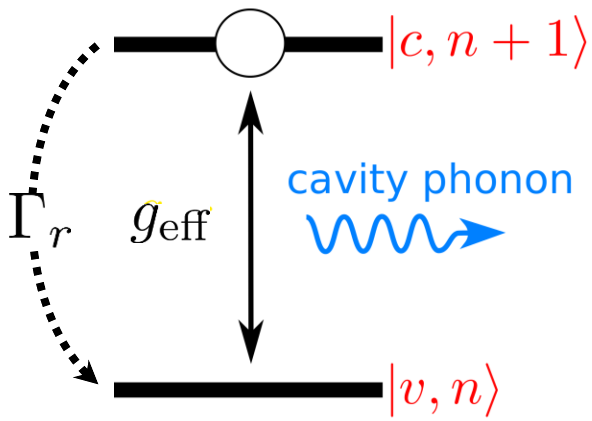


A. Effektive spontaneous emission of phonons at transition into the excited state

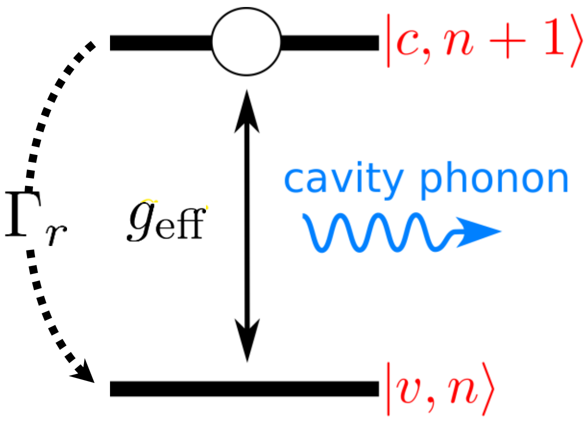
B. Radiative decay pumps the ground state (upper laser level)

Analogy to incoherently pumped 2-level laser

Mu et al., Phys. Rev. A (1992)

Kabuss et. al., Phys. Rev. B (2013)

effective semiclassical treatment



Semiclassical treatment: phonons are supposed in coherent state

$$N = \langle b^\dagger b \rangle \rightarrow \langle b^\dagger \rangle \langle b \rangle \qquad \langle |i\rangle\langle j| b^{(\dagger)} \rangle \rightarrow \langle |i\rangle\langle j| \rangle \langle b^{(\dagger)} \rangle$$

$B \equiv \langle b \rangle$	phonon-field amplitude
$P \equiv \langle |c\rangle\langle v| \rangle$	polarisation
$R \equiv \langle |c\rangle\langle c| \rangle - \langle |v\rangle\langle v| \rangle$	inversion

Mu et al., Phys. Rev. A (1992) Kabuss et. al., Phys. Rev. B (2013)

phonon laser equations

$$\dot{B} = [-i\omega_{ph} - \kappa]B - ig_{\text{eff}}P$$
$$\dot{P} = [i\tilde{\omega} - \gamma]P - ig_{\text{eff}}RB$$
$$\dot{R} = -2\Gamma_r(1 + R) + i2g_{\text{eff}}\left(P^*B - PB*\right)$$

Kabuss et. al., Phys. Rev. B (2013)

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$$\Delta + \frac{2\Omega^2}{\Delta} - \frac{g^2}{\omega_{ph}}$$

Reason for self-quenching behavior

Kabuss et. al., Phys. Rev. B (2013)

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$$\Delta + \frac{2\Omega^2}{\Delta} - \frac{g^2}{\omega_{ph}}$$

Reason for self-quenching behavior

Phonon number

$$N = \frac{\Gamma_r(W(\Omega) - 2\kappa)}{2\kappa_r W(\Omega)}$$

Kabuss et. al., Phys. Rev. B (2013)

phonon laser equations

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$$\Delta + \frac{2\Omega^2}{\Delta} - \frac{g^2}{\omega_{ph}}$$

Reason for self-quenching behavior

Phonon number

$$N = \frac{\Gamma_r(W(\Omega) - 2\kappa)}{2\kappa_r W(\Omega)}$$

Inversion

$$R = -\frac{\Gamma_r}{\Gamma_r + 2NW(\Omega)}$$

Inversion negative

thresholds

$$T_{\mp} = \left[\mathcal{G} \mp \sqrt{\mathcal{G}^2 - \Delta^2(\bar{\omega} + \gamma^2)}\right]^{\frac{1}{2}}$$

Kabuss et. al., Phys. Rev. B (2013)

phonon laser equations

$$\dot{B} = [-i\omega_{ph} - \kappa]B - ig_{\text{eff}}P$$
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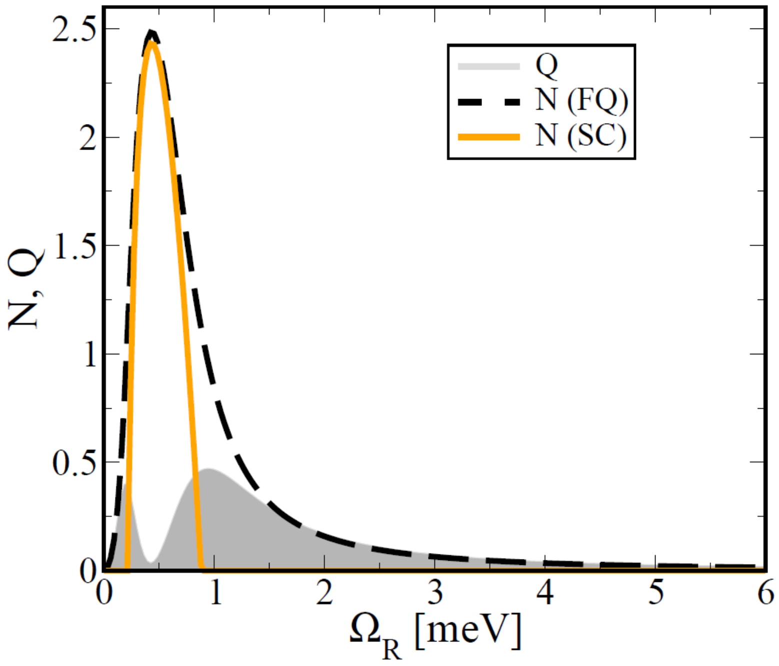
Kabuss et. al., Phys. Rev. B (2013)

Comparison to full QM model



Models are in good agreement ✓

Laser threshold and lasing window are easy to determine

Carfeful with loss and disspation

Kabuss et. al., Phys. Rev. B (2013)

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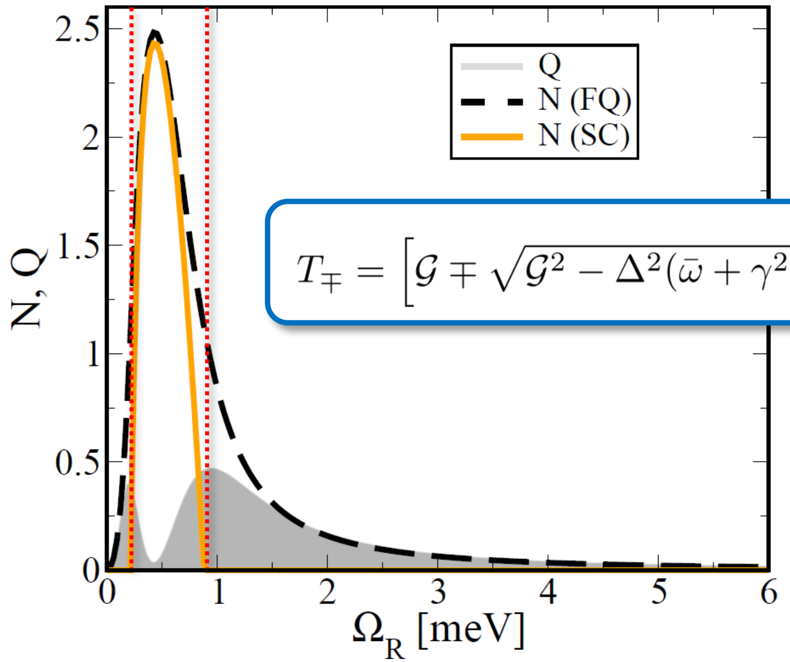

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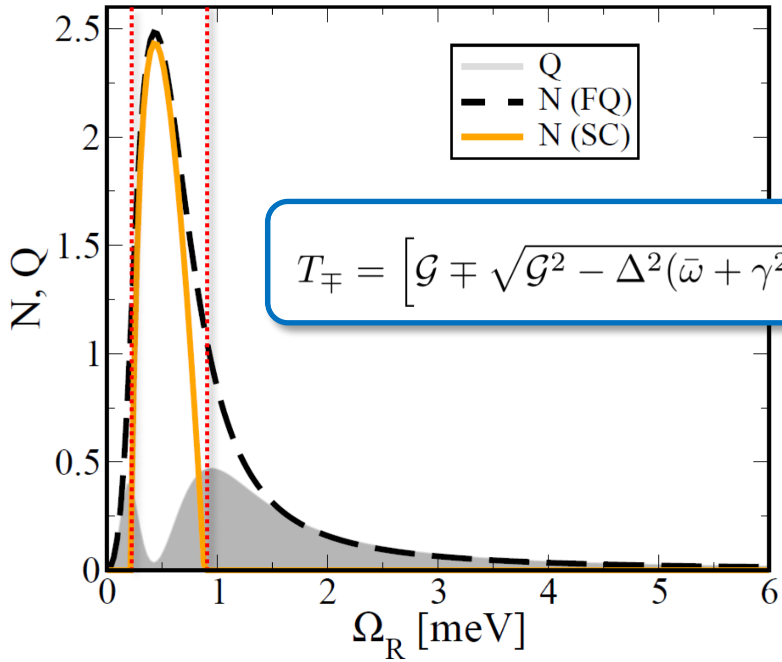

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Laser threshold and lasing window are easy to determine

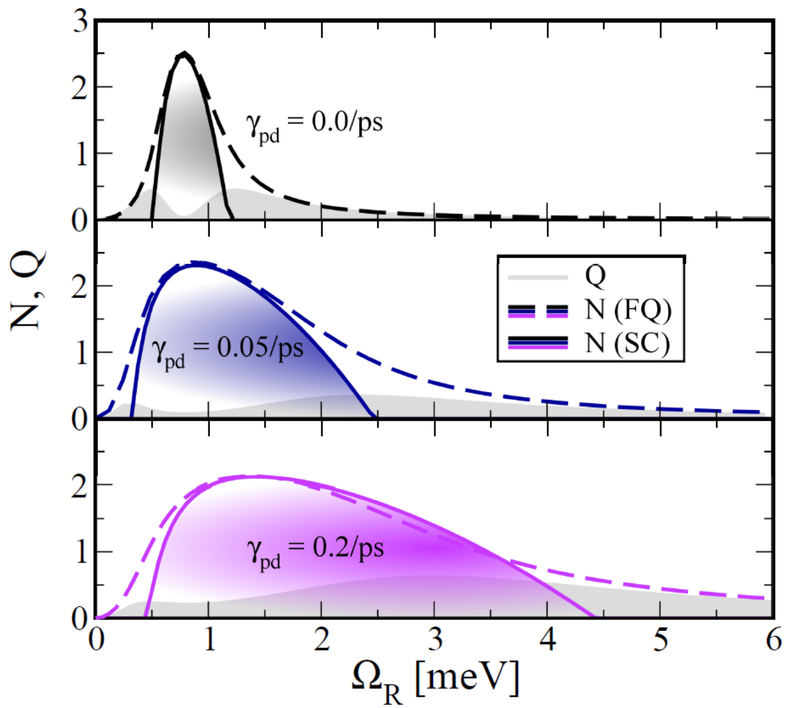
Carfeful with loss and disspation

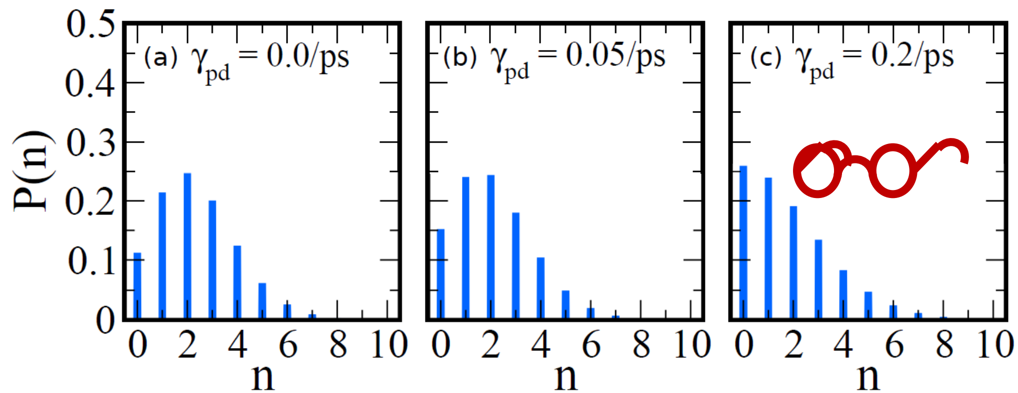
!

$$\Gamma_r, \gamma_{pd} \ll \omega_{ph}$$

Kabuss et. al., Phys. Rev. B (2013)

radiative decay and pure dephasing





but more super Poissonian statistics for too large γ_{pd}

Enhancement of lasing window

Conclusion

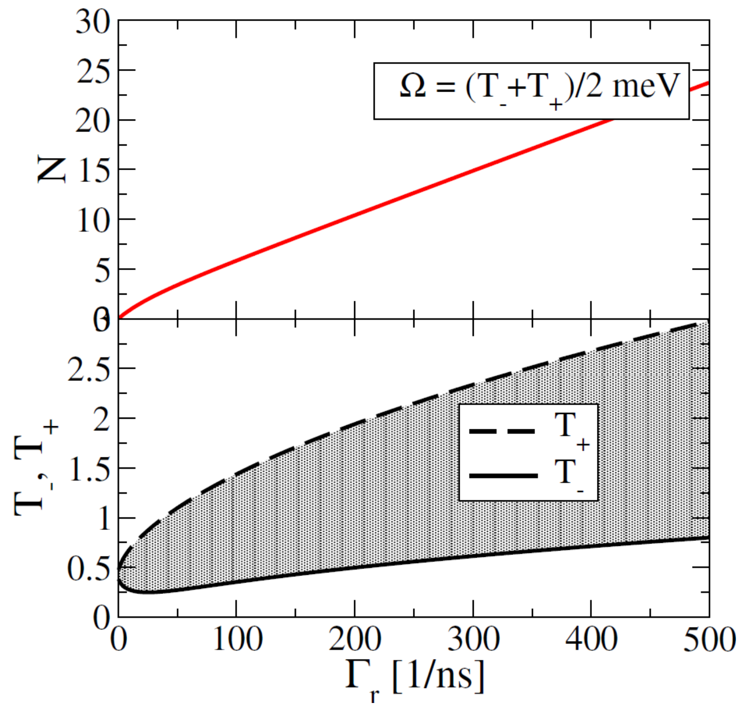
- QD coupled to acoustic cavity can be operated as a phonon laser
- → different types of phonon statistics can be generated
- Effective semiclassical treatment for evaluating a possible phonon laser system
- Dissipative processes, such as radiative decay and pure dephasing can have beneficial effects with respect to phonon lasing

Thank you for your intention!

DFG Deutsche Forschungsgemeinschaft

SFB 910

radiative decay and pure dephasing



Phonon number

$$N = \frac{\Gamma_r (W(\Omega) - 2\kappa)}{2\kappa_r W(\Omega)}$$

$$T_{\mp} = \left[\mathcal{G} \mp \sqrt{\mathcal{G}^2 - \Delta^2(\bar{\omega} + \gamma^2)} \right]^{\frac{1}{2}}$$

Phonon lasing window scales with Γ_r

Effective approach: Eliminating any but the anti-Stokes induced Raman process leads to simple laser equations

$$H_{eff} = e^{iS} H e^{-iS}, \quad [iS, H_0] + H_I = 0$$

$$H_{eff} = \frac{\hbar\tilde{\omega}}{2}\sigma_z + \hbar\omega_{ph} b^\dagger b + \hbar g_{eff}(\sigma b + \sigma^\dagger b^\dagger)$$

$$\begin{aligned}
0 &= -2n\kappa V^{(n|n)} + 2\Gamma_r C^{(n|n)} + g_{eff}\tilde{P}^{(n|n+1)}, \\
0 &= -(2n\kappa - 2\Gamma_r)C^{(n|n)} - g_{eff}\tilde{P}^{(n|n+1)} - n g_{eff}\tilde{P}^{(n-1|n)}, \\
0 &= \frac{[((2n-1)\kappa + \gamma)^2 + (\tilde{\omega}(\ \ , g) + \omega)^2]}{[(2n-1)\kappa + \gamma]}\tilde{P}^{(n-1|n)} \\
&\quad + g_{eff} C^{(n|n)} - g_{eff} V^{(n|n)} - n g_{eff} V^{(n-1|n-1)}.
\end{aligned}$$

Semiclassical approach: Faktorization of expectation values

$$\begin{aligned}
0 &= [-i(\omega_{ph} - \omega) - \kappa]B - i g_{eff} P, \\
0 &= [i\tilde{\omega}(\ \ , g) + \omega - \gamma]P - i g_{eff} R B, \\
0 &= -2\Gamma_r(1 + R) + i2 g_{eff}(P^* B - P B^*).
\end{aligned}$$

[3]PRB 88, 064305 (2013)

$$H_{eff} = e^{iS} H e^{-iS}, \quad [iS, H_0] + H_I = 0$$

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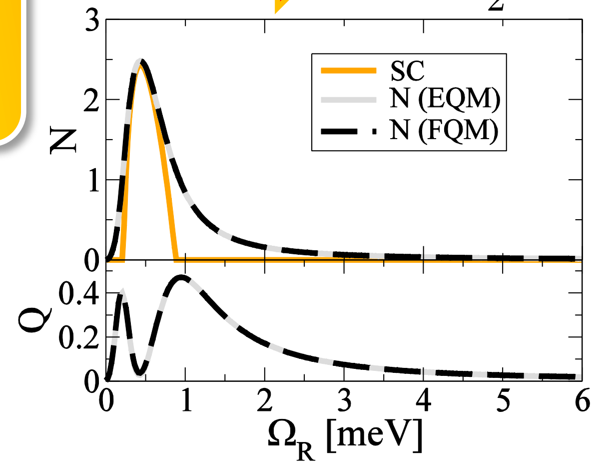
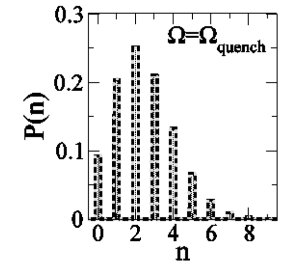
Effective approach: Eliminating any but the anti-Stokes induced Raman process leads to simple laser equations





- Semiclassical

$$N = \frac{\Gamma_r}{2\kappa W(\ \ )}(W(\ \ ) - 2\kappa)$$

Semiclassical theory can serve as an estimate for threshold behavior, etc.

- Quantum mechanical

True quantum effects, such as phonon anti-bunching can only be described by the two quantum mechancal models

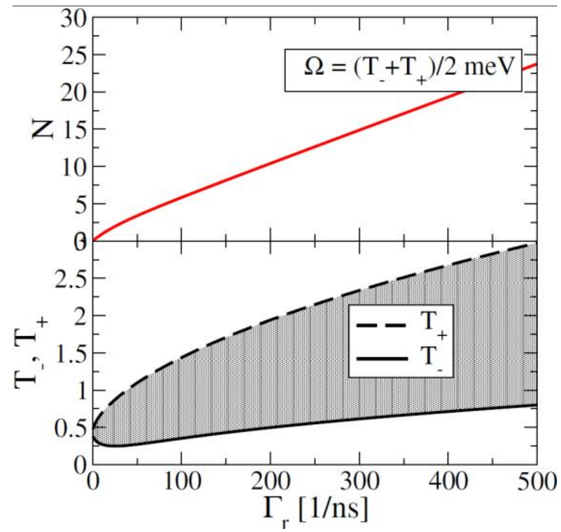
$$N^{(n|n)} = \Pi^n_m B^m,$$

$$B^m = \frac{c_m}{b_m + a_m \frac{c_{m+1}}{b_{m+1} + a_{m+1}\frac{c_{m+2}}{b_{m+2+\dots}}}}$$

[2] PRL, 109, 054301 (2012), [3] PRB 88, 064305 (2013)

Semiclassical

$$N = \frac{\Gamma_r}{2\kappa W(\ )}(W(\ ) - 2\kappa)$$



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$$N^{(n|n)} = \Pi_m^n B^m,$$
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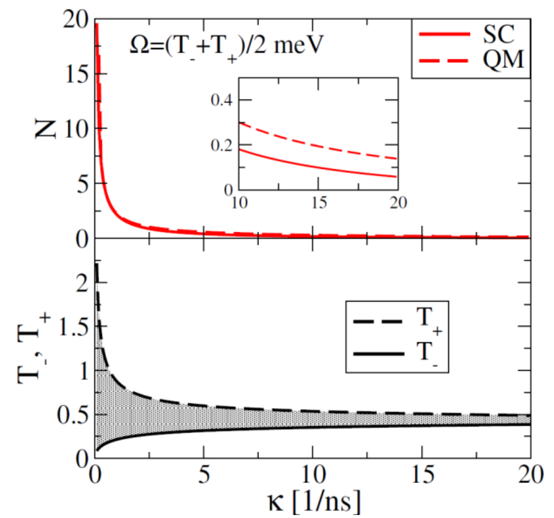
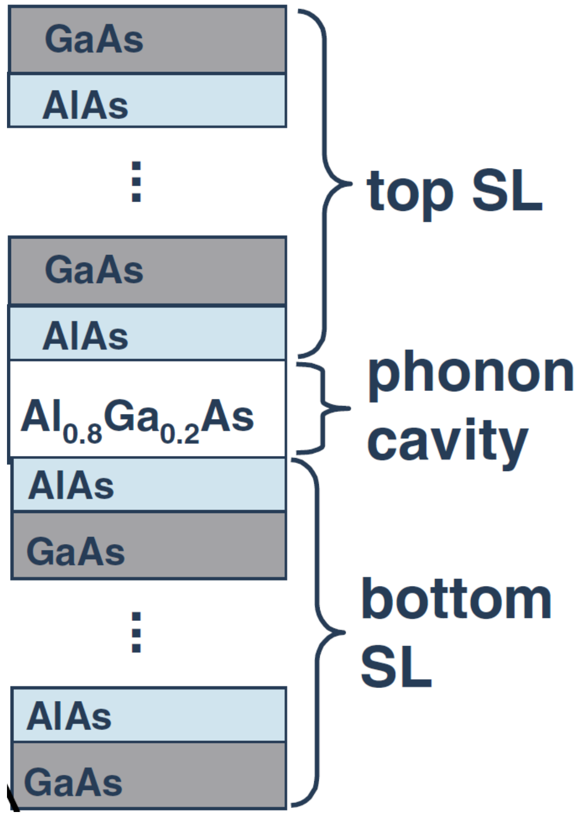


Scheme of a phonon cavity



Phonon cavity is composed of two superlattices and a spacer

spacer thickness $d = m\lambda_c/2$

taken from Trigo et al, PRL 89, 227402 (2002)